Chapter 8 - Motion

Introduction

- In the physical world, one of the most common phenomena is motion. The branch of Physics, which deals with the behavior of moving objects, is known as mechanics.
- Mechanics is further divided into two sections namely Kinematics and Dynamics.
- Kinematics deals with the study of motion without taking into account the cause of motion.
- Dynamics is concerned with the cause of motion, namely force.

Motion and Rest

- An object is said to be in motion if it changes its position with respect to its surroundings in a given time.
- An object is said to be at rest if it does not change its position with respect to its surroundings.
- A frame of reference is another object or scene with respect to which we compare an object's position.

Figure 1 **Figure 2**

Look at the figures. In figure1, the car is to the right of the tree. In figure 2, after 2 seconds, the car is to the left of the tree. As the tree does not move, the car must have moved from one place to another. Therefore, here the tree is considered as the frame of reference.

Types of Motion

There are three types of motion:

- Translatory motion
- Rotatory motion
- Vibratory motion

Translatory Motion

- In translatory motion the particle moves from one point in space to another. This motion may be along a straight line or along a curved path.
- Motion along a straight line is called rectilinear motion.
- Motion along a curved path is called curvilinear motion.
- Example: A car moving on a straight road

Rectilinear Motion

Example: A car negotiating a curve

Curvilinear Motion

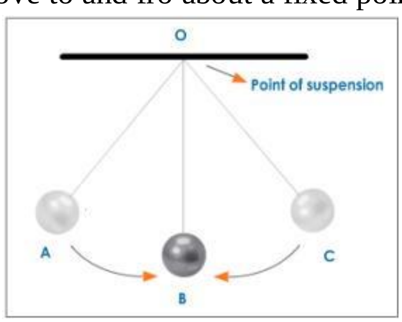
Rotatory Motion

In rotatory motion, the particles of the body describe concentric circles about the axis of motion.

Rotatory Motion

Vibratory Motion

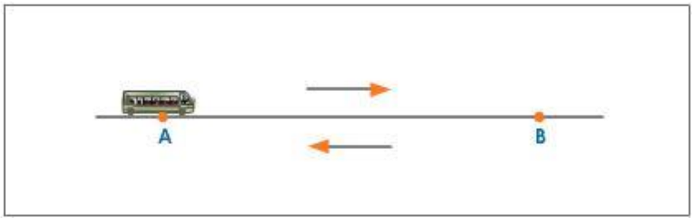
In vibratory motion the particles move to and fro about a fixed point.



Simple Pendulum

Distance and Displacement

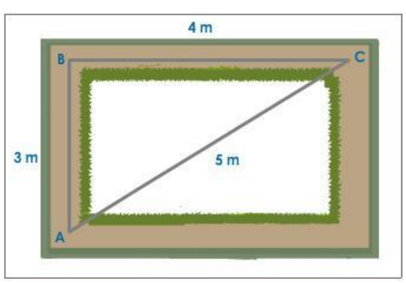
The distance between terminus A and terminus B is 150 km. A bus travels from terminus A to terminus B. The distance covered by the bus is 150 km. The bus travelling on the same route returns from terminus B to the terminus A. Thus the total distance covered by the bus during the trip from A to B and then from B to A is 150 km + 150 km = 300 km.



A Bus Moving from A to B and Again from B to A

- The distance covered by a moving object is the actual length of the path followed by the object.
- Distance is a scalar quantity. SI unit of distance is meter.
- The position of the bus changed when it moved from the terminus A to terminus B. There is a displacement of 150 km from A to B. The displacement by the return trip is also 150 km.
- Displacement is the shortest distance covered by a moving object from the point of reference (initial position of the body), in a specified direction.
- Note:
- But the displacement when the bus moves from A $\rightarrow$B and then from B $\rightarrow$A is zero. SI unit of displacement is meter.
- Displacement is a vector, i.e., the displacement is given by a number with proper units and direction.
- To drive home, the difference between displacement and distance let us consider a few more examples.

Suppose a person moves 3 meters from A to B and 4 meters from B to C as shown in the figure. The total distance travelled by him is 7 meters. But is he actually 7 meters from his initial position? No, he is only 5 meters away from his initial position i.e., he is displaced only by 5 m, which is the shortest distance between his initial position and final position.



