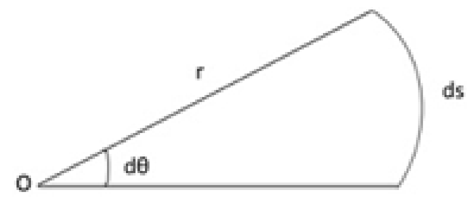
Chapter 2
Units and Measurements

Units

A unit is an internationally accepted standard for measurements of quantities.

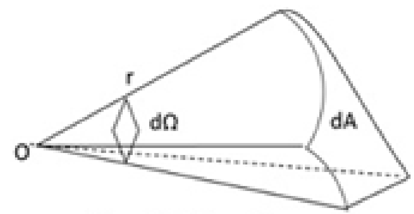
- Measurement consists of a numeric quantity along with a relevant unit.
- Units for Fundamental or base quantities (like length, time etc.) are called **Fundamental units**.
- Units which are combination of fundamental units are called **Derived units**.
- Fundamental and Derived units together form a **System of Units**.
- Internationally accepted system of units is **SystèmeInternationale d' Unites (French for International system of Units)** or **SI**. It was developed and recommended by General Conference on Weights and Measures in **1971**.
- SI lists **7 base units** as in the table below. Along with it, there are two units - **radian or rad** (unit for plane angle) and **steradian or sr** (unit for solid angle). They both are **dimensionless**.

Base Quantity	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermo dynamic Temperature	kelvin	K
Amount of Substance	mole	mol
Luminous intensity	candela	cd



$d\theta = ds/r$ radian

Plane angle. Unit - Radian

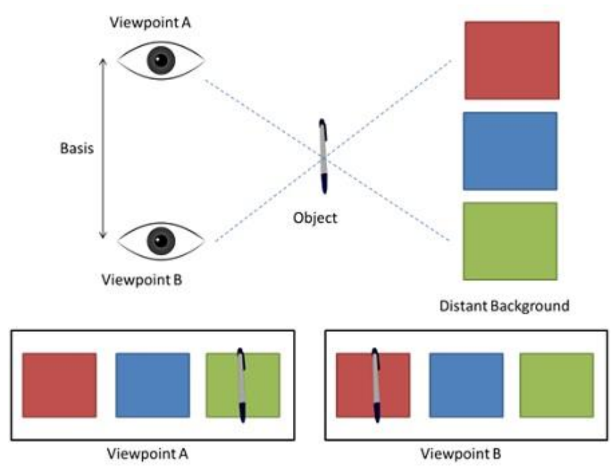


$d\Omega = dA/r^2$ steradian

Solid angle. Unit - Steradian

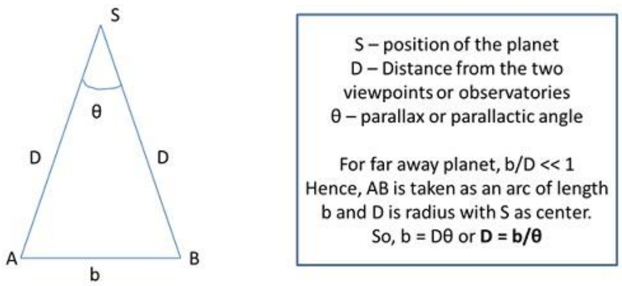
2.3.1. Measuring large Distances – Parallax Method

- **Parallax** is a displacement or difference in the apparent position of an object viewed along two different lines of sight, and is measured by the angle or semi-angle of inclination between those two lines. Distance between the two viewpoints is called **Basis**.



Parallax. From viewpoint A the pen appears over green box while from viewpoint B the pen appears over red box.

Measuring distance of a planet using parallax method



Parallax method to determine distance of a planet

Similarly, α = d/D

Where **α** = **angular size** of the planet (angle subtended by d at earth) and **d** is the **diameter** of the planet.αis angle between the direction of the telescope when two diametrically opposite points of the planet are viewed.

2.3.2.Measuring very small distances

To measure distances as low as size of a molecule, **electron microscopes** are used. These contain electrons beams controlled by electric and magnetic fields.

- Electron microscopes have a resolution of 0.6 Å or Agstroms.
- Electron microscopes are able to resolve atoms and molecules while using tunneling microscopy, it is possible to estimate size of molecule.

