Activity 21

Objective

To verify that the ratio of the areas of a parallelogram and a triangle on the same base and between the same parallels is 2:1.

Material Required

Plywood sheet of convenient size, graph paper, colour box, a pair of wooden strips, scissors, cutter, adhesive, geometry box.

Method of Construction

1. Take a rectangular plywood sheet.
2. Paste a graph paper on it.
3. Take any pair of wooden strips or wooden scale and fix these two horizontally so that they are parallel.
4. Fix any two points A and B on the base strip (say Strip I) and take any two points C and D on the second strip (say Strip II) such that AB = CD.
5. Take any point P on the second strip and join it to A and B [see Fig. 1].

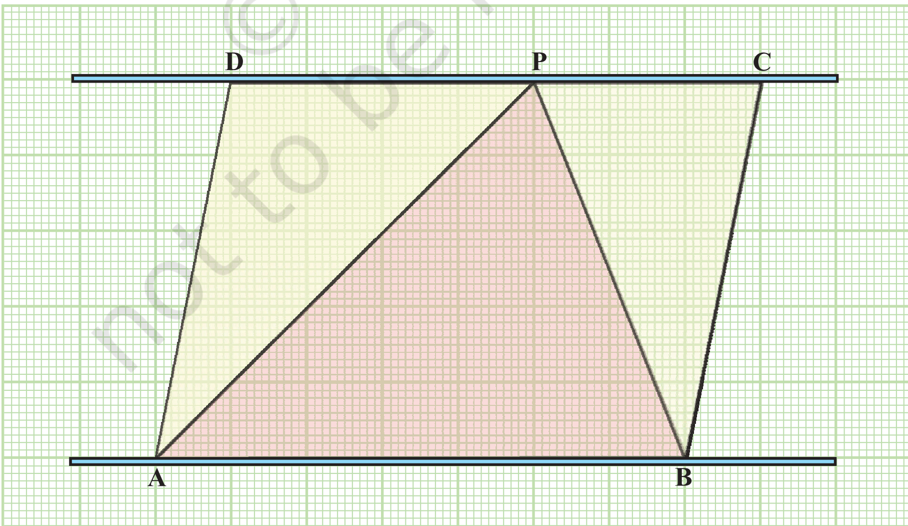


Fig. 1

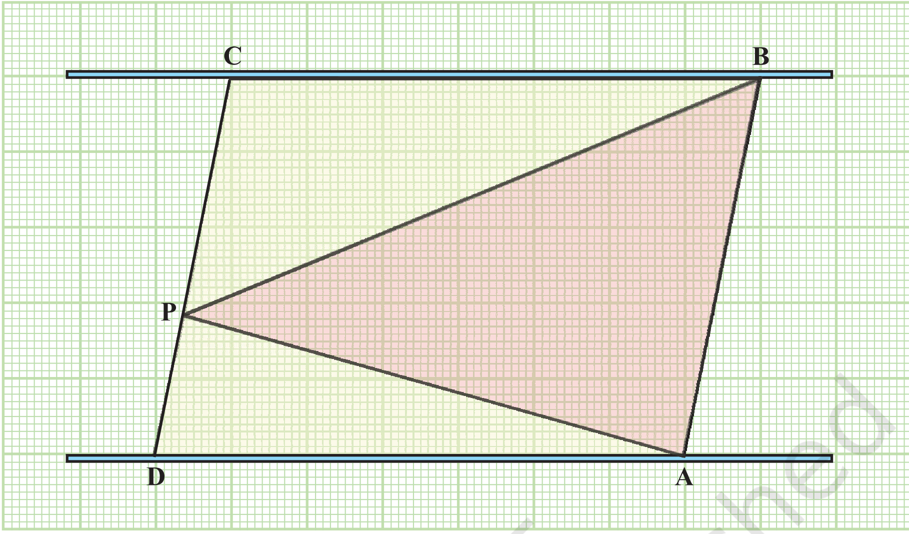


Fig. 2

DEMONSTRATION

1. AB is parallel to CD and P is any point on CD.
2. Triangle PAB and parallelogram ABCD are on the same base AB and between the same parallels.
3. Count the number of squares contained in each of the above triangle and parallelograms, keeping half square as $\frac{1}{2}$ and more than half as 1 square, leaving those squares which contain less than half square.
4. See that area of the triangle PAB is half of the area of parallelograms ABCD.

OBSERVATION

1. The number of squares in triangle PAB =...............
2. The number of squares in parallelogram ABCD =............... .

   So, the area of parallelogram ABCD = 2 [Area of triangle PAB]

   Thus, area of parallelogram ABCD : area of DPAB = ... : ....

APPLICATION

This activity is useful in deriving formula for the area of a triangle and also in solving problems on mensuration.

NOTE

You may take different triangles PAB by taking different positions of point P and the two parallel strips as shown in Fig. 2.

One should study Mathematics because it is only through Mathematics that nature can be conceived in hormonious form.

– Birkhoff

Activity 22

Objective

To verify that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Material Required

Cardboard, coloured drawing sheets, scissors, sketch pens, adhesive, geometry box, transparent sheet.

Method of Construction

1. Take a rectangular cardboard of a convenient size and paste a white paper on it.
2. Cut out a circle of suitable radius on a coloured drawing sheet and paste on the cardboard.
3. Take two points B and C on the circle to obtain the arc BC [see Fig. 1].
4. Join the points B and C to the centre O to obtain an angle subtended by the arc BC at the centre O.
5. Take any point A on the remaining part of the circle. Join it to B and C to get ∠BAC subtended by the arc BC on any point A on the remaining part of the circle [see Fig. 1].
6. Make a cut-out of ∠BOC and two cut-outs of angle BAC, using transparent sheet [see Fig. 2].