Distance and Displacement

Note:

In this example we can make use of the Pythagoras theorem to determine the displacement.

Now let us consider an object changing its position, with respect to a fixed point called the origin 0. x_i and x_f are the initial and final positions of the object. Then the displacement of the object $= x_f - x_i$

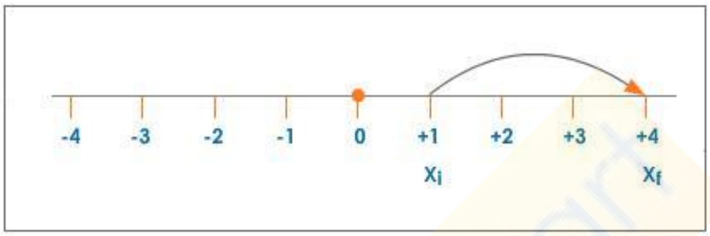
Case 1

Suppose the object is moving from +1 to +4,

then displacement $= x_f - x_i$

= +4 - (+1)

= +3



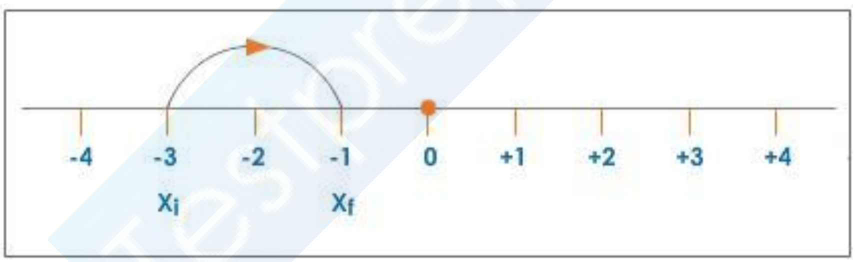
Displacement: Case1

Case 2

If the object is moving from -3 to -1, then displacement $= x_f - x_i$

= -1 - (-3)

= 2



Displacement: Case 2

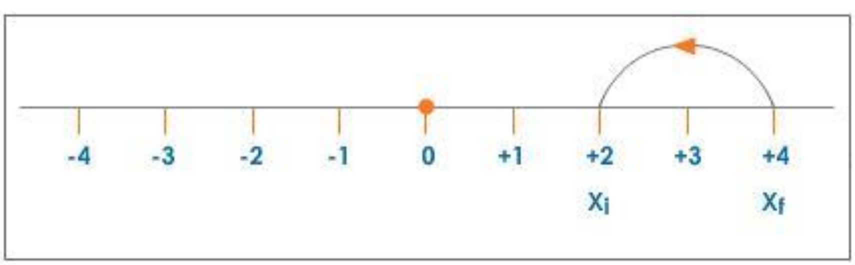
Case 3

If the object is moving from +4 to +2,

then displacement $= x_f - x_i$

= +2 - (+4)

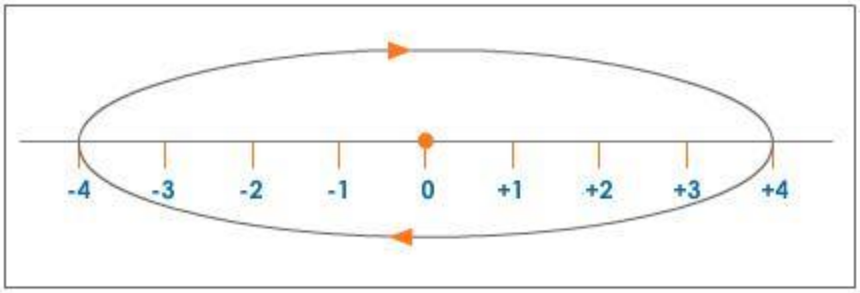
= -2



Displacement: Case 3

Case 4

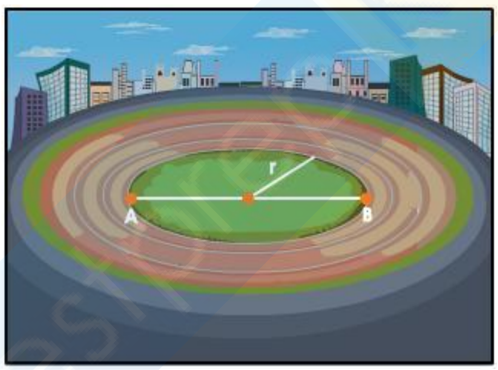
If the object follows the path as shown in the figure then the final position and the initial position are the same i.e., the displacement is zero.



