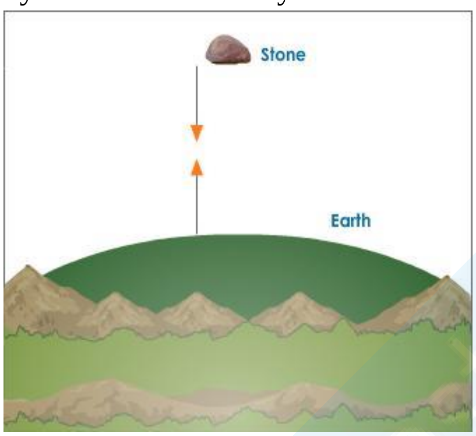
Chapter 10 - Force of Gravitation

Drop a stone from a height. What do you observe? The stone which was initially at rest starts moving towards the ground and attains a maximum speed just before it touches the ground. The stone is not moving with uniform speed. Its speed keeps changing at every instant, which means that the stone is accelerating.

According to Newton's second law of motion, a force is required to produce an acceleration in a body. No force was exerted on the stone. Where did the force come from? The solution for this problem was given by Sir Isaac Newton, when he saw an apple falling from a tree. His argument was that the Earth attracts the apple and the apple attracts the Earth. The force exerted by the Earth on the apple is very large and hence the apple lands on Earth. Whereas the apple is not able to pull the Earth, because the force exerted by it on the Earth is very small. Thus we can conclude that the acceleration produced in the stone is due to the large force of attraction exerted by Earth which is very massive.



A Stone Falling Towards the Earth

It is clear from the above example that it is this force of attraction that holds our complex universe together, keeps the moon revolving around the Earth and holds all the planets in its orbit around the Sun and it is this force which helps us to walk properly on the surface of the Earth. This type of force of attraction which exists between any two objects in the universe is known as force of gravitation or gravitation.

The force of attraction or force of gravitation between Earth (or any planet) and any other material objects in the universe is known as force of gravity or gravity.

The Universal Law of Gravitation or Newton's Law of Gravitation

Sir Isaac Newton gave a mathematical relation to calculate the force of gravitation and this relation is known as the universal law of gravitation.

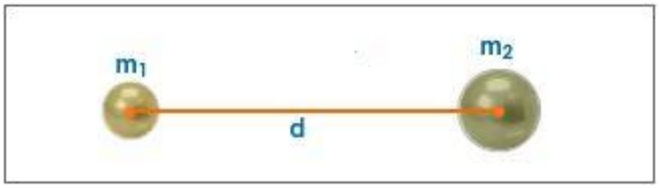
According to this law “Every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, the direction of the force being along the line joining the masses".

Consider two objects of masses m_1 and m_2 separated by a distance d. According to the law of gravitation, the force of gravitation F is proportional to the product of the masses,

$F \propto m_1 m_2$ ...(1)

and inversely proportional to the square of the distance between the masses,

$$F \propto \frac{1}{d^2} \quad ...(2)$$



Two Objects of Masses m_1 and m_2 Separated by a Distance d

Inversely proportional is always represented as directly proportional to the reciprocal of that quantity.

Combining equation (1) and equation (2), we get

$$F \propto \frac{m_1 m_2}{d^2}$$

$$F \propto \frac{G m_1 m_2}{d^2} \quad ...(3)$$

Where G is a constant of proportionality called the universal gravitational constant. G is called universal constant because its value remains the same throughout the universe and is independent of masses of the objects.

Universal Gravitational Constant

The mathematical form of Newton's Law of Gravitation is

$$F = \frac{G m_1 m_2}{d^2}$$

If $m_1 = m_2 = 1$, $d = 1$, then

$$F = \frac{G \times 1 \times 1}{1^2}$$

or F = G

Hence universal gravitational constant may be defined as the force of gravitation existing between two unit masses separated by unit distance.

