Linear Inequalities

- An inequality is a relation that holds between two values when they are different.
- E.g. $x < 5$. Here there is a relation between x & 5.
- 2 real numbers or algebraic expressions related by symbol '<', '>', '≤' or '≥' form inequality.
- The study of inequalities is very useful in solving problems in the field of science, mathematics, statistics, optimization problems, economics, psychology, etc.

Example of inequality in daily life:

Alex has 200 Rs & wants to buy some pen & eraser. How many pen & eraser she can buy?
We can write this statement mathematically using inequalities. Let the number of pen Alex can buy be x & number of eraser be y. Then, $7x + 23y \leq 200$.

Notations

- The notation $a < b$ means that a is **less than** b.
- The notation $a > b$ means that a is **greater than** b.
- The notation $a \neq b$ means that a is **not equal to** b
- The notation $a \leq b$ means that a is **less than or equal to** b
- The notation $a \geq b$ means that a is **greater than or equal to** b

Types of Inequalities:

- **Numerical inequalities**: Relationship between numbers. E.g. $3 < 5$
- **Literal or variable inequalities**: Relationship between variables or variable & number. E.g. $x < 5$
- **Double Inequalities**: Relationship from two side. E.g. $2 < x < 5$
- **Strict inequalities:** An inequality that uses the symbols < or >. The symbols ≤ and ≥ are not used. g. $x < 5$; $3<5$
- **Slack inequalities**. An inequality that uses the symbols ≤ or ≥ E.g. $x \leq 5$
- **Linear inequalities in one variable:** An inequality which involves a linear function in one variable E.g. $x < 5$;
- **Linear inequalities in two variables**: An inequality which involves a linear function in two variable E.g. $3x + 2y < 5$;
- **Quadratic inequalities**: An inequality which involves a quadratic function .E.g. $x^2 + 2x \leq 5$

Solution for linear inequality in one variable
Solution & Solution Set

- **Solution**: Values of x, which make inequality a true statement. E.g. 3 is a solution for $x<7$
- **Solution Set**: The set of values of x is called its solution set. E.g.: {1,2,3,4,5,6} is solution set for $x<7$ where x is natural Number

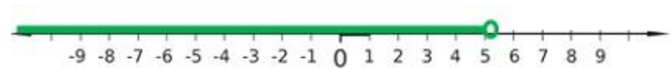
Rules of Inequality:

- Equal numbers may be added to (or subtracted from) both sides of an inequality without affecting the sign of inequality. E.g. $x < 7$ is same as $x + 2 < 7 + 2$
- Both sides of an inequality can be multiplied (or divided) by the same **positive** number without affecting the sign of inequality. E.g. : $x + y < 7$ is same as $(x + y) \times 3 < 7 \times 3$
- But when both sides are multiplied or divided by a **negative** number, then the sign of inequality is reversed. e.g: $x + y < 7$ is same as $(x + y) \times (-3) > 7 \times (-3)$

Question: Solve $30x < 160$ when (i) x is a natural number, (ii) x is an integer, (iii) x is real number
Solution:
Dividing the inequality by 30 as per rule 2.

$$\frac{30x}{30} < \frac{160}{30}$$
$$Or\ x < \frac{16}{3}$$

<u>Case 1:</u> x is a natural number. Then solution set is {1,2,3,4,5}
<u>Case 2:</u> x is an integer. Then solution set is {……-4,-3,-32,-1,0,1,2,3,4,5}
<u>Case 3:</u> x is a real number. Then solution set is $(-\infty, 16/3)$.
We can represent case 3 solution using number line



Question: Solve $7x + 2 \leq 5x + 8$. Show the graph of the solutions on number line.

Solution: Subtracting 2 from both side we get

$7x \leq 5x + 6$

Subtracting 5x from both side we get

$2x \leq 6$

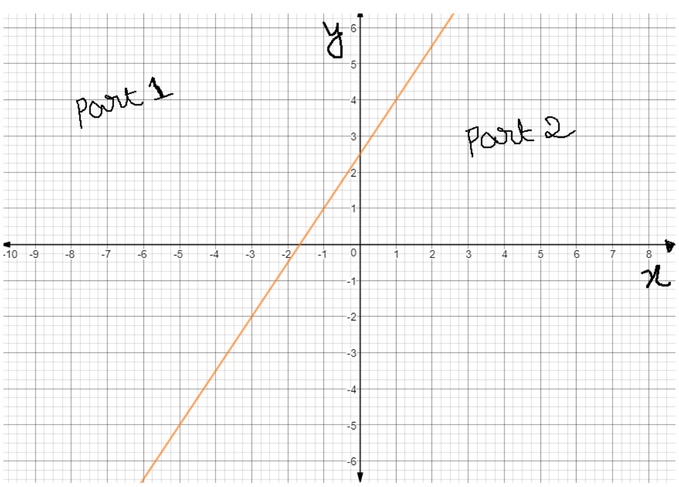
Dividing 2 both side we get

$x \leq 3$

We can represent this in Number line below.