Displacement: Case 4

From the above examples, we can conclude that the displacement of a body is positive if its final position lies on the right side of the initial position and negative if its final position is on the left side of its initial position. When a moving object comes back to the original position the displacement is said to be zero.

Imagine an athlete running along a circular track of radius r in a clockwise direction starting from A.



A Circular Track of Radius r

What is the distance covered by the athlete when he reaches the point B?

The distance covered by the athlete when he reaches the point B is

equal to half of the circumference of the circular track $= \frac{2\pi r}{2} = \pi r$.

Displacement = AB = 2r = Diameter of the circle (the shortest distance between the initial and final positions).

Suppose the athlete reaches the initial point A, then the distance covered is equal to the circumference of the circular track i.e., $2\pi r$. But the displacement is zero as the initial and final positions of the athlete are the same.

Difference between Distance and Displacement

Distance	Displacement
It is the actual length of the path covered by a moving object	It is the shortest distance between the initial and final positions of the moving object
It is a scalar quantity	It is a vector quantity

Motion

Uniform Motion and Non-uniform Motion

The distances covered by car A and car B with respect to time is given below.

Car A

Time in Seconds	**0**	**5**	**10**	**15**	**20**	**25**	**30**	**35**
Distance covered in meters	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**

Car B

Time in Seconds	**0**	**5**	**10**	**15**	**20**	**25**	**30**	**35**
Distance covered in meters	**0**	**10**	**15**	**20**	**30**	**60**	**65**	**75**

The car A travels equal distances in equal intervals of time whereas the car B does not cover equal distances in equal intervals of time. That is, motion of car A is an example of uniform motion whereas that of car B is that of non-uniform motion.

When a body covers equal distances in equal intervals of time then the body is said to describe uniform motion.

When a body moves unequal distances in equal intervals of time or vice-versa, then the body is said to describe non-uniform motion.

Speed

Ram and Krishna run in different races covering different distances. Ram takes 20 minutes to cover 1000 m and Krishna takes 10 minutes to cover 700 m. Who is faster?

To find out who is faster we shall calculate the distance covered by them in one minute.

$$\text{Distance covered by Ram in one minute} = \frac{1000\text{ m}}{20\text{ min}} = 50\text{ m/min.}$$

$$\text{Distance covered by Krishna in one minute} = \frac{700\text{ m}}{10\text{ min}} = 70\text{ m/min.}$$

Krishna covered more distance in unit time. We conclude that Krishna is faster. Speed can be defined as the distance covered by a moving object in unit time

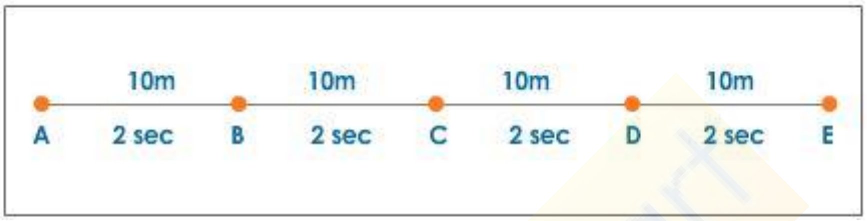
$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{S}{t}$$

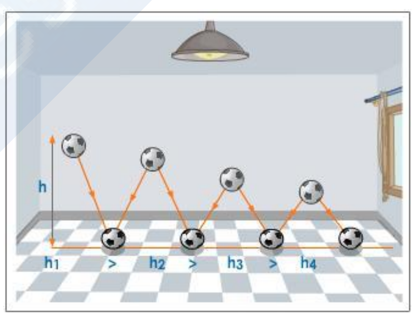
Where S is the distance covered and t is the time taken.

SI unit of speed is m/s or m s^{-1}. Speed is a scalar quantity.