SI unit of gravitational constant

$$G = \frac{Fd^2}{m_1 m_2}$$

SI unit of force F is N, SI unit of distance is metres and that of mass is kg.

$$\text{SI unit of G} = \frac{Nm^2}{kg \times kg}$$

SI unit of G = N m^2/kg^2 or N m^2 kg^{-2}

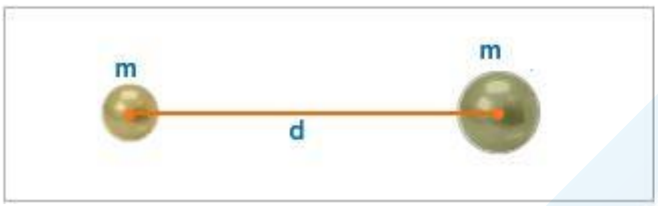
The experimental value of G equal to 6.6734 x 10^{11} Nm^2/kg^2 was measured by Sir Henry Cavendish in 1798.

Dependence of Gravitational Force on Mass

According to Newton's law of gravitation, the force of attraction is directly proportional to the mass of the body.

Consider two objects of mass m separated by a distance d, then the force between them is given by the relation

$$F_1 = \frac{Gmm}{d^2} = \frac{Gm^2}{d^2}$$



Two Objects of Mass m Separated by a Distance d

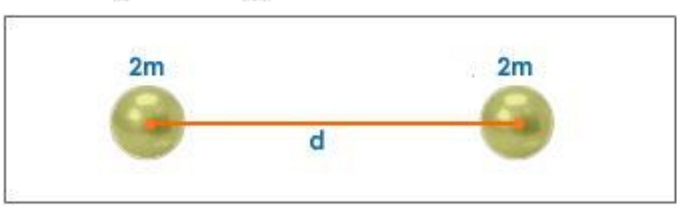
When the mass of one of the two objects is doubled, force of attraction is given by the relation

$$F_2 = \frac{Gm_2 m}{d^2} = \frac{2Gm^2}{d^2} = 2F_1$$

Two Objects of Mass m and 2m Separated by a Distance d

When the masses of both bodies are doubled, the force of attraction is

$$F_3 = \frac{G_2 m_2 m}{d^2} = \frac{4Gm^2}{d^2} = 4F_1$$



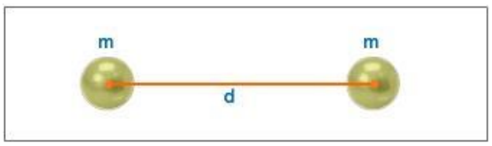
Two Objects of Mass 2m Separated by a Distance d i.e., whenever the mass increases the force of attraction also increases.

Dependence of Gravitational Force on Distance

According to the universal law of gravitation, the force of attraction between two bodies is inversely proportional to the square of the distance between the objects.

Force of attraction between two bodies of mass m separated by a distance d:

$$F_1 = \frac{Gm_1m_2}{d^2} = \frac{Gmm}{d^2}$$

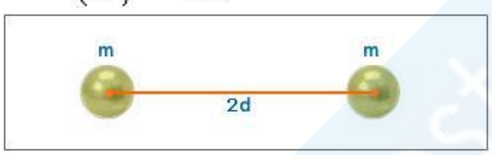


Two Bodies of Mass m Separated by a Distance d

$$F_1 = \frac{Gm^2}{d^2}$$

The force of attraction when the distance is doubled:

$$F_2 = \frac{Gmm}{(2d)^2} = \frac{Gm^2}{4d^2}$$



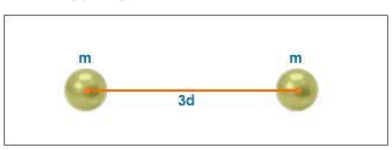
Two Bodies of Mass m Separated by a Distance 2d

$$F_2 = \frac{1}{4}\frac{Gm^2}{d^2}$$

$$F_2 = \frac{1}{4}F_1$$

Force of attraction when the distance between the bodies is increased three times:

$$F_3 = \frac{Gmm}{(3d)^2} = \frac{Gm^2}{9d^2}$$



Force of Attraction between Two Bodies of Mass m

Separated by a Distance 3d

$$F_3 = \frac{1}{9}\frac{Gm^2}{d^2}$$

$$F_3 = \frac{1}{9}F_1$$

That is, when the distance is doubled, the force will be reduced to 1/4th of the original value of force and when the distance is increased three times, the force will be reduced to 1/9th of the original value of force. From the above example, we can arrive at the conclusion that the force of attraction between the bodies varies inversely as the square of the distance between them.