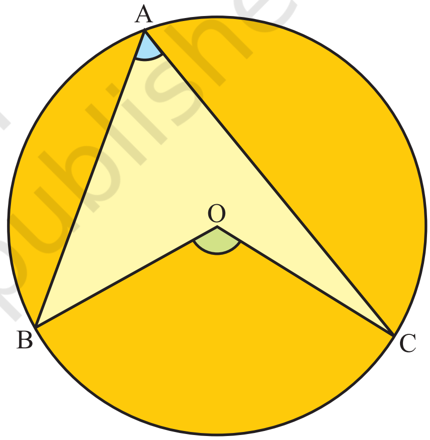


Fig. 1

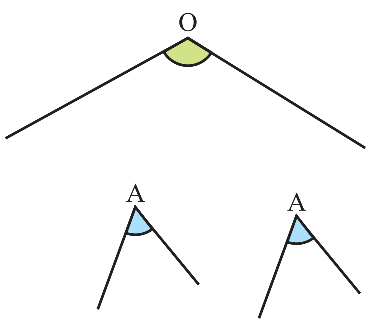


Fig. 2

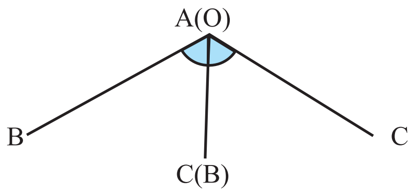


Fig. 3

DEMONSTRATION

Place the two cut-outs of ∠BAC on the cut-out of angle BOC, adjacent to each as shown in the Fig. 3. Clearly, 2 ∠BAC = ∠BOC, i.e., the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

OBSERVATION

Measure of ∠BOC = ...

Measure of ∠BAC = ...

Therefore, ∠BOC = 2 ....

APPLICATION

This property is used in proving many other important results such as angles in the same segment of a circle are equal, opposite angles of a cyclic quadrilateral are supplementary, etc.

Activity 23

Objective

To verify that the angles in the same segment of a circle are equal.

Material Required

Geometry box, coloured glazed papers, scissors, cardboard, white paper and adhesive.

Method of Construction

1. Take a cardboard of suitable size and paste a white paper on it.
2. Take a sheet of glazed paper and draw a circle of radius *a* units on it [see Fig. 1].
3. Make a cut-out of the circle and paste it on the cardboard.
4. Take two points A and B on the circle and join them to form chord AB [see Fig. 2].
5. Now take two points C and D on the circle in the same segment and join AC, BC, AD and BD [see Fig. 3].
6. Take replicas of the angles ∠ACB and ∠ADB.

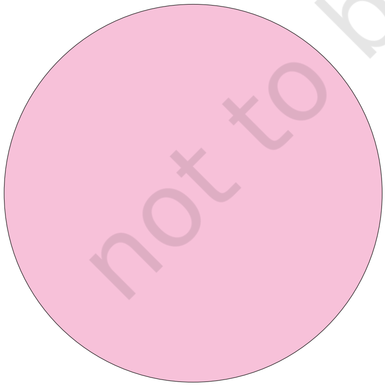


Fig. 1

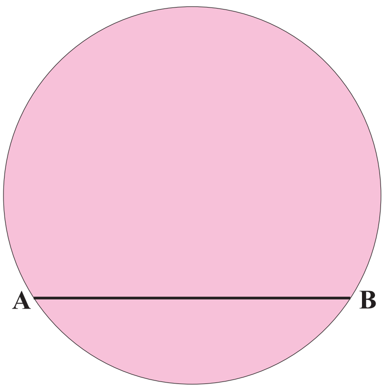


Fig. 2

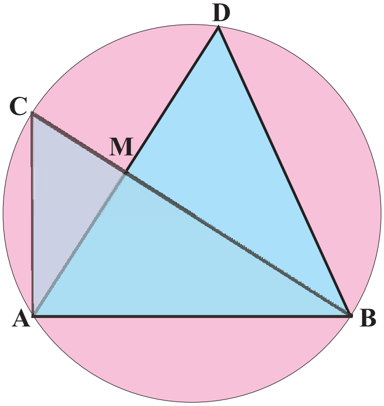


Fig. 3

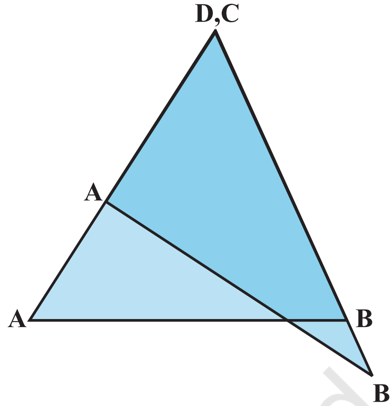


Fig. 4

Demonstration

Put the cut-outs of ∠ACB and ∠ADB on each other such that vertex C falls on vertex D [see Fig. 4]. In Fig. 4, ∠ACB covers ∠ADB completely. So, ∠ACB = ∠ADB.

Observation

On actual measurement:

∠ACB = ---------------, ∠ADB = ---------------

So, ∠ACB = ∠ADB. Thus, angles in the same segment are ---------.

Application

This result may be used in proving other theorems/riders of geometry related to circles.

Activity 24

Objective

To verify that the opposite angles of a cyclic quadrilateral are supplementary.

Material Required

Chart paper, geometry box, scissors, sketch pens, adhesive, transparent sheet.

Method of Construction

1. Take a chart paper and draw a circle of radius on it.
2. In the circle, draw a quadrilateral so that all the four vertices of the quadrilateral lie on the circle. Name the angles and colour them as shown in Fig. 1.
3. Make the cut-outs of the angles as shown in Fig. 2.