Uniform Speed

The figure shows the distance covered by a ball at every 2 second interval.





Variable Speed

Example: A rubber ball dropped from a certain height (h_1) on reaching the ground bounces up to a height less than the initial one (h_2). It continues to bounce but the height to which it raises keeps decreasing (h_3, h_4). The distance covered by the ball in unit time decreases. The speed of the ball varies from point to point.

Such a speed is called a variable speed.

An object is said to be moving with variable speed or non-uniform speed if it covers equal distances in unequal intervals of time or vice-versa**.**

Average Speed and Instantaneous Speed

When we travel in a vehicle the speed of the vehicle changes from time to time depending upon the conditions existing on the road. In such a situation, the speed is calculated by taking the ratio of the total distance travelled by the vehicle to the total time taken for the journey. This is called the average speed.

If an object covers a distance S_1 in time t_1, distance S_2 in time t_2 and distance S_n in time t_n then the average speed is given by,

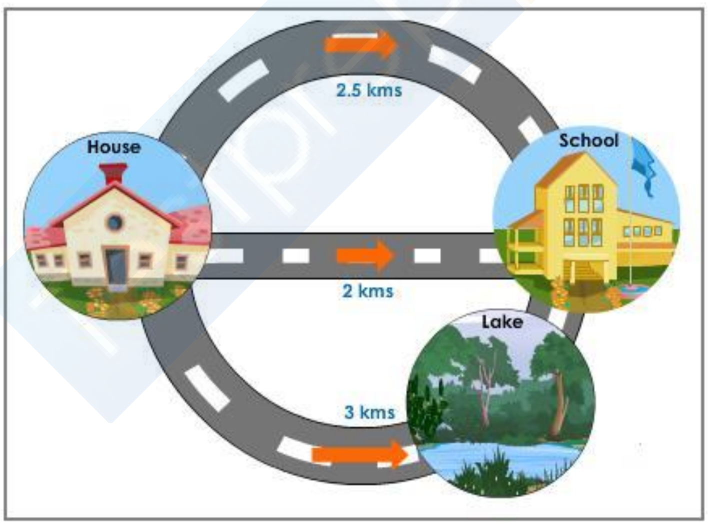
$$\text{Average speed} = \frac{S_1 + S_2 + S_3 + \ldots + S_n}{t_1 + t_2 + t_3 + \ldots + t_n} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

When we say that the car travels at an average speed of 60 km/h it does not mean that the car would be moving with the speed of 60 km/h throughout the journey. The actual speed of the car may be less than or greater than the average speed at a particular instant of time.

The speed of a moving body at any particular instant of time is called instantaneous speed.

Velocity

Figure below gives different routes that Shyam can choose from his house to school.



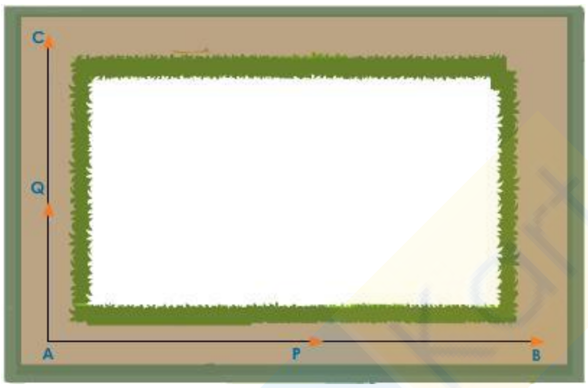
Different Routes to School

Shyam goes regularly to school by car, with an average speed of 60 km/h. Is it possible to find out the time required to reach the destination? Yes, you can definitely find out the time using the relation,

$$\text{speed} = \frac{\text{distance}}{\text{time}},$$

But you are not sure of the route which he would have taken. Thus, by just giving the speed of a moving object it is not possible to locate the exact position of the object at a given time. Thus, there arises a need to define a quantity, which has both magnitude and direction.

Consider two objects P and Q starting from A. Let them cover equal distances in equal intervals of time i.e., their speed is the same. Can you tell where each of them would be after say 20 seconds? P and Q can move in any direction. To locate the exact position of P and Q we require their direction of motion also.