Gravitational Force between two Light Objects

Let us now calculate the force of gravitation existing between two unit masses separated by a unit distance.

m_1 = 1 kg

m_2 = 1 kg

d = 1m

G = 6.6734 x 10^{-11} Nm^2/kg^2

$$F = \frac{Gm_1m_2}{d^2}$$

$$F = \frac{6.6734 \times 10^{-11} \times 1 \times 1}{1^2}$$

$$F = 6.6734 \times 10^{-11}\ N$$

This is a very weak force.

To find Force existing between two objects of masses 60kg and 100kg separated by a distance of 1m:

$$F = \frac{Gm_1m_2}{d^2}$$

$$F = \frac{6.6734 \times 10^{-11} \times 60 \times 100}{1^2}$$

$$F = 6.6734 \times 10^{-8} \times 6$$

F = 40.04 x 10^{-8} N

This is also a very weak force.

From the above two examples, it is clear why we do not experience the force exerted by one object (on the surface of Earth) on the other.

Gravitational Force Between Massive Objects

Let us now calculate the force of attraction between an object of mass 50 kg and Earth. The mass of the Earth is about 6 x 10^{24}kg. The distance between the Earth and the object is approximately 64 x 10^5m. The force of attraction between the object and the Earth

$$F = \frac{Gm_1m_2}{d^2}$$

$$= \frac{6.6734 \times 10^{-11} \times 6 \times 10^{24} \times 50}{\left(64 \times 10^5\right)^2}$$

$$= \frac{6.6734 \times 6 \times 5 \times 10^{-11} \times 10^{25}}{(64)^2 \times 10^{10}}$$

$$= \frac{6.6734 \times 6 \times 5 \times 10^{-11} \times 10^{25} \times 10^{-10}}{64 \times 64}$$

$$= \frac{6.6734 \times 6 \times 5 \times 10^4}{64 \times 64}$$

$$= \frac{6.6734 \times 3 \times 10^5}{64 \times 64}$$

$= 488.7$ N

This force is strong, we cannot ignore it.

Now let us calculate the force of attraction between Earth and the Sun.

The mass of the Earth = 6×10^{24} kg

The mass of the Sun = 1.99×10^{30} kg

The distance between the Earth and the Sun = 15×10^{10}m

$$\therefore \quad F = \frac{Gm_1m_2}{d^2}$$

$$F = \frac{6.6734 \times 10^{-11} \times 6 \times 10^{24} \times 1.99 \times 10^{30}}{\left(15 \times 10^{10}\right)^2}$$

$$F = \frac{6.6734 \times 6 \times 1.99 \times 10^{-11} \times 10^{24} \times 10^{30} \times 10^{-20}}{225}$$

