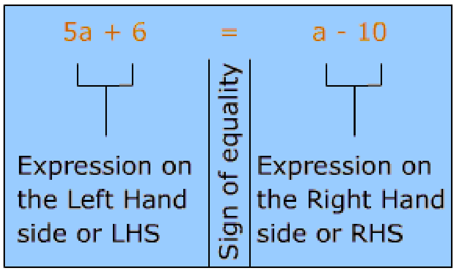
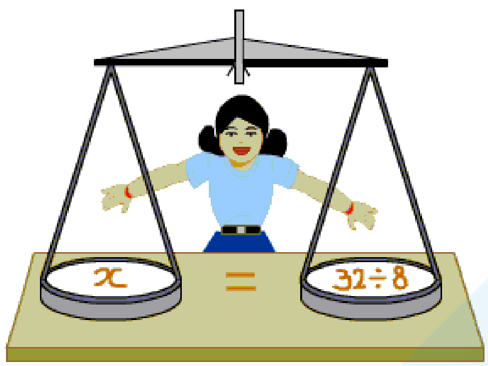
Linear Equations in Two Variables

A simple linear equation is a statement of equality between two algebraic expressions involving an unknown quantity called the variable. In a linear equation the power of the variable is always equal to 1. The two sides of an equation are called Left-Hand Side (LHS) and Right Hand Side (RHS). They are written on either side of the equality sign.



The two sides of an equation are like the two pans of a balance.



Equations of Condition

Study the following equations:

(a) 4x + 2 = 14

(b) x - 7 = 5 - 2x

(c) 3a - 8 = a + 12

The equation 4x + 2 = 14 is true only when x = 3,

Similarly, x - 7 = 5 - 2x is true only when x = 4, and

3a - 8 = a + 12 is true only when a = 10.

The two expressions (LHS and RHS) are equal only for a particular value of the variable x. These equations are called **equations of condition.**

Identical Equations or Identities

Now study the equations given below:

(a) x + 3 + x + 4 = 2x + 7

2x + 7 = 2x + 7 (By simplification)

[any value given to 'x' always satisfies the equation]

Similarly,

(b) $(3a + 4) + 2(a - 1) = 5a + 2$
i.e., $5a + 2 = 5a + 2$ (By simplification)
[any value given to 'a' always satisfies the equation]
The two expressions (LHS and RHS) are always equal for any value we give to the variable.
Equations that are true for any value of the variable are called **Identical equations** or **Identities.**

Solving Linear Equations

The process of finding the value of the unknown quantity for which the equation is true, is called solving the equation. The value so found is called the **root** or **solution** of the equation.
The process of solving a simple equation depends upon the following axioms:

Addition Property

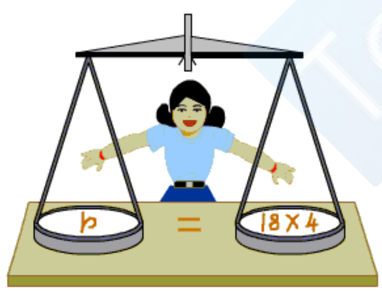
If any number is added to both sides of an equation, then the equality of the equation remains unchanged.
i.e., if $x = y$ then $x + a = y + a$

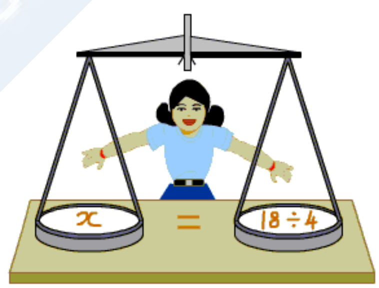
Subtraction Property

If any number is subtracted from both sides of an equation, then the equality of 0the equation remains unchanged.
i.e., if $x = y$, then $x - a = y - a$

Multiplication and Division Property

The following are also true.
If $x = y$, then $x \times a = y \times a$ and, $x \div a = y \div a$
where a is a non-zero constant.

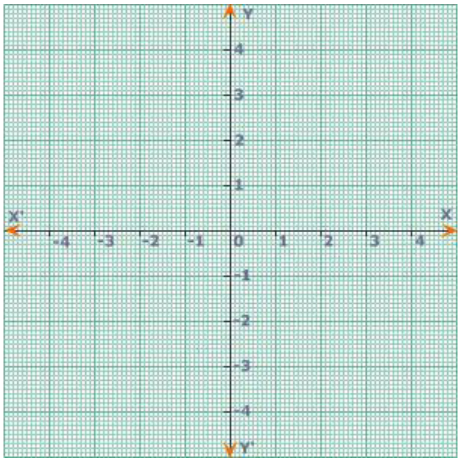




Graphical representation of a Linear Equation in one variable

Rectangular Axes

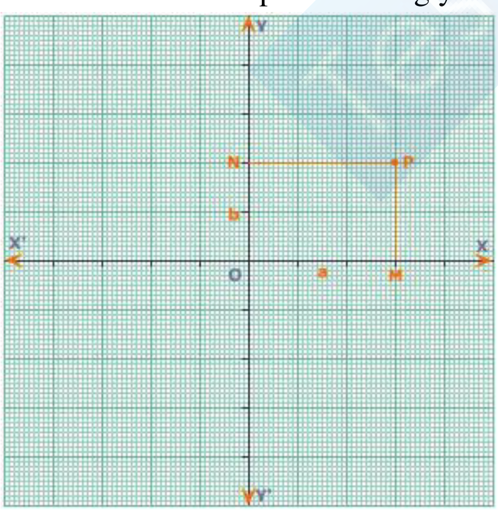
The position of a point in a plane is fixed by selecting two axes of reference which are formed by combining two number lines at right angles so that their zeros coincide.