Pictorial Representation of the Position of the Objects P and Q

Thus another physical quantity called velocity is introduced to give us the idea of speed as well as direction.
Velocity is defined as the distance covered by a moving object in a particular direction in unit time or speed in a particular direction.

$$\text{Velocity} = \frac{\text{distance travelled in a specified direction}}{\text{time taken}}$$

$$v = \frac{S}{t} \text{ [where 'S' is the distance covered and 't' is the time taken]}$$

SI unit of velocity is m/s (meter/second).
[$\because$ SI unit of distance is meter and that of time is second]
Velocity is a vector quantity.

Note:
Velocity is defined as the distance travelled in a specified direction in unit time. The distance travelled in a specified direction is displacement.
Therefore, velocity can be defined as the rate of change of displacement.

Uniform Velocity and Non-Uniform Velocity
Imagine that two athletes Ram and Shyam are running with a uniform speed of 5 m/s. Ram moves in a straight line and Shyam moves on a circular track. For a layman both Ram and Shyam are moving with uniform velocity but for a physicist only Ram is running with uniform velocity because there is no change in his speed and direction of motion. Whereas in the case of Shyam

who is running along a circular track the direction of motion is changing at every instant as circle is polygon with infinite sides and Shyam has to change his direction at every instant.

Thus a body is said to be moving with uniform velocity if it covers equal distances in equal intervals of time in a specified direction.

A body is said to be moving with variable velocity if it covers unequal distances in equal intervals of time and vice-versa in a specified direction or if it changes the direction of motion.

Acceleration

All of us know that a car moving on road does not have a uniform velocity. Either the speed or the direction changes. Whenever a vehicle is speeding i.e., when the speed is increased we say that the vehicle is accelerating.

To get an idea of acceleration, let us study the change in velocity of a train moving from Bangalore to Mysore. The train, which was initially at rest starts moving, its velocity slowly increases and after a certain time interval it attains a constant velocity. As the next station approaches its velocity gradually decreases and finally the train comes to rest.

When the train starts from rest its speed increases from zero and we say that the train is accelerating. After sometime the speed becomes uniform and we say that it is moving with uniform speed that means the train is not accelerating. But as the train is nearing Mysore it slows down, which means the train is accelerating in negative direction. Again the train stops accelerating when it comes to a halt at Mysore.

Thus, it is clear that the term 'acceleration' need not always mean that the speed of a moving body is always increasing, it can also decrease, remain the same or become zero.

In general, acceleration is defined as the rate of change of velocity of a moving body with time.

This change could be a change in the speed of the object or its direction of motion or both.

Now let us get a mathematical formula for calculating acceleration.

Let an object moving with an initial velocity 'u' attain a final velocity 'v' in time 't', then acceleration 'a' produced in the object is

Acceleration = Rate of change of velocity with time

$$= \frac{\text{change in velocity}}{\text{time}}$$

$$a = \frac{v - u}{t}$$

Unit of Acceleration

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}} = \frac{v}{t}$$

The SI unit of velocity is m/s and time is s

$\therefore$ SI unit of acceleration is $\frac{m/s}{s} = m/s^2$

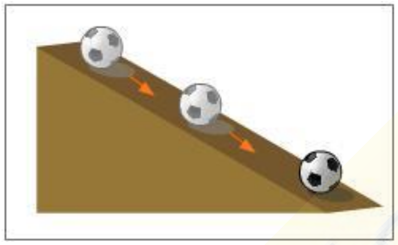
Acceleration is a vector quantity.

Different Types of Acceleration

From the above example, it is very clear that acceleration is of different types depending on the change in velocity.

Positive Acceleration

If the velocity of an object increases, then the object is said to be moving with positive acceleration.



Example of Positive Acceleration

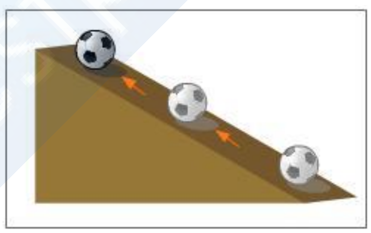
Example: A ball rolling down on an inclined plane.

Negative Acceleration

If the velocity of an object decreases, then the object is said to be moving with negative acceleration. Negative acceleration is also known as retardation or deceleration.