Estimating size of molecule of Oleic acid

Oleic acid is a soapy liquid with large molecular size of the order of $10^{-9}m$. The steps followed in determining the size of molecule are:

- Dissolve 1 cm^3 of oleic acid in alcohol to make a solution of 20 cm^3.Take 1 cm^3 of above solution and dilute it to 20 cm^3, using alcohol. Now, the concentration of oleic acid in the solution will be (1/(20x20)) cm^3 of oleic acid/ cm^3 of solution.
- Sprinkle lycopodium powder on the surface of water in a trough and put one drop of above solution. The oleic acid in the solution will spread over water in a circular molecular thick film.
- Measure the diameter of the above circular film using below calculations.
- If n –Number of drops of solution in water, V – Volume of each drop, t – Thickness of the film, A – Area of the film

Total volume of n drops of solution = $nV\ cm^3$

Amount of Oleic acid in this solution

$$= nV\left(\frac{1}{20\times 20}\right) cm^3$$

Thickness of the film = $t\ = \frac{Volume\ of\ the\ film}{Area\ of\ the\ film}$

$$t\ = \frac{\mathrm{nV}}{20\times 20\mathrm{A}}\ \mathrm{cm}.$$

Special Length units

Unit name	**Unit Symbol**	**Value in meters**
fermi	f	10^{-15} m
angstrom	Å	10^{-10}m
astronomical unit(average distance of sun from earth)	AU	1.496 X 10^{11} m
light year(distance travelled by light in 1 year with velocity $3\times 10^8 m/s$)	ly	9.46 X 10^{11} m
parsec(distance at which average radius of earth's orbits subtends an angle of 1 arc second)	pc	3.08 x 10^{16} m

Measurement of Mass

Mass is usually measured in terms of kg but for atoms and molecules, **unified atomic mass unit (u)** is used.

1 u = 1/12 of the mass of an atom of carbon-12 isotope including mass of electrons (1.66 x 10^{-27} kg)

Apart from using **balances** for normal weights, mass of planets is measured using **gravitational methods** and mass of atomic particles are measured using **mass spectrograph** (radius of trajectory is proportional to mass of charged particle moving in uniform electric and magnetic field).

Range of Mass

Object	**Mass (kg)**
Electron	10^{-30}
Proton	10^{-27}
Red Blood Cell	10^{-13}
Dust particle	10^{-9}
Rain drop	10^{-6}
Mosquito	10^{-5}
Grapes	10^{-3}
Human	10^{2}
Automobile	10^{3}
Boeing 747 aircraft	10^{8}
Moon	10^{23}
Earth	10^{25}
Sun	10^{30}
Milky way Galaxy	10^{41}
Observable Universe	10^{55}

<u>Measurement of Time</u>

Time is measured using a clock. As a standard, **atomic standard of time** is now used, which is measured by **Cesium or Atomic clock**.

- In Cesium clock, a second is equal to 9,192,631,770 vibrations of radiation from the transition between two hyperfine levels of cesium-133 atom.
- Cesium clock works on the vibration of cesium atom which is similar to vibrations of balance wheel in a regular wristwatch and quartz crystal in a quartz wristwatch.
- National standard time and frequency is maintained by 4 atomic clocks. Indian standard time is maintained by a Cesium clock at National Physical Laboratory (NPL), New Delhi.
- Cesium clocks are very accurate and the uncertainty is very low 1 part in 10^{13} which means not more than 3 μs are lost or gained in a year.

Range of Time

Event	**Time Interval (s)**
Life span of most unstable particle	10^{-24}
Period of x-rays	10^{-19}
Period of light wave	10^{-15}
Period of radio wave	10^{-6}
Period of sound wave	10^{-3}
Wink on an eye	10^{-1}
Travel time of light from moon to earth	10^{0}

Travel time of light from sun to earth	10^2
Rotation period of the earth	10^5
Revolution period of the earth	10^7
Average human life span	10^9
Age of Egyptian pyramids	10^{11}
Time since dinosaur extinction	10^{15}
Age of Universe	10^{17}

Accuracy and Precision of Instruments

- Any uncertainty resulting from measurement by a measuring instrument is called an **error**. They can be systematic or random.
- **Accuracy** of a measurement is how close the measured value is to the true value.
- **Precision** is the resolution or closeness of a series of measurements of a same quantity under similar conditions.
- If the true value of a certain length is 3.678 cm and two instruments with different resolutions, up to 1 (less precise) and 2 (more precise) decimal places respectively, are used. If first measures the length as 3.5 and the second as 3.38 then the first has more accuracy but less precision while the second has less accuracy and more precision.