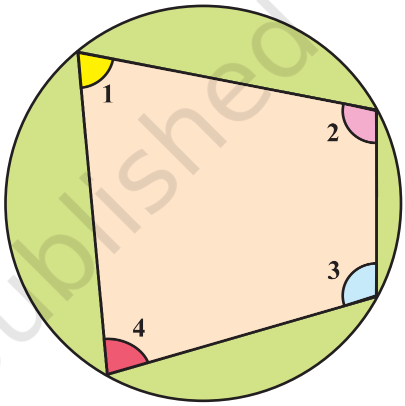


Fig. 1

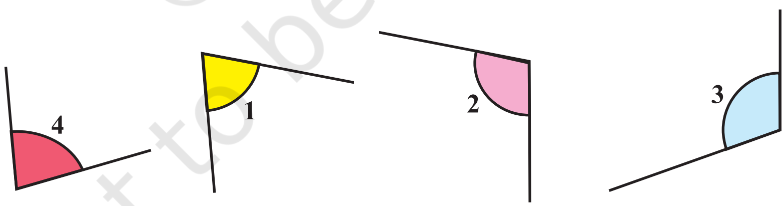


Fig. 2

Fig. 3

Demonstration

Paste cut-outs of the opposite angles $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$ to make straight angles as shown in Fig. 3. Thus $\angle 1 + \angle 3 = 180°$ and $\angle 2 + \angle 4 = 180°$.

Observation

On actual measurement:

$\angle 1$ = ...; $\angle 2$ = ...; $\angle 3$ = ...; $\angle 4$ = ...

So, $\angle 1 + \angle 3$ = ...; $\angle 2 + \angle 4$ = ...;

Therefore, sum of each pair of the opposite angles of a cyclic quadrilateral is ....

Application

The concept may be used in solving various problems in geometry.

Activity 25

Objective

To find the formula for the area of a trapezium experimentally.

Material Required

Hardboard, thermocol, coloured glazed papers, adhesive, scissors.

Method of Construction

1. Take a piece of hardboard for the base of the model.
2. Cut two congruent trapeziums of parallel sides a and b units [see Fig. 1].
3. Place them on the hardboard as shown in Fig. 2.

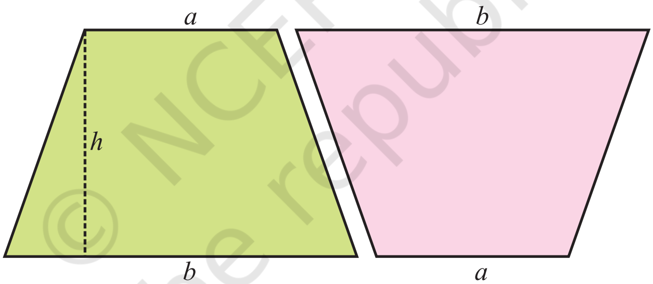


Fig. 1

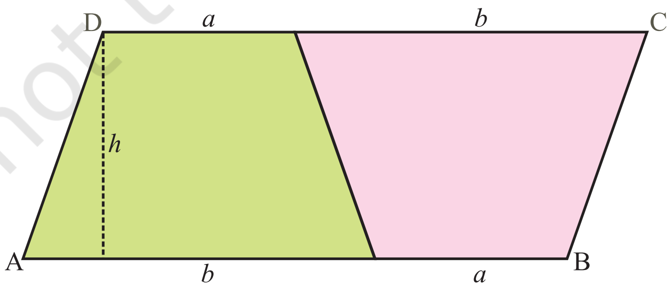


Fig. 2

DEMONSTRATION

1. Figure formed by the two trapeziums [see Fig. 2] is a parallelogram ABCD.
2. Side AB of the parallelogram = (a + b) units and its corresponding altitude = h units.
3. Area of each trapezium $= \frac{1}{2}$ (area of parallelogram) $= \frac{1}{2}(a+b)\times h$

Therefore, area of trapezium $= \frac{1}{2}(a+b)\times h$

$= \frac{1}{2}$ (sum of parallel sides) × perpendicular distance.

Here, area is in square units.

OBSERVATION

Lengths of parallel sides of the trapezium = -------,-------.

Length of altitude of the parallelogram = --------.

Area of parallelogram = --------------.

Area of the trapezium $= \frac{1}{2}$ (Sum of ---------- sides) × ---------.

APPLICATION

This concept is used for finding the formula for area of a triangle in coordinate geometry. This may also be used in finding the area of a field which can be split into different trapeziums and right triangles.

Activity 26

Objective

To form a cube and find the formula for its surface area experimentally.

Material Required

Cardboard, ruler, cutter, cellotape, sketch pen/pencil.

Method of Construction

1. Make six identical squares each of side a units, using cardboard and join them as shown in Fig. 1 using a cellotape.
2. Fold the squares along the dotted markings to form a cube [see Fig. 2].