Example:

(1) A ball moving up an inclined plane.



Example of Negative Acceleration

(2) A ball thrown vertically upwards is moving with a negative acceleration as the velocity decreases with time.

Example of Negative Acceleration

Zero Acceleration

If the change in velocity is zero, i.e., either the object is at rest or moving with uniform velocity, then the object is said to have zero acceleration.

Example: a parked car, a train moving with a constant speed of 90 km/hr

Uniform Acceleration

If the change in velocity in equal intervals of time is always the same, then the object is said to be moving with uniform acceleration.

Example: a body falling from a height towards the surface of the earth.

Non-uniform or Variable Acceleration

If the change in velocity in equal intervals of time is not the same, then the object is said to be moving with variable acceleration.

Motion

Distance-Time Table and Distance-Time Graph

Mr. X is travelling from New Delhi to Agra in a bus and records his observations.

Distance in km	0	10	20	30	40	50	60
Time	10.00a.m	10.15a.m	10.30a.m	10.45a.m	11.00a.m	11.15a.m	11.30a.m

The above table tells us that the bus is covering equal distances in equal intervals of time. The bus is moving with uniform speed. In such an event, we can calculate the distance covered by the bus, at any given time.

Consider an object moving with uniform speed v from its initial position x_i to final position x_f in time t.

$$\text{Then, uniform speed} = \frac{\text{total distance}}{\text{time taken}}$$

$$v = \frac{x_f - x_i}{t}$$

$$x_f - x_i = vt \; \dots (1)$$

The equation (1) gives the relation between distance, time and average speed.
This relation can be used for making distance-time tables and also to determine the position of any moving object at any given time.
But it is a long and tedious process especially when we have to determine the position after a long time or when we have to compare the motion of two objects.
In such situations we can make use of graphs like distance-time graph.
A distance-time graph is a line graph showing the variation of distance with time.
In a distance-time graph, time is taken along x-axis and distance along y-axis.

Distance-Time Graph for Non - Uniform Motion

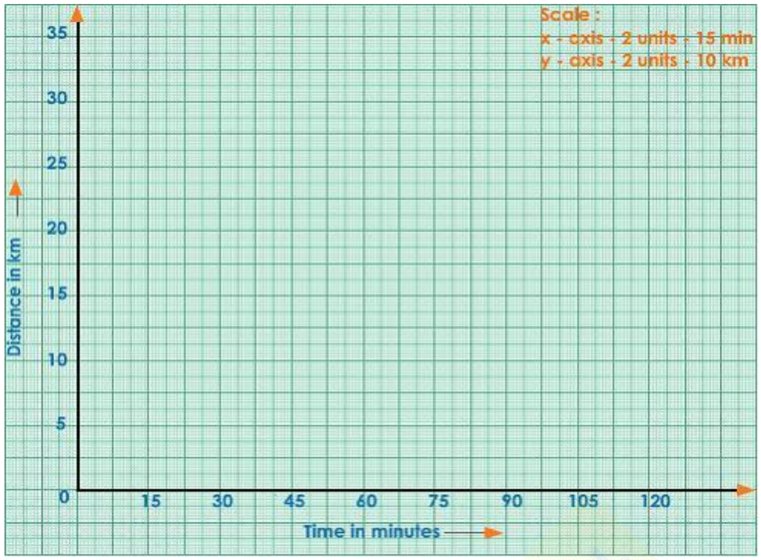
Now, let us see the nature of distance-time graph for a non-uniform motion. The following table gives the distance covered by a bus after every 15 minutes.

Distance covered in km	**0**	**5**	**15**	**20**	**25**	**30**	**35**
Time in minutes	**0**	**15**	**30**	**45**	**60**	**75**	**90**