Types of Errors- Systematic Errors

Errors which can either be positive or negative are called **Systematic errors**. They are of following types:

1. **Instrumental errors**: These arise from imperfect design or calibration error in the instrument. Worn off scale, zero error in a weighing scale are some examples of instrument errors.
2. **Imperfections in experimental techniques**: If the technique is not accurate (for example measuring temperature of human body by placing thermometer under armpit resulting in lower temperature than actual) and due to the external conditions like temperature, wind, humidity, these kinds of errors occur.

3. Personal errors: Errors occurring due to human carelessness, lack of proper setting, taking down incorrect reading are called personal errors.

These errors can be removed by:

- Taking proper instrument and calibrating it properly.
- Experimenting under proper atmospheric conditions and techniques.

Removing human bias as far as possible

Random Errors

Errors which occur at random with respect to sign and size are called **Random errors**.

- These occur due to unpredictable fluctuations in experimental conditions like temperature, voltage supply, mechanical vibrations, personal errors etc.

Least Count Error

Smallest value that can be measured by the measuring instrument is called its **least count**. **Least count error** is the error associated with the resolution or the least count of the instrument.

- Least count errors can be minimized by using instruments of higher precision/resolution and improving experimental techniques (taking several readings of a measurement and then taking a mean).

Errors in a series of Measurements

Suppose the values obtained in several measurement are $a_1, a_2, a_3, \ldots, a_n$.

Arithmetic mean, $a_{mean} = (a_1 + a_2 + a_3 + \ldots + a_n)/n$

$$a_{mean} = \sum_{i=1}^{n} \frac{a_i}{n}$$

- **Absolute Error**: The magnitude of the difference between the true value of the quantity and the individual measurement value is called absolute error of the measurement. It is denoted by **|Δ*a*|** (or Mod of Delta a). The mod value is always positive even if Δa is negative. The individual errors are:

$\Delta a_1 = a_{mean} - a_1$

$\Delta a_2 = a_{mean} - a_2,$

… … …

… … …

$\Delta a_n = a_{mean} - a_n$

- **Mean absolute error** is the arithmetic mean of all absolute errors. It is represented by Δa_{mean}.

$$\Delta a_{mean} = \frac{|\Delta a1| + |\Delta a2| + |\Delta a3| + \dots + |\Delta an|}{n}$$

$$\Delta a_{mean} = \sum_{i=1}^{n} \frac{|\Delta a_i|}{n}$$

For single measurement, the value of 'a' is always in the range $a_{mean} \pm \Delta a_{mean}$

So, $a = a_{mean} \pm \Delta a_{mean}$

Or, $a_{mean} - \Delta a_{mean} \leq a \leq a_{mean} + \Delta a_{mean}$

- **Relative Error**: It is the ratio of **mean absolute error** to the **mean value** of the quantity measured.

$$Relative\ Error = \frac{\Delta a_{mean}}{a_{mean}}$$

- **Percentage Error:** It is the relative error expressed in percentage. It is denoted by δa.

$$\delta a = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%$$

Combinations of Errors

If a quantity depends on two or more other quantities, the combination of errors in the two quantities helps to determine and predict the errors in the resultant quantity. There are several procedures for this.

Suppose two quantities A and B have values as A ± ΔA and B ± ΔB. Z is the result and ΔZ is the error due to combination of A and B.

Criteria	**Sum or Difference**	**Product**	**Raised to Power**
Resultant value Z	Z = A ± B	Z = AB	$Z = A^k$
Result with error	Z ± ΔZ = (A ± ΔA) + (B ± ΔB)	Z ± ΔZ = (A ± ΔA) (B ± ΔB)	$Z \pm \Delta Z = (A \pm \Delta A)^k$
Resultant error range	± ΔZ = ± ΔA ± ΔB	ΔZ/Z = ΔA/A ± ΔB/B	
Maximum error	ΔZ = ΔA + ΔB	ΔZ/Z = ΔA/A + ΔB/B	ΔZ/Z = k(ΔA/A)
Error	Sum of absolute errors	Sum of relative errors	k times relative error

Significant Figures

Every measurement results in a number that includes reliable digits and uncertain digits. Reliable digits plus the first uncertain digit are called **significant digits or significant figures**.These indicate the precision of measurement which depends on least count of measuring instrument. Example, period of oscillation of a pendulum is 1.62 s. Here 1 and 6 are reliable and 2 is uncertain. Thus, the measured value has three significant figures.