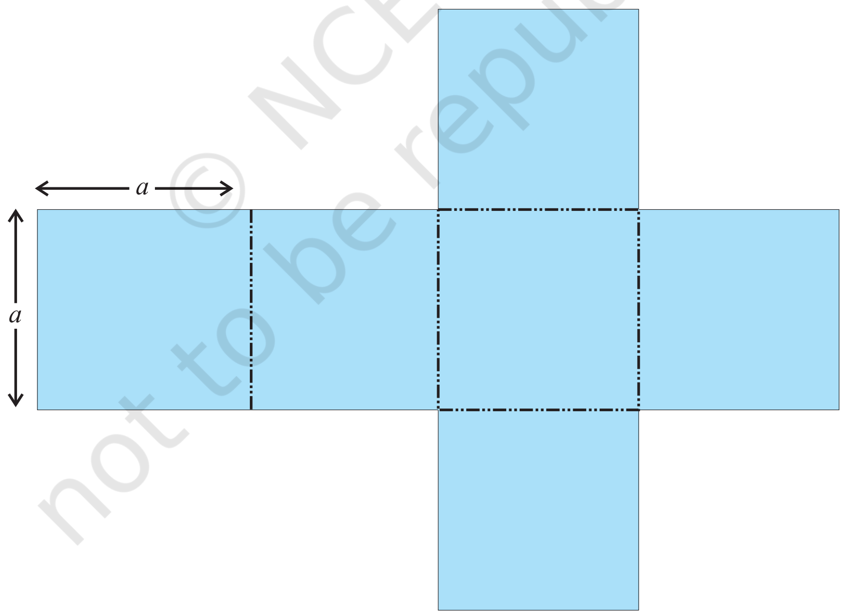


Fig. 1

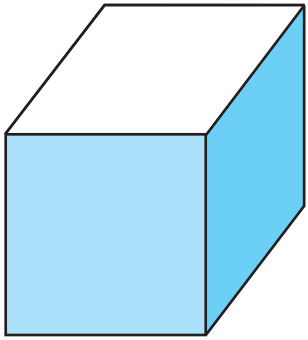


Fig. 2

DEMONSTRATION

1. Each face of the cube so obtained is a square of side a units. Therefore, area of one face of the cube is a^2 square units.
2. Thus, the surface area of the cube with side a units = $6a^2$ square units.

OBSERVATION

On actual measurement:

Length of side a = ...

Area of one square / one face = a^2 = ...

So, sum of the areas of all the squares = ...+............+..........+ ....+ .... + ...

Therefore, surface area of the cube = $6a^2$

APPLICATION

This result is useful in estimating materials required for making cubical boxes needed for packing.

NOTE

Instead of making six squares separately as done in the activity, a net of a cube be directly prepared on the cardboard itself.

Activity 27

OBJECTIVE

To form a cuboid and find the formula for its surface area experimentally.

MATERIAL REQUIRED

Cardboard, cellotape, cutter, ruler, sketch pen/pencil.

METHOD OF CONSTRUCTION

1. Make two identical rectangles of dimensions a units × b units, two identical rectangles of dimensions b units × c units and two identical rectangles of dimensions c units × a units, using a cardboard and cut them out.
2. Arrange these six rectangles as shown in Fig. 1 to obtain a net for the cuboid to be made.
3. Fold the rectangles along the dotted markings using cello-tape to form a cuboid [see Fig. 2].

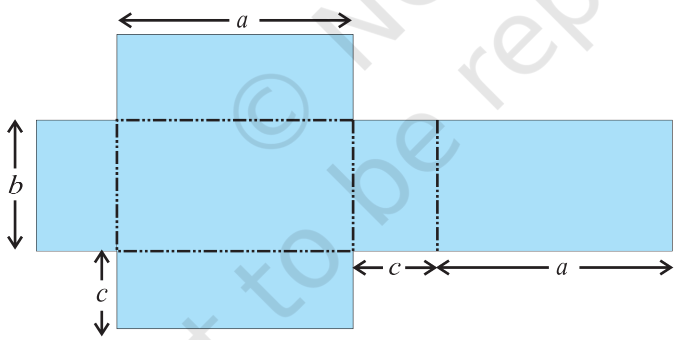


Fig. 1

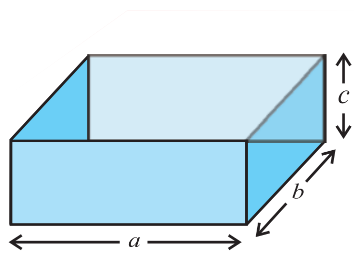


Fig. 2

Demonstration

Area of a rectangle of dimensions (a units × b units) = ab square units.

Area of a rectangle of dimensions (b units × c units) = bc square units.

Area of a rectangle of dimensions (c units × a units) = ca square units.

Surface area of the cuboid so formed

= $(2 \times ab + 2 \times bc + 2 \times ca)$ square units = $2\,(ab + bc + ca)$ square units.

Observation

On actual measurement:

a = ..., b = ..., c = ...,

So, ab = ..., bc = ..., ca = ...,

$2ab$ = ..., $2bc$ = ..., $2ca$ = ...

Sum of areas of all the six rectangles = ...

Therefore, surface area of the cuboid = $2\,(ab+bc+ca)$

Application

This result is useful in estimating materials required for making cuboidal boxes/almirahs, etc.

Note

Instead of making six rectangles separately, as done in the activity, a net of a cuboid be directly prepared on the cardboard itself.

Activity 28

Objective

To form a cone from a sector of a circle and to find the formula for its curved surface area.

Material Required

Wooden hardboard, acrylic sheets, cellotape, glazed papers, sketch pens, white paper, nails, marker.

Method of Construction

1. Take a wooden hardboard of a convenient size and paste a white paper on it.
2. Cut out a circle of radius l from a acrylic sheet [see Fig. 1].
3. Cut out a sector of angle q degrees from the circle [see Fig. 2].
4. Bring together both the radii of the sector to form a cone and paste the ends using a cellotape and fix it on the hardboard [see Fig. 3].

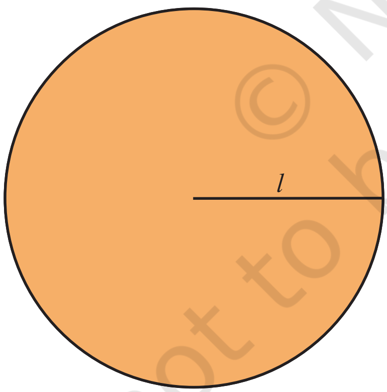


Fig. 1

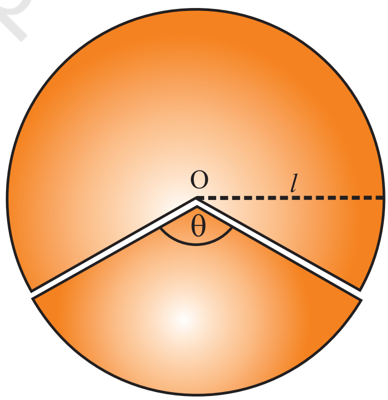


Fig. 2

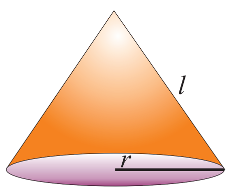


Fig. 3

Demonstration

1. Slant height of the cone = radius of the circle = l.
2. Radius of the base of the cone = r.
3. Circumference of the base of the cone = Arc length of the sector = $2\pi r$.
4. Curved surface area of the cone = Area of the sector

$$= \frac{\text{Arc length}}{\text{Circumference of the circle}} \times \text{Area of the circle}$$

$$= \frac{2\pi r}{2\pi l} \times \pi l^2 = \pi r l.$$

Observation

On actual measurement :