$$F = \frac{6.6734 \times 6 \times 1.99 \times 10^{23}}{225}$$

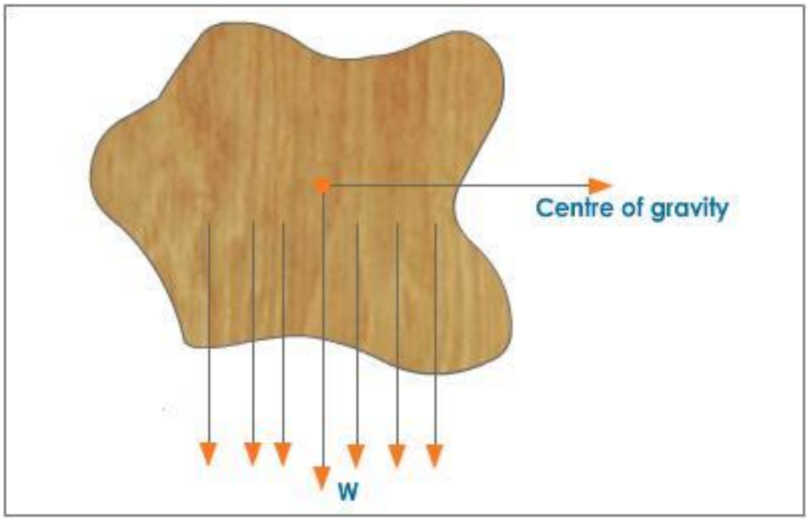
$= 3.541 \times 10^{22}$ N

This force is very large and it is this force which keeps the planets in their respective orbits.

Conclusion: Gravitational force between two objects of ordinary size is very small, while the force is very large when at least one of the objects is massive

Centre of Gravity

We know that the earth attracts every particle towards the centre. A body can be considered to be made up of number particles. As the size of the body is small when compared to that of the earth, the gravitational pull acting on these particles can be regarded to be parallel to each other as shown in the figure.



Centre of Gravity of an Irregular Lamina

A single force passing through a fixed point called the centre of gravity of the body can replace these parallel forces acting vertically in the downward direction. The resultant force is equal to the weight of the body.

Thus centre of gravity is the point through which the weight of the body acts irrespective of the position of the body.

For bodies which are of regular shape and which have uniform density, the centre of gravity lies at the geometrical centre of the body.

Application of Newton's Law of Gravitation

One of the important applications of Newton's law is to estimate masses of binary stars. A binary star is a system of two stars orbiting round their common centre of mass.

Any irregularity in the motion of a star indicates that it might be another star or a planet going round the stars. This regularity in the motion of a star is called a wobble.

Mass and Weight

Mass and weight are commonly mistaken as the same, but they are two different quantities. Now let us try to find out the differences between them.

Mass of a body is **defined** as the **amount of matter** contained in it. The SI unit of mass is kilogram (kg). Mass is a scalar quantity. The amount of matter contained in a body does not change with time or from place to place i.e., mass of a body remains the same throughout the universe. However, two different bodies can have different masses. Mass of a body is measured using a pan balance.

Weight is defined **as the force** with **which an object** is **pulled** towards **the centre** of the **Earth**.

Weight of a body = force exerted by the Earth = mg (according to Newton's second law of motion)

W = mg

SI unit of weight is Newton.
For example, the weight of a body having a mass of 1kg is
W = mg
W = 1×9.8 = 9.8 N
We know that kg. wt. is commonly used as the unit of weight.
1 kg weight is the **force with which an object** of **mass 1kg** is pulled **towards the Earth.**
W = mg
1 kg wt = 1×9.8 = 9.8 N
1 kg wt = 9.8 N
Weight is measured using spring balance. Weight varies from place to place as it depends on acceleration due to gravity. A body weighs more at the poles than at the equator and a body's weight will become zero at the centre of the Earth as acceleration due to gravity is zero at the center of the Earth

Difference between Mass and Weight

Mass	Weight
It is the amount of matter contained in an object	It is the force with which an object is pulled towards the Earth
The mass of a body is constant throughout the universe	Weight varies from place to place as **g** varies
Mass can never be equal to zero	Weight can be equal to zero
Mass is a scalar quantity	Weight is a vector quantity
SI unit of mass is kg	SI unit of weight is newton

To show that weight of a body on moon is 1/6th its weight on Earth
Let m be the mass of a body on Earth. Its weight on Earth is given by the equation

$$W_e = mg_e$$

$$W_e = \frac{mGM_e}{R_e^2} \quad ----(1) \quad \left(g_e = \frac{GM_e}{R_e^2}\right)$$

The weight of the same body on moon (W_m) is given by,

$$W_m = mg_m$$

$$W_m = \frac{mGM_m}{R_m^2} \quad ----(2) \quad \left(g_m = \frac{GM_m}{R_m^2}\right)$$