The horizontal number line is called x-axis and the vertical number line is called y-axis.
The point of intersection of the two number lines is called origin.
The two number lines together are called rectangular axes.

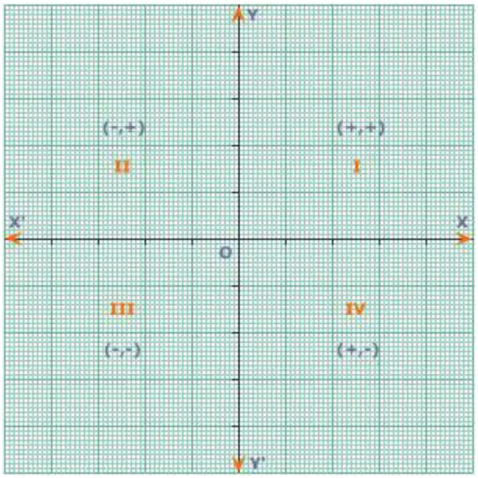
Coordinates

The position of a point with respect to the rectangular axes by means of a pair of numbers is called coordinates.
The distance OM of point P along x-axis is called x-coordinate or abscissa.
The distance ON of point P along y-axis is called ordinate or y-coordinate.



If OM=a and ON=b then position of the point P is denoted by (a, b).
Coordinates of the origin is (0, 0).
Coordinates of any point on the x-axis is (x, 0).
Coordinates of any point on the y-axis is (0, y).

Quadrants
The rectangular axes divide the plane into four regions called quadrants.
By convention the quadrants are numbered as I, II, III, IV in the anti-clockwise direction.

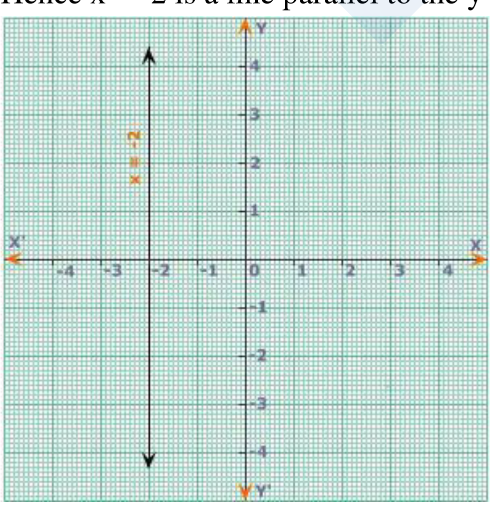


- In the I quadrant, any point will have both the coordinates positive.
- In the II quadrant, x-coordinate is negative while y-coordinate is positive.
- In the III quadrant, both x-coordinate and y-coordinate are negative.
- In the IV quadrant, x-coordinate is positive while the y-coordinate is negative.

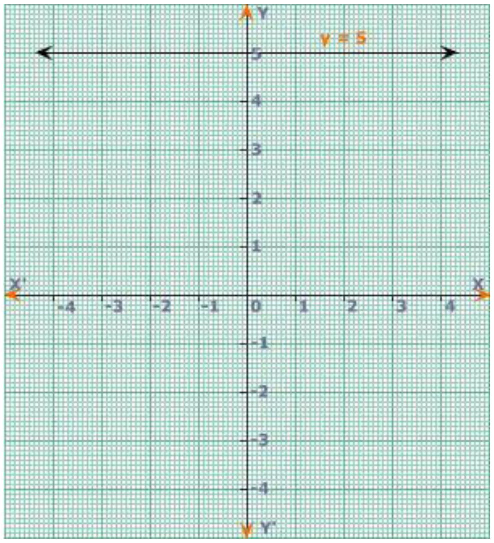
To Plot the Graph of a Linear Equation in one Variable
ax + b = 0 is a linear equation in one variable.
Consider the equation 2x + 4 = 0
2x = -4

$$x = \frac{-4}{2}$$

x = -2
Since this equation is independent of y, for all values of y, x = -2
Hence x = -2 is a line parallel to the y-axis at x = -2.



Consider the equation y - 5 = 0
y - 5 = 0
y = 5
Since this equation is independent of x, for all values of x, y = 5.
Hence the graph of y = 5 is a line parallel to the x-axis at y = 5.



The graph of x = a is a line parallel to y-axis at x=a. The graph of y=b is a line parallel to x-axis at y = b.

Linear equations in two variables

A linear equation in two variables is of the form ax + by = c, where $a \neq 0, b \neq 0.$

Example:

2x + 3y = 5, x - 2y = 6, -6x + y =8
A pair of values of x and y that satisfy a given linear equation in two variables is said to be its solution.

Graph of a Linear Equation in two variables

To plot the graph of a linear equation in two variables

- Re-write the given equation expressing one term in terms of the other.
- Draw the x and y - axes. Choose a suitable scale so as to locate the selected point on the graph.
- Plot the selected points.
- Draw a straight line joining the points.

Plot the graph of
2x+3y=9
2x=9-3y

$$x = \frac{9-3y}{2}$$

(Expressing one variable in terms of the other)

x	3	6	-6
y	1	-1	7

Put y = 1,

$$x = \frac{9-3(1)}{2} = \frac{9-3}{2} = \frac{6}{2} = 3$$

x=3

Put y = -1,

$$x = \frac{9-3(-1)}{2} = \frac{9+3}{2} = \frac{12}{2} = 6$$

x=6

Put y = 7,

$$x = \frac{9-3(7)}{2} = \frac{9-21}{2} = \frac{-12}{2}$$

x=-6