The slant height l of the cone = ---------------------, r = -----------------------

So, arc length l = ---------------------,

Area of the sector = ---------------, Curved surface area of the cone = ---------
----

Therefore, curved surface area of the cone = Area of the sector.

Here, area is in square units.

Application

The result is useful in

1. estimating canvas required to make a conical tent
2. estimating material required to make Joker's cap, ice cream cone, etc.

Activity 29

Objective

To find the relationship among the volumes of a right circular cone, a hemisphere and a right circular cylinder of equal radii and equal heights.

Material Required

Cardboard, acrylic sheet, cutter, a hollow ball, adhesive, marker, sand or salt.

Method of Construction

1. Take a hollow ball of radius, say, a units and cut this ball into two halves [see Fig. 1].
2. Make a cone of radius a and height a by cutting a sector of a circle of suitable radius using acrylic sheet and place it on the cardboard [see Fig. 2].
3. Make a cylinder of radius a and height a, by cutting a rectangular sheet of a suitable size. Stick it on the cardboard [see Fig. 3].

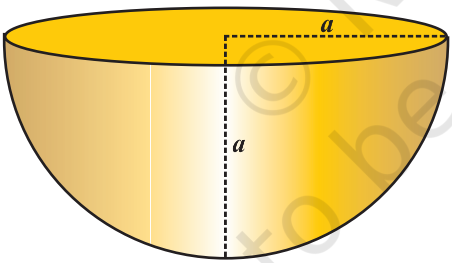


Fig. 1

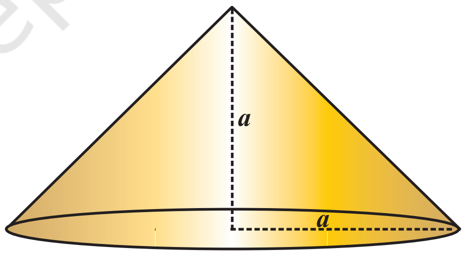


Fig. 2

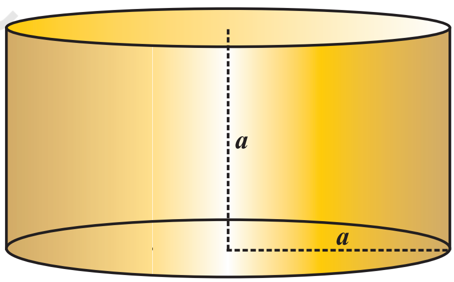


Fig. 3

DEMONSTRATION

1. Fill the cone with sand (or salt) and pour it twice into the hemisphere. The hemisphere is completely filled with sand.

   Therefore, volume of cone $= \frac{1}{2}$ volume of hemisphere.

2. Fill the cone with sand (or salt) and pour it thrice into the cylinder. The cylinder is completely filled with sand.

   Therefore, volume of cone $= \frac{1}{3}$ volume of cylinder.

3. Volume of cone : Volume of hemisphere : Volume of cylinder = 1:2:3

OBSERVATION

Radius of cone = Height of the cone = ----------.

Volume of cone $= \frac{1}{2}$ Volume of --------------.

Volume of cone $= \frac{1}{3}$ Volume of --------------.

Volume of cone : Volume of a hemisphere = -------- : ----------

Volume of cone : Volume of a cylinder = -------- : ----------

Volume of cone : Volume of hemisphere : Volume of cylinder = -------- : ---------- : ----------

APPLICATION

1. This relationship is useful in obtaining the formula for the volume of a cone and that of a hemisphere/sphere from the formula of volume of a cylinder.

2. This relationship among the volumes can be used in making packages of the same material in containers of different shapes such as cone, hemisphere, cylinder.

Activity 30

OBJECTIVE

To find a formula for the curved surface area of a right circular cylinder, experimentally.

MATERIAL REQUIRED

Coloured chart paper, cellotape, ruler.

METHOD OF CONSTRUCTION

1. Take a rectangular chart paper of length l units and breadth b units [see Fig. 1].
2. Fold this paper along its breadth and join the two ends by using cellotape and obtain a cylinder as shown in Fig. 2.

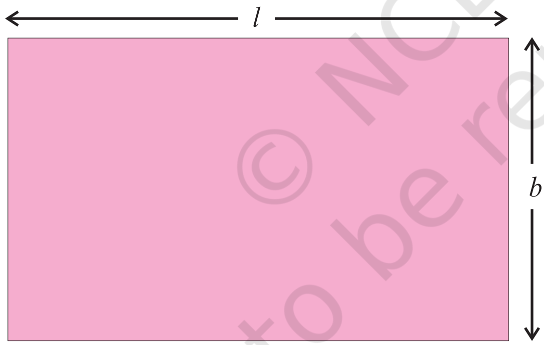


Fig. 1

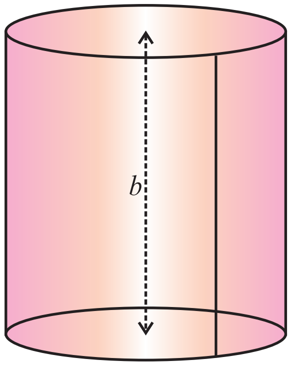


Fig. 2

Demonstration

1. Length of the rectangular paper = l = circumference of the base of the cylinder = $2\pi r$, where r is the radius of the cylinder.
2. Breadth of the rectangular paper = b = height (h) of the cylinder.
3. The curved surface area of the cylinder is equal to the area of the rectangle = $l \times b = 2\pi r \times h = 2\pi rh$ square units.