Dividing equation (2) by equation (1) we get,

$$\frac{W_m}{W_e} = \frac{mGM_m}{R_m^2} \times \frac{R_e^2}{mGM_e}$$

$$\frac{W_m}{W_e} = \frac{M_m R_e^2}{R_m^2 M_e}$$

$$\frac{W_m}{W_e} = \left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)^2 \quad ----(3)$$

But we know that $M_e = 100\ M_m$ and $R_e = 4\ R_m$.

$$\therefore \quad \frac{M_m}{M_e} = \frac{1}{100} \text{ and } \frac{R_e}{R_m} = 4$$

Substituting the values of $\frac{M_m}{M_e}$ and $\frac{R_e}{R_m}$ in equation (3), we get

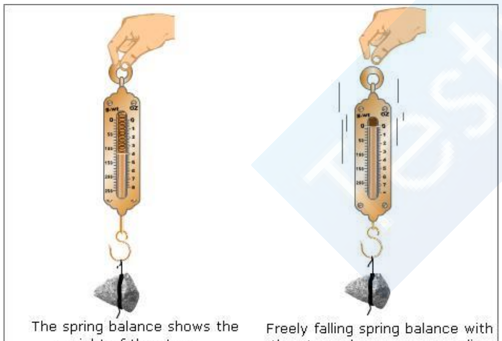
$$\frac{W_m}{W_e} = \frac{1}{100} \times 4^2 = \frac{16}{100}$$

$$\frac{W_m}{W_e} = \frac{4}{25} = \frac{1}{6.25} = \frac{1}{6}$$

$W_m = \frac{1}{6} W_e$ i.e., the weight of a body on moon is $1/6^{th}$ its weight on Earth.

Weightlessness

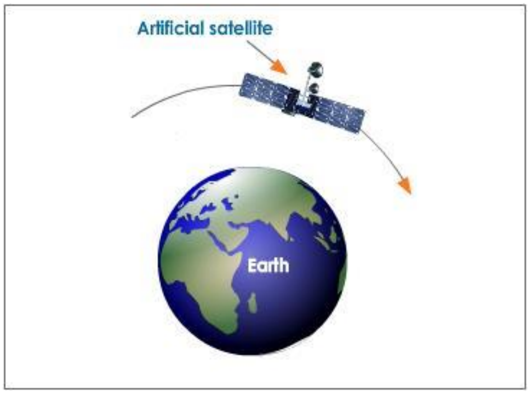
We often hear that an astronaut experiences weightlessness in space. What does this mean? Let us perform a simple experiment to demonstrate weightlessness. Suspend a stone from a spring balance, the pointer of the spring balance shows the weight of the stone. Allow the spring balance along with the stone to fall freely. The spring balance records zero weight indicating that the stone is weightless. Does it mean that the weight of the stone is zero? No, actually the stone is in the state of weightlessness as it is falling freely. A body becomes conscious of the weight, whenever its weight is opposed by some other object.



The spring balance shows the weight of the stone

Freely falling spring balance with the stone shows a zero reading

Now, let us try to explain why an astronaut in a spaceship experiences weightlessness. When the astronaut in the spaceship is orbiting the Earth, then both, the astronaut and the spaceship are in a state of free fall towards the Earth. During a free fall, both travel downwards with the same acceleration, equal to the acceleration due to gravity. As a result, the astronaut does not exert any force on the sides or floor of the spaceship, and the sides and floor of the spaceship do not push the astronaut up. The astronaut therefore experiences weightlessness while orbiting around the Earth in a spaceship.



A Satellite / Spaceship Orbiting the Earth

Density

0.5 kg of cotton occupies more space than 0.5 kg of iron. The particles of iron are closely packed while that of cotton is loosely packed. The amount of iron packed in a unit volume is more. This explains as to why iron is heavier than the same volume of cotton.

In Physics, we describe the lightness or heaviness of different substances by using the word density.

The density of a substance is defined as the mass of the substance per unit volume.

Density = $\frac{\text{mass of the substance}}{\text{volume of the substance}}$

D = $\frac{M}{v}$.

