PROBABILITY

Probability is the measure of uncertainty of various phenomenon, numerically. It can have positive value from 0 to 1.

The words 'probably', 'doubt', 'most probably', 'chances', etc., used in the statements above involve an element of uncertainty.

$$Probability = \frac{no. of\ favorable\ outcome}{total\ no. of\ outcomes}$$

Approach to Probability:

i. **Statistical approach** : Observation & data collection
ii. **Classical approach**: Only Equal probable events
iii. **Axiomatic approach:** For real life events. It closely relates to set theory.

2. Random Experiments:

An experiment is called random experiment if it satisfies the following two conditions:

(i) It has more than one possible outcome.
(ii) It is not possible to predict the outcome in advance.

Outcomes: a possible result of a random experiment is called its outcome.

Sample space: Set of all possible outcomes of a random experiment is called sample space. It is denoted by the symbol S. Example: In Toss of a coin, Sample space is Head, Tail.

Sample point: Each element of the sample space is called a sample point. E.g. in toss of a coin, Head is a Sample point.

3. Event:

It is the set of favorable outcome.
Any subset E of a sample space S is called an event.
E.g. Event of getting odd outcome in a throw of a die

Occurrence of an event: the event E of a sample space S is said to have occurred if the outcome ω of the experiment is such that $\omega \in E$. If the outcome ω is such that $\omega \notin E$, we say that the event E has not occurred.

Types of Event

i. Impossible and Sure Events
ii. Simple Event
iii. Compound Event

Impossible and Sure Events:

The empty set ϕ and the sample space S describe events. **Impossible event** is denoted by ϕ, while the whole sample space, S, is called the **Sure Event**.
E.g. in rolling a die, impossible event is that number is more than 6 & Sure event is the event of getting number less than or equal to 6.

Simple Event:

If an event E has only one sample point of a sample space, it is called a simple (or elementary) event.
In a sample space containing n distinct elements, there are exactly n simple events.
E.g. in rolling a die, Simple event could be the event of getting 4.

Compound Event:

If an event has more than one sample point, it is called a Compound event.
E.g. in rolling a die, Simple event could be the event of getting even number

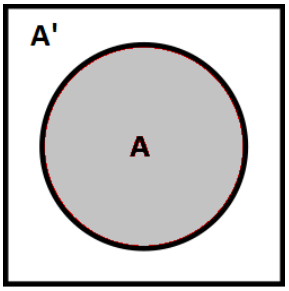
Algebra of Events

i. Complementary Event
ii. Event 'A or B'
iii. Event 'A and B'
iv. Event 'A but not B

Complementary Event

Complementary event to A= 'not A'
Example: If event A= Event of getting odd number in throw of a die, that is {1, 3, 5}
Then, Complementary event to A = Event of getting even number in throw of a die, that is {2, 4, 6}

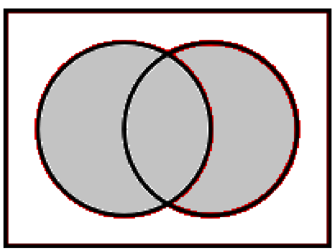
$A' = \{\omega : \omega \in S \; and \; \omega \notin A\} = S - A$ (Where S is the Sample Space)



Event (A or B)
Union of two sets A and B denoted by A ∪ B contains all those elements which are either in A or in B or in both.

When the sets A and B are two events associated with a sample space, then 'A ∪ B' is the event 'either A or B or both'. This event 'A ∪ B' is also called 'A or B'.

Event 'A or B' = A ∪ B = {ω : ω ∈ A or ω ∈ B}.

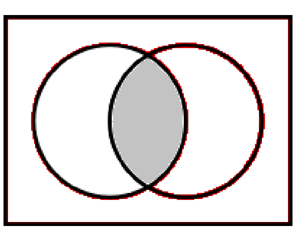


A ∪ B

Event 'A and B'

Intersection of two sets A ∩ B is the set of those elements which are common to both A and B. i.e., which belong to both 'A and B'.
If A and B are two events, then the set A ∩ B denotes the event 'A and B'.

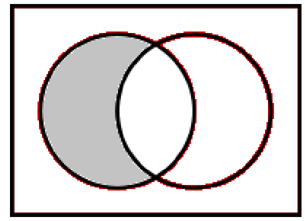
Thus, A ∩ B = {ω : ω ∈ A and ω ∈ B}



A ∩ B

Event 'A but not B'
A–B is the set of all those elements which are in A but not in B. Therefore, the set A–B may denote the event 'A but not B'.

A – B = A ∩ B'



A ∩ B'

Mutually exclusive events
Events A and B are called mutually exclusive events if occurrence of any one of them excludes occurrence of other event, i.e., if they cannot occur simultaneously.
Example: A die is thrown. Event A = All even outcome & event B = All odd outcome. Then A & B are mutually exclusive events, they cannot occur simultaneously.
Simple events of a sample space are always mutually exclusive.

Exhaustive events
Lot of events that together forms sample space.
Example: A die is thrown. Event A = All even outcome & event B = All odd outcome. Even A & B together forms exhaustive events as it forms Sample Space.

<u>4. Axiomatic Approach to Probability:</u>
It is another way of describing probability. Here Axioms or rules are used.

Let S be sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S and range is the interval[0,1] satisfying the following axioms

i) For any event E, $P[E] \geq 0$
ii) $P[S] = 1$
iii) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

It follows from (iii) that $P(\phi) = 0$. Let $F = \phi$ and $E = \phi$ be two disjoint events,

$\therefore P(E \cup \phi) = P(E) + P(\phi)$ or $P(E) = P(E) + P(\phi)$ $i.e., P(\phi) = 0$

Let S be a sample space containing outcomes $\omega_1, \omega_2, \cdots \omega_n$, i.e., $S = \{\omega_1, \omega_2, \cdots \omega_n\}$ then

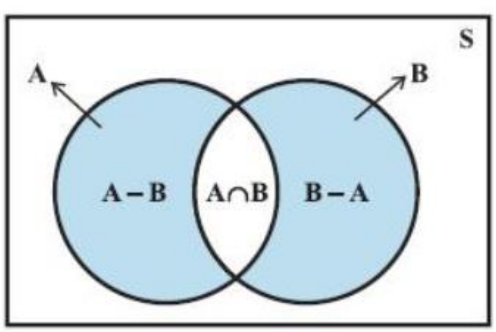
i. $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$
ii. $P(\omega_1) + P(\omega_2) + \cdots + P(\omega_n) = 1$
iii. For any event E, $P(E) = \Sigma P(\omega_i), \omega_i \in A$
iv. P (ϕ) = 0

Probabilities of equally likely outcomes:
Let P(ωi) = p, for all ωi ∈ S where 0 ≤ p ≤ 1,
then $p = \frac{1}{n}$ where n= number of elements.
Let S be a sample space and E be an event, such that $n(S) = n$ and $n(E) = m$. If each outcome is equally likely, then it follows that

$$P(E) = \frac{m}{n} = \frac{Number\ of\ Outcomes\ favourable\ to\ E}{Total\ possible\ outcomes}$$

Probability of the event 'A or B' :
P(A∪B) = P(A) + P(B) – P(A∩ B)

Probability of the event 'A and B'
P(A∩B) = P(A) + P(B) – P(AU B)



Probability of the event 'Not A'
P(A') = P(not A) = 1 – P(A)