Rules for determining number of significant figures

- All non-zero digits are significant.
- All zeros between two non-zero digits are significant irrespective of decimal place.
- For a value less than 1, zeroes after decimal and before non-zero digits are not significant. Zero before decimal place in such a number is always insignificant.

- Trailing zeroes in a number without decimal place are insignificant.
- Trailing zeroes in a number with decimal place are significant.

<u>Cautions to remove ambiguities in determining number of significant figures</u>

- **Change of units should not change number of significant digits**. Example, 4.700m = 470.0 cm = 4700 mm. In this, first two quantities have 4 but third quantity has 2 significant figures.
- **Use scientific notation to report measurements**. Numbers should be expressed in powers of 10 like **a x 10^{b}** where b is called **order of magnitude**. Example, $4.700\ m = 4.700 \times 10^2\ cm = 4.700 \times 10^3\ mm = 4.700 \times 10^{-3} km$
  In all the above, since power of 10 are irrelevant, number of significant figures are 4.
- Multiplying or dividing exact numbers can have infinite number of significant digits. Example, radius = diameter / 2. Here 2 can be written as 2, 2.0, 2.00, 2.000 and so on.

<u>Rules for Arithmetic operation with Significant Figures</u>

Type	**Multiplication or Division**	**Addition or Subtraction**
Rule	The final result should retain as many significant figures as there in the **original number with the lowest number of significant digits.**	The final result should retain as many decimal places as there in the **original number with the least decimal places.**
Example	Density = Mass / Volume if mass = 4.237 g (**4 significant figures**) and Volume = 2.51 cm^3(**3 significant figures**) Density = 4.237 g/2.51 cm^3 = 1.68804 g cm^{-3} = 1.69 g cm^{-3} (**3 significant figures**)	Addition of 436.32 (2 digits after decimal), 227.2 (1 digit after decimal) & .301 (3 digits after decimal) is = 663.821 Since 227.2 is precise up to only 1 decimal place, Hence, the final result should be 663.8

<u>Rules for Rounding off the uncertain digits</u>

Rounding off is necessary to reduce the number of insignificant figures to adhere to the rules of arithmetic operation with significant figures.

Rule Number	**Insignificant Digit**	**Preceding Digit**	**Example (rounding off to two decimal places)**
1	Insignificant digit to be dropped is more than 5	Preceding digit is raised by 1.	Number – 3.137 Result – 3.14
2	Insignificant digit to be dropped is less than 5	Preceding digit is left unchanged.	Number – 3.132 Result – 3.13

3	Insignificant digit to be dropped is equal to 5	If preceding digit is even, it is left unchanged.	Number – 3.125 Result – 3.12
4	Insignificant digit to be dropped is equal to 5	If preceding digit is odd, it is raised by 1.	Number – 3.135 Result – 3.14

Rules for determining uncertainty in results of arithmetic calculations

To calculate the uncertainty, below process should be used.

- Add a lowest amount of uncertainty in the original numbers. Example uncertainty for 3.2 will be ± 0.1 and for 3.22 will be ± 0.01. Calculate these in percentage also.
- After the calculations, the uncertainties get multiplied/divided/added/subtracted.
- Round off the decimal place in the uncertainty to get the final uncertainty result.

Example, for a rectangle, if length l = 16.2 cm and breadth b = 10.1 cm

Then, take l = 16.2 ± 0.1 cm or 16.2 cm ± 0.6% and breadth = 10.1 ± 0.1 cm or 10.1 cm ± 1%.

On Multiplication, area = length x breadth = 163.62 cm^2 ± 1.6% or 163.62 ± 2.6 cm^2.
Therefore after rounding off, area = 164 ± 3 cm^2.
Hence 3 cm^2 is the uncertainty or the error in estimation.

Rules

1. For a set experimental data of 'n' significant figures, the result will be valid to 'n' significant figures or less (only in case of subtraction).