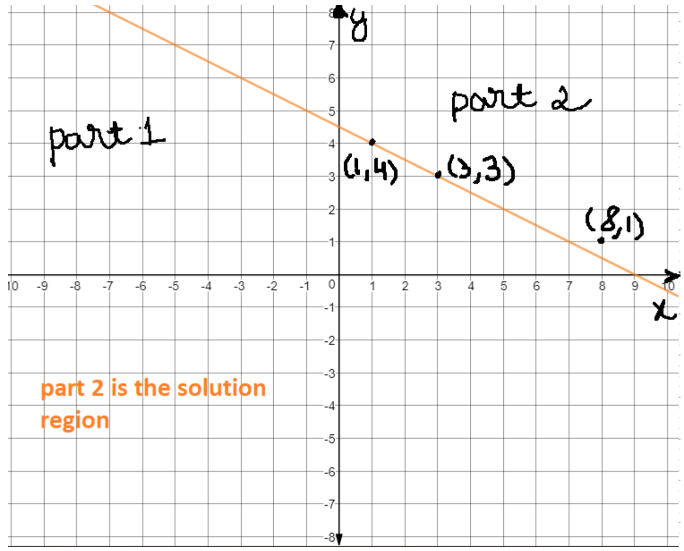
Graphical Solution of Linear Inequalities in 2 variables

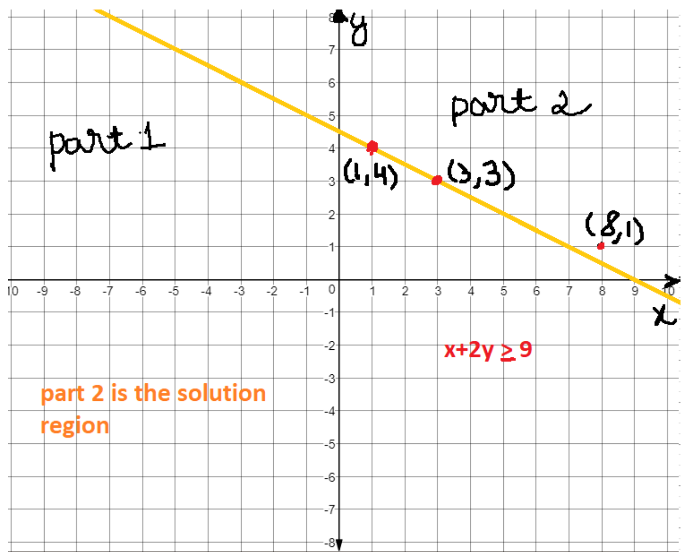
A line divides the Cartesian plane into two equal parts. A point in Cartesian plane will either lie on a line or will lie in either of half planes. The region containing all the solutions of an inequality is called the solution region.



In order to identify the half plane represented by an inequality, it is just sufficient to take any point (*a*, *b*) (not online) and check whether it satisfies the inequality or not. If it satisfies, then the inequality represents the half plane and shade the region which contains the point, otherwise, the inequality represents that half plane which does not contain the point within it. For convenience, the point (0, 0) is preferred.

Example: $x + 2y > 9$





Steps for find solution region for a linear inequality in 2 variables

1. Replace the inequality sign with equal sign & plot the graph. In this case plot graph for x+2y =9. Green line represent x+2y =9
2. Take any point on the graph. Here we took (8, 1) & check if satisfies the linear inequality. In this case x+2y >9. If yes, then the region where this assumed point lies is the solution region.
3. In case of Slack inequality (≥ or ≤) use solid line, since the points on the line is included in the solution set.
4. In case of Strict inequality (> or <) use dotted line, since points on the line is not included in solution set.

In case of multiple linear inequalities, the region common to all the inequalities is the solution region.

Question: Solve the following system of inequalities graphically $5x + 4y \leq 40$, $x > 2$ & $y \geq 3$

Solution: Steps

1. Draw lines for $5x + 4y = 40$ & $x = 2$ & $y = 3$
2. Find the solution region for each of these linear inequalities
3. Find common region. Common region is the solution region.