Where, 'D' represents the density, 'M' mass and 'V' volume.

SI unit of density is kg/m^3.

The density of a substance remains the same under certain specified conditions. Thus the density of a substance is one of its characteristic properties and this property can be used to determine the purity of any substance.

Relative Density of a Substance

In order to determine the density of a substance or an object, we find out the

Mass and volume of the substance and use the relation D = $\frac{M}{v}$. This is possible only if the object has a regular shape.

It is difficult to measure the dimensions an object with irregular shape. In such cases we express the density of the object in comparison with that of water.

The relative density of a substance is the ratio of the density of the substance to the density of water at 4^oC. We take the relative density of water as one.

What is meant by the statement relative density of gold is 19.3?

It means that gold is 19.3 times denser than an equal volume of water. Those objects whose relative density is less than one will float in water and those greater than one will sink.

$$\text{Relative density of a substance} = \frac{\text{density of the substance}}{\text{density of water at } 4^{0}\text{C}}$$

$$= \frac{\left(\dfrac{\text{mass of the substance}}{\text{volume of the substance}}\right)}{\dfrac{\text{mass of water}}{\text{volume of water}}}$$

$$= \frac{\text{mass of the substance}}{\text{volume of the substance}} \times \frac{\text{volume of water}}{\text{mass of water}}$$

Now, if we take equal volumes of the substance and of water, then the above becomes,

$$\text{Relative density of a substance} = \frac{\text{mass of the substance}}{\text{mass of an equal volume of water}}$$

As the relative density is a ratio of two similar quantities it has no unit.

Thrust and Pressure

In the beginning of this chapter, we have defined the force as an external agent, which changes the direction of motion, speed or shape of the body. All along we were discussing only about the forces acting at a point on a body. Let us now consider the forces acting over an area.

Suppose you have to fix a poster on your class bulletin board, you have to apply force on the head of the drawing pin, i.e., the force is perpendicular to the surface of the bulletin board and this force which is acting perpendicular to the surface is called thrust.

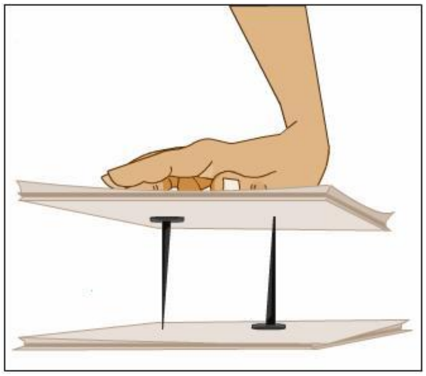
Force Applied is Perpendicular to the Board

Thrust is the force acting on a body perpendicular to its surface.

S.I unit of thrust is Newton (N).

Now let us see whether there is a relation between the force applied (thrust) and the area on which it is acting.

Hold a pin erect on a pile of papers. Keep another pin next to it upside down in such a way that its flat head rests on the pile. Press both these pins down by placing a flat object such as duster. We observe that pin which is erect pierces through the pile of papers.



Applying Force on Both the Pins, the Erect Pin Pierces Through the Pile of Papers

This is because in the case of the erect pin the force is acting on a small area whereas in the case of the second pin the force is acting over a large area.
Hold your bag with a strap made of a thin but strong string. Now lift the same bag with another strap made of a wide cloth band. You will find that carrying a school bag with a wide cloth band is more comfortable than carrying the same with a thin strip. This is because in the second case the weight of the books is distributed over a large area of the shoulder exerting less force.

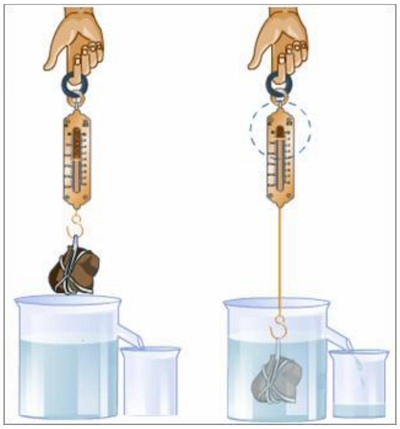
Thus from the above examples it is clear that the effectiveness of the force applied depends on the area on which it is acting.