Example 12.9 - 7.06 = 5.84 or 5.8 (rounding off to lowest number of decimal places of original number).

2. The relative error of a value of number specified to significant figures depends not only on n but also on the number itself.

Example, accuracy for two numbers 1.02 and 9.89 is ±0.01. But relative errors will be:

For 1.02, (± 0.01/1.02) x 100% = ± 1%

For 9.89, (± 0.01/9.89) x 100% = ± 0.1%

Hence, the relative error depends upon number itself.

3. Intermediate results in multi-step computation should be calculated to one more significant figure in every measurement than the number of digits in the least precise measurement.
   Example:1/9.58 = 0.1044

Now, 1/0.104 = 9.56 and 1/0.1044 = 9.58

Hence, taking one extra digit gives more precise results and reduces rounding off errors.

Dimensions of a Physical Quantity

Dimensions of a physical quantity are powers (exponents) to which base quantities are raised to represent that quantity. They are **represented by square brackets** around the quantity.

- Dimensions of the 7 base quantities are – Length [L], Mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd] and amount of substance [mol].

  Examples, Volume = Length x Breadth x Height
  $= [L] \times [L] \times [L] = [L]^3 = [L^3]$

  Force = Mass x Acceleration
  $= [M][L]/[T]^2 = [MLT^{-2}]$
- The other dimensions for a quantity are always 0. For example, for volume only length has 3 dimensions but the mass, time etc have 0 dimensions. Zero dimension is represented by superscript 0 like $[M^0]$.

Dimensions do not take into account the magnitude of a quantity

Dimensional Formula and Dimensional Equation

Dimensional Formula is the expression which shows how and which of the base quantities represent the dimensions of a physical quantity.
Dimensional Equation is an equation obtained by equating a physical quantity with its dimensional formula.

Physical Quantity	**Dimensional Formula**	**Dimensional Equation**
Volume	$[M^0 L^3 T^0]$	$[V] = [M^0 L^3 T^0]$
Speed	$[M^0 L T^{-1}]$	$[\upsilon] = [M^0 L T^{-1}]$
Force	$[M L T^{-2}]$	$[F] = [M L T^{-2}]$
Mass Density	$[M L^{-3} T^0]$	$[\rho] = [M L^{-3} T^0]$

Dimensional Analysis

- Only those physical quantities which have same dimensions can be added and subtracted. This is called **principle of homogeneity of dimensions**.
- Dimensions can be multiplied and cancelled like normal algebraic methods.
- In mathematical equations, quantities on both sides must always have same dimensions.
- Arguments of special functions like trigonometric, logarithmic and ratio of similar physical quantities are dimensionless.
- Equations are uncertain to the extent of dimensionless quantities.

Example Distance = Speed x Time. In Dimension terms, $[L] = [LT^{-1}] \times [T]$

Since, dimensions can be cancelled like algebra, dimension [T] gets cancelled and the equation becomes [L] = [L].

Applications of Dimensional Analysis

Checking Dimensional Consistency of equations

- A **dimensionally correct equation** must have same dimensions on both sides of the equation.
- A dimensionally correct equation need not be a correct equation but a dimensionally incorrect equation is always wrong. It can test dimensional validity but not find exact relationship between the physical quantities.

Example, $x = x_0 + v_0 t + \left(\frac{1}{2}\right) a t^2$

Or, Dimensionally, $[L] = [L] + [LT^{-1}][T] + [LT^{-2}][T^2]$

Where, x – Distance travelled in time t,
x_0 – starting position,
v_0 - initial velocity,
a – uniform acceleration.

Dimensions on both sides will be [L] as [T] gets cancelled out. Hence this is dimensionally correct equation.

Deducing relation among physical quantities

- To deduce relation among physical quantities, we should know the dependence of one quantity over others (or independent variables) and consider it as product type of dependence.
- Dimensionless constants cannot be obtained using this method.

Example, $T = k\, l^x g^y m^z$

Or $[L^0M^0T^1] = [L^1]^x [L^1T^{-2}]^y [M^1]^z = [L^{x+y}T^{-2y} M^z]$

Means, x+y = 0, -2y = 1 and z = 0. So, x = ½, y = -½ and z = 0

So the original equation reduces to $T = k\sqrt{\frac{l}{g}}$