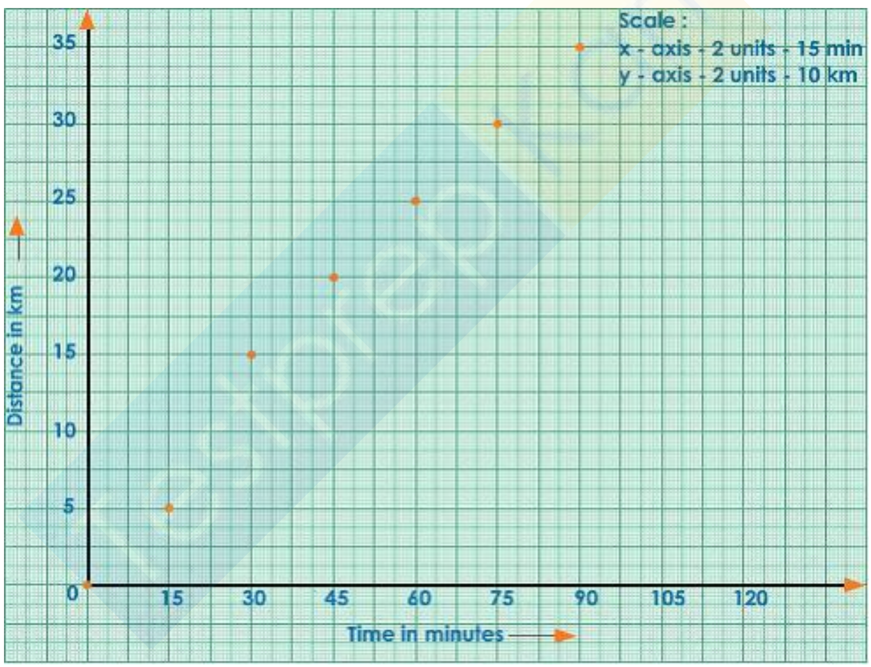
From the above table we can conclude that the motion is non-uniform i.e., it covers unequal distances in equal intervals of time.

- Take time along x-axis and distance along y-axis.

- Analyse the given data and make a proper choice of scale for time and distance.



- Plot the points.

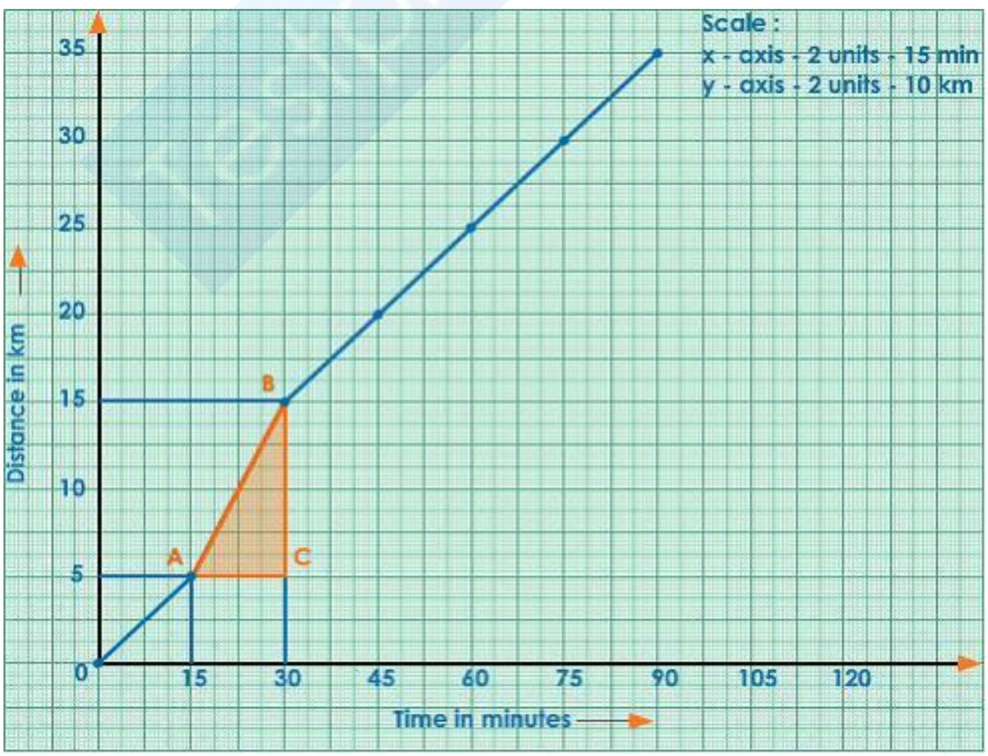


- Join the points.