Now there arises a need to define a new physical quantity called pressure.
Pressure is defined as the force acting on a unit area.
Pressure = $\frac{\text{force}}{\text{area}} = \frac{\text{thrust}}{\text{area}}$
S.I unit of pressure is N/m^2. N/m^2 is known as Pascal (Pa) in honor of the French Scientist Blaise Pascal.
1 N/m^2 = 1 Pascal

Pascal is a very small unit and hence often we use kilopascal. Pressure depends on two factors namely, force applied and area over which force acts.

Buoyancy and Archimedes' Principle
It is a matter of common experience that bodies appear lighter when immersed in water or any other liquid. While bathing we notice that the mug of water suddenly appears heavier as soon as it comes above the water surface. Similarly, when a fish is pulled out of water, it appears to be heavier in air than inside the water. Now let us see why it is so.

Objects appear to be less heavy in water or in any liquid because the liquid or water exerts an upward force on the objects immersed in it. Now by performing an experiment let us find out whether there is an apparent loss of weight when immersed in water.
Take a stone and tie it to one end of the spring balance. Suspend the spring balance as shown in the figure.



Experimental Set up to Prove Archimedes' Principle
Note the reading on the spring balance. Let it be W_1. Now, slowly dip the stone in the water in a container and note the reading on the spring balance. The reading shown on the spring balance goes on decreasing until it is completely immersed in water. The reading on the spring balance gives us the weight of the stone. Since the reading goes on decreasing, we can infer that the weight of the object is decreasing when it is lowered in water. The apparent loss of weight shows that a type of force is acting on the object in the upward direction thereby decreasing the weight.
Thus the upward force acting on an object immersed in a liquid resulting in the apparent loss of weight of the object is called the buoyant force.
The tendency of a liquid to exert an upward force on an object placed in it thereby making it float or rise is called buoyancy.

Factors Affecting the Buoyant Force
We know that when an iron nail is placed on the surface of water it sinks whereas ship made up of iron floats. This is because size or volume of the ship is more.
Similarly when an iron nail and a cork of some mass is placed on water, the iron nail sinks because the density of iron nail is more than the density of water and whereas density of the cork is less than that of the water. Thus if the density of the liquid is more than the density of the material of the body then the body floats due to the buoyant force exerted by it and vice-versa.

From the above examples we can infer that the buoyant force experienced by a body when submerged in a liquid depends on the volume of the body and the density of the liquid.

Archimedes' Principle

Archimedes' studied the upthrust acting on a body, when it is partially or completely immersed in a fluid by performing several experiments and then stated the following principle known as the Archimedes' Principle.

According to this principle, when a body is partially or wholly immersed in a fluid, it experiences an upthrust (buoyant force) equal to the weight of the liquid displaced.

Experiment to Verify Archimedes' Principle

Take a clean and dry beaker and find it's mass (m) using a physical balance. Now find the weight of a stone by suspending it from a spring balance. Fill an Eureka can (Eureka can is a beaker having a spout near the top) with water filled till the spout. Place the beaker of mass 'm' under the spout. Gently lower the solid, suspended from spring balance, into the Eureka can, till the stone is completely immersed in water. When the stone is immersed in water it displaces a certain amount of water. The spring balance records lesser value thereby showing that the solid experiences an upthrust. The displaced water is collected in the beaker. Using the physical balance the mass of the water and beaker is determined. Let it be m_1.

∴ Amount of water displaced = $m_1 - m$

If we compare the apparent loss of weight of the solid in water, with the amount of water displaced, it is found that they are equal. This experiment thus verifies Archimedes' Principle.

Application of Archimedes' Principle

It is used in designing ships and submarines. The lactometers and hydrometers used for measuring the purity of a sample of milk and for determining the density of the liquids are based on this principle.