Observation

On actual measurement :

l = ..., b = ...,

$2\pi r = l$ = ..., $h = b$ = ...,

Area of the rectangular paper = $l \times b$ = ...

Therefore, curved surface area of the cylinder = $2\pi rh$.

Application

This result can be used in finding the material used in making cylindrical containers, i.e., powder tins, drums, oil tanks used in industrial units, overhead water tanks, etc.

Activity 31

Objective

To obtain the formula for the surface area of a sphere.

Material Required

A ball, cardboard/wooden strips, thick sheet of paper, ruler, cutter, string, measuring tape, adhesive.

Method of Construction

1. Take a spherical ball and find its diameter by placing it between two vertical boards (or wooden strips) [see Fig. 1]. Denote the diameter as *d*.
2. Mark the topmost part of ball and fix a pin [see Fig. 2].
3. Taking support of pin, wrap the ball (spirally) with string completely, so that on the ball no space is left uncovered [see Fig. 2].
4. Mark the starting and finishing points on the string, measure the length between these two marks and denote it by *l*. Slowly, unwind the string from the surface of ball.
5. On the thick sheet of paper, draw 4 circles of radius '*r*' (radius equal to the radius of ball).

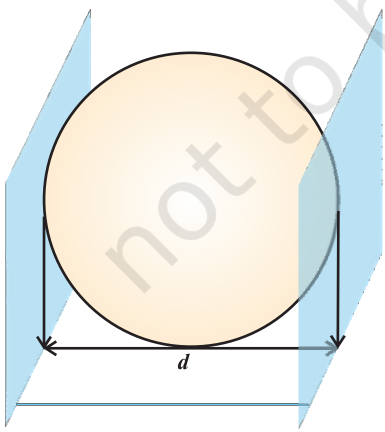


Fig. 1

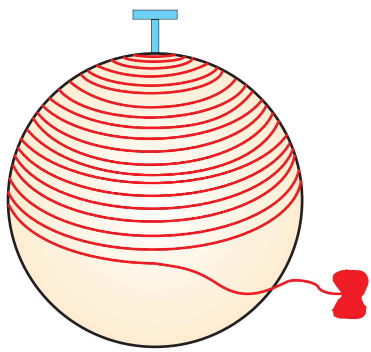


Fig. 2

6. Start filling the circles [see Fig. 3] one by one with string that you have wound around the ball.

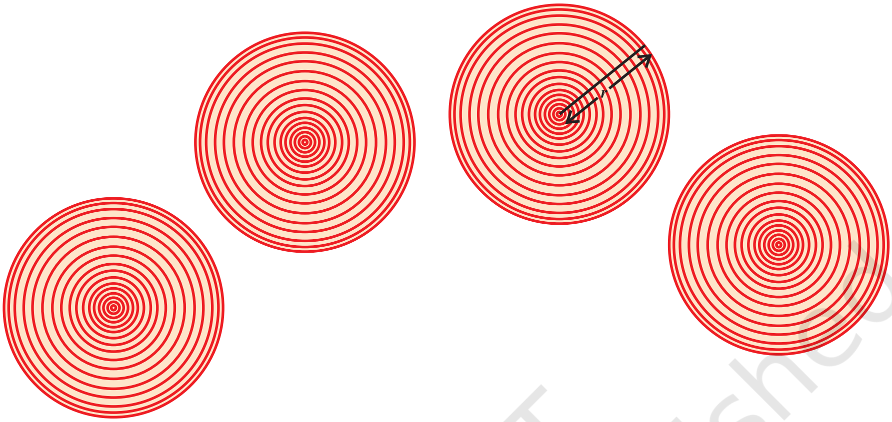


Fig. 3

DEMONSTRATION

Let the length of string which covers a circle (radius r) be denoted by a.

The string which had completely covered the surface area of ball has been used completely to fill the region of four circles (all of the same radius as of ball or sphere).

This suggests:

Length of string needed to cover sphere of radius r = 4 × length of string needed to cover one circle

i.e., $l = 4a$

or, surface area of sphere = 4 × area of a circle of radius r

So, surface area of a sphere = $4\pi r^2$

OBSERVATION

Diameter d of the spherical ball =................ units

radius r =................ units

Length l of string used to cover ball = ... units

Length a of string used to cover one circle =.............. units

So $l = 4 \times$ ____

Surface area of a sphere of radius $r = 4 \times$ Area of a circle of radius _____ $= 4\pi r^2$.

APPLICATION

This result is useful in finding the cost of painting, repairing, constructing spherical and hemispherical objects.

PRECAUTIONS

- Measure diameter of ball carefully.
- Wrap the ball completely so that no space is left uncovered.
- Thinner the string more is the accuracy.

Activity 32

Objective

To draw histograms for classes of equal widths and varying widths.

Material Required

Graph paper, geometry box, sketch pens, scissors, adhesive, cardboard.

Method of Construction

1. Collect data from day to day life such as weights of students in a class and make a frequency distribution table.

   Case I : For classes of equal widths

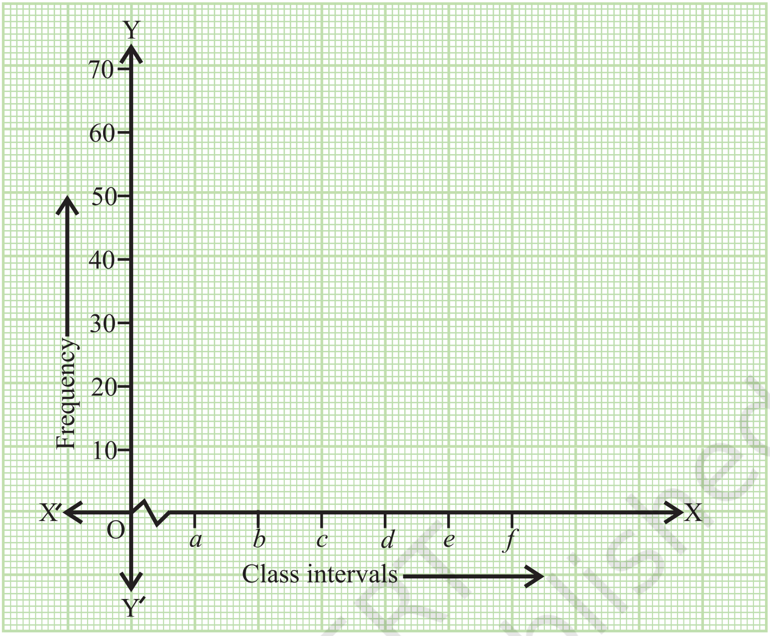
Class	*a*-*b*	*b*-c	*c*-*d*	*d*-e	*e*-f
Frequency	f_1	f_2	f_3	f_4	f_5

   Case II : For classes of varying widths

   Here : $d - f = 2\ (a - b)$

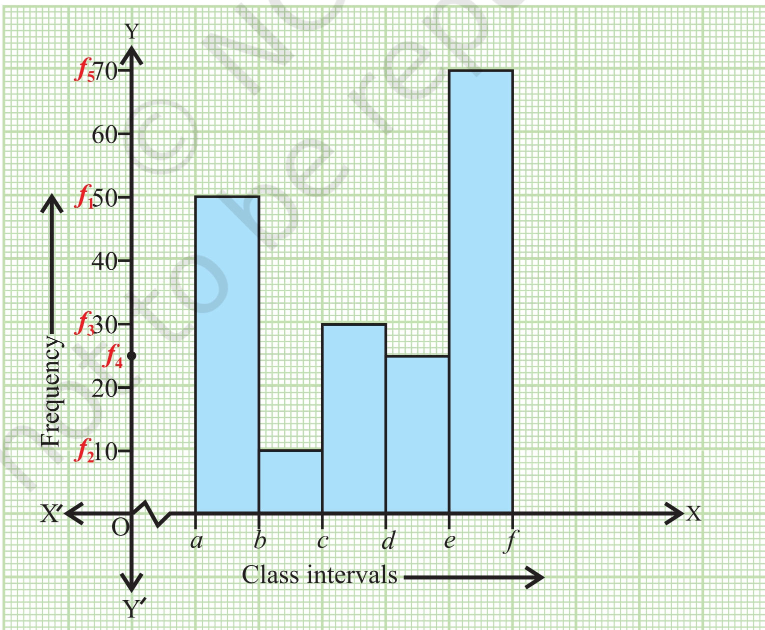
Class	***a*-*b* (width *x*)**	***b*-c (width *x*)**	***c*-*d* (width *x*)**	***d*-*f* (width 2*x*)**
Frequency	f_1	f_2	f_3	f_4
Modified frequency	f_1	f_2	f_3	$F' = \frac{f_4}{2}$

2. Take a graph paper (20 cm × 20 cm) and paste it on a cardboard.
3. Draw two perpendicular axes X′OX and YOY′ on the graph paper.
4. Mark classes on *x*-axis and frequencies on *y*-axis at equal distances as shown in Fig. 1.



Y
70
60
50
40
30
20
10
Frequency
X′
O
X
a
b
c
d
e
f
Class intervals
Y′

Fig. 1



Y
f_5 70
60
f_1 50
40
f_3 30
f_4
20
f_2 10
Frequency
X′
O
X
a
b
c
d
e
f
Class intervals
Y′

Fig. 2

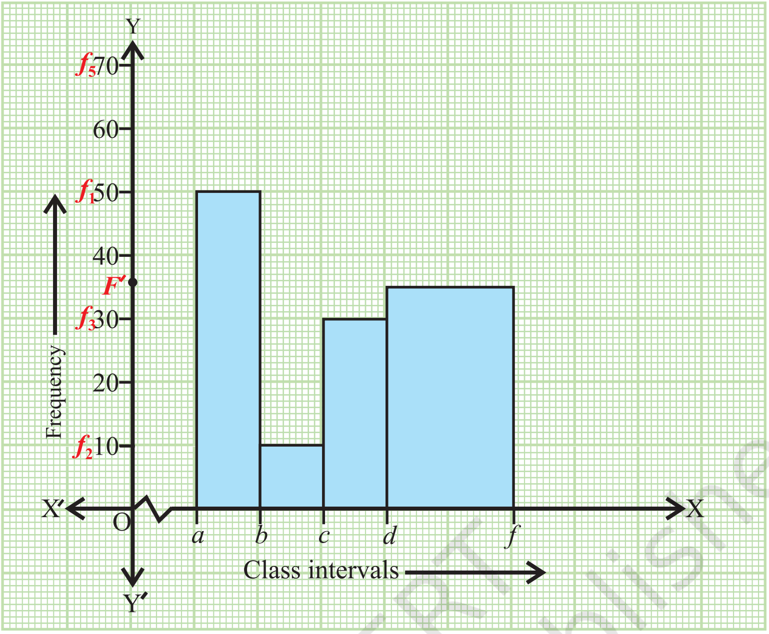


Fig. 3

5. On intervals (*a*-*b*), (*b*-*c*), (*c*-*d*), (*d*-*e*), (*e*- *f*), draw rectangles of equal widths and of heights f_1, f_2, f_3, f_4 and f_5, respectively, as shown in Fig. 2.
6. On intervals (*a*-*b*), (*b*-*c*), (*c*-*d*), (*d*-*f*), draw rectangles of heights f_1, f_2, f_3, and F' as shown in Fig. 3.

Demonstration

1. Different numerical values can be taken for *a, b, c, d, e* and *f*.
2. With these numerical values, histograms of equal widths and varying widths can be drawn.

Observation

Case I

1. The intervals are

a-*b* = ..., *b*-*c* = ..., *c*-*d* = ...,

d-e = ..., $e - f$ = ...

2. f_1 = ..., f_2 = ..., f_3 = ...,

f_4 = ..., f_5 = ...

Case II

1. a-b = ..., b-c = ..., c-d = ...,

d-f = ...,

2. f_1 = ..., f_2 = ..., f_3 = ...,

f_4 = ..., $F' = \frac{f_4}{2}$ = ...

APPLICATION

Histograms are used in presenting large data in a concise form pictorially.

Activity 33

OBJECTIVE

To find experimental probability of unit's digits of telephone numbers listed on a page selected at random of a telephone directory.

MATERIAL REQUIRED

Telephone directory, note book, pen, ruler.

METHOD OF CONSTRUCTION

1. Take a telephone directory and select a page at random.
2. Count the number of telephone numbers on the selected page. Let it be 'N'.
3. Unit place of a telephone number can be occupied by any one of the digits 0, 1, ..., 9.
4. Prepare a frequency distribution table for the digits, at unit's place using tally marks.
5. Write the frequency of each of the digits 0, 1, 2, ...8, 9 from the table.
6. Find the probability of each digit using the formula for experimental probability.

DEMONSTRATION

1. Prepare a frequency distribution table (using tally marks) for digits 0, 1, ..., 8, 9 as shown below:

Digit	0	1	2	3	4	5	6	7	8	9
Tally marks **Frequency**	n_0	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9

2. Note down frequency of each digit (0, 1, 2, 3,...,9) from the table.

   Digits 0, 1, 2, 3, ..., 9 are occuring respectively n_0, n_1, n_2, n_3, ..., n_9 times.

3. Calculate probability of each digit considering it as an event 'E' using the formula

$$P(E) = \frac{\text{Number of trials in which the event occured}}{\text{Total number of trials}}$$

4. Therefore, respective experimental probability of occurence of 0, 1, 2, ..., 9 is given by

$$P(0) = \frac{n_0}{N},\ P(1) = \frac{n_1}{N},\ P(2) = \frac{n_2}{N}, ...,\ P(9) = \frac{n_9}{N}.$$

Observation

Total number of telephone numbers on a page (N) = ...

Number of times 0 occurring at unit's place (n_0) = ...

Number of times 1 occurring at unit's place (n_1) = ...

Number of times 2 occurring at unit's place (n_2) = ...

--------------------3 ------------------------ (n_3) = ...

-------------------- 4 ---------------------- (n_4) = ...

Number of times 9 occurring at unit's place (n_9)= ....

Therefore, experimental probability of occurence of 0 $= P(0) = \frac{n_0}{N} =$ ....,

Experimental probability of occurence of 1 $= P(1) = \frac{n_1}{N} =$ ....

$P(2) = \frac{n_2}{N} =$ ...., ...,

:

$P(9) = \frac{n_9}{N} =$ ....

APPLICATION

Concept of experimental probability is used for deciding premium tables by insurance companies, by metreological department to forecast weather, for forecasting the performance of a company in stock market.

The mathematics experience of the students is incomplete if he never had the opportunity to solve a problem invented by himself.

– G. Polya

Activity 34

Objective

To find experimental probability of each outcome of a die when it is thrown a large number of times.

Material Required

Die, note book, pen.

Method of Construction

1. Divide the whole class in ten groups say G_1, G_2, G_3, ..., G_{10} of a suitable size.
2. Allow each group to throw a die 100 times and ask them to note down the observations, i.e., the number of times the outcomes 1, 2, 3, 4, 5 or 6 come up.
3. Count the number of times 1 has appeared in all the groups. Denote it by *a*. Similarly, count the number of times each of 2, 3, 4, 5 and 6 has appeared.

   Denote them by *b, c, d, e* and *f* respectively.
4. Find the probability of each outcome 'E' using the formula :

$$P(E) = \frac{\text{Number of times an outcome occured}}{\text{Total number of trials}}$$

Demonstration

1. There are 10 groups and each group throws a die 100 times. So, the total number of trials is 1000.
2. Total number of times 1 has appeared is *a*

   Therefore, experimental probability of 1 is $P(1) = \frac{a}{1000}$

   Similarly, experimental probability of 2 is $P(2) = \frac{b}{1000}$, of 3 is $P(3) = \frac{c}{1000}$, of 4 is $P(4)=\frac{d}{1000}$,

of 5 is $P(5)=\frac{e}{1000}$, of 6 is $P(6)=\frac{f}{1000}$

OBSERVATION

Fill in the results of your experiment in the following table:

Outcome\ Group	Number of times a number comes up on a die						Total
	1	2	3	4	5	6	
G_1	----	----	----	----	----	----	100
G_2	----	----	----	----	----	----	100
G_3	----	----	----	----	----	----	100
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
G_{10}	----	----	----	----	----	----	100
Total	***a* = ----**	***b* = ----**	***c* = ----**	***d* = ----**	***e* = ----**	***f* = ----**	**1000**

Therefore,

$P(1)=\frac{.......}{1000}$, $P(2)=\frac{.......}{1000}$, $P(3)=\frac{.......}{1000}$, $P(4)=\frac{.......}{1000}$,

$P(5)=\frac{.......}{1000}$, $P(6)=\frac{.......}{1000}$.

APPLICATION

Concept of probability is used by several statistical institutions to estimate/ predict next action based on available data.