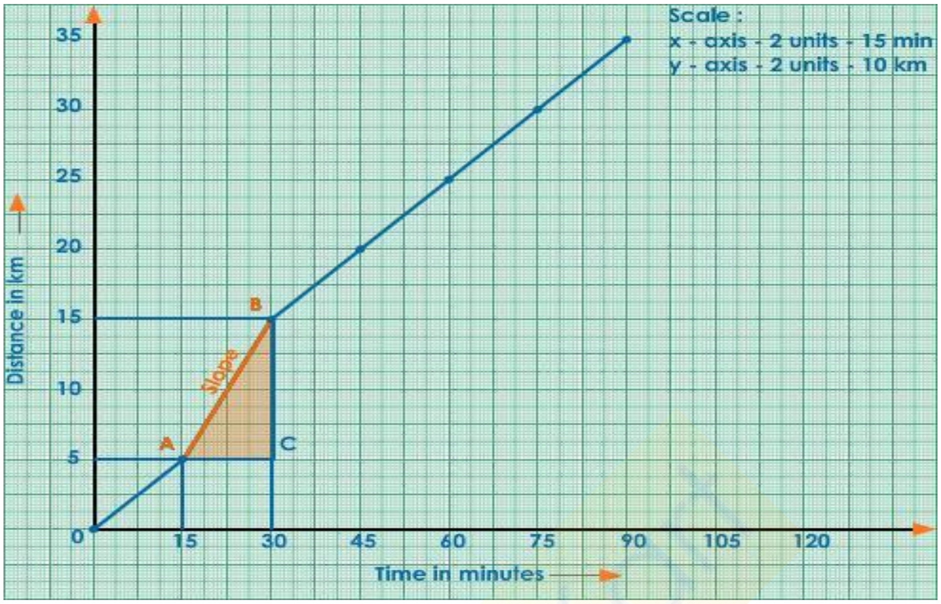
- Consider any two points (A, B) on the graph.

- Draw perpendicular from A to B to x and y axes.

Join A to C to get a right angled ACB.

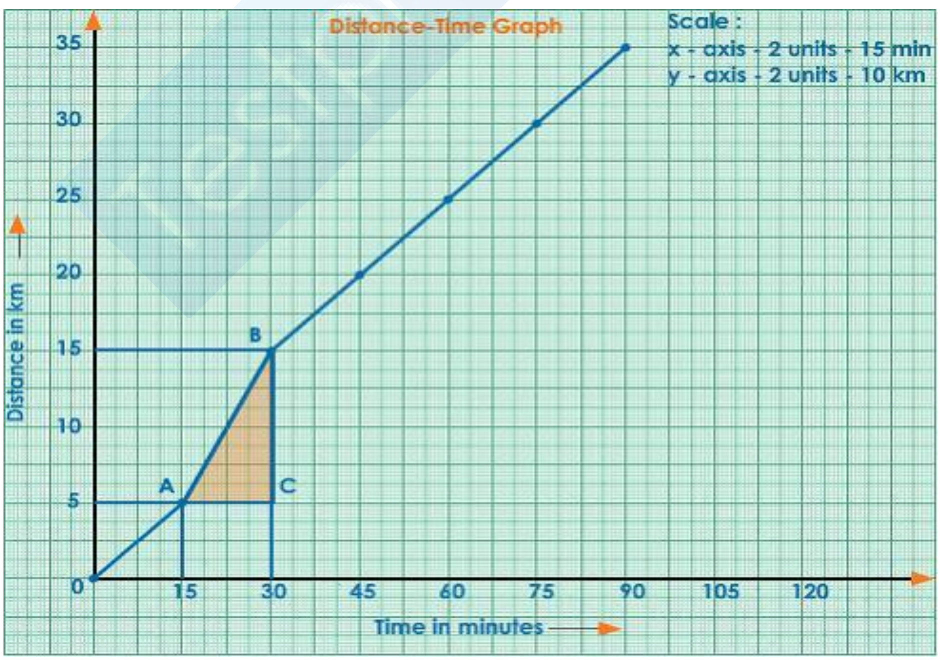


The slope of the graph, AB= $\frac{BC}{AC} = \frac{S}{t}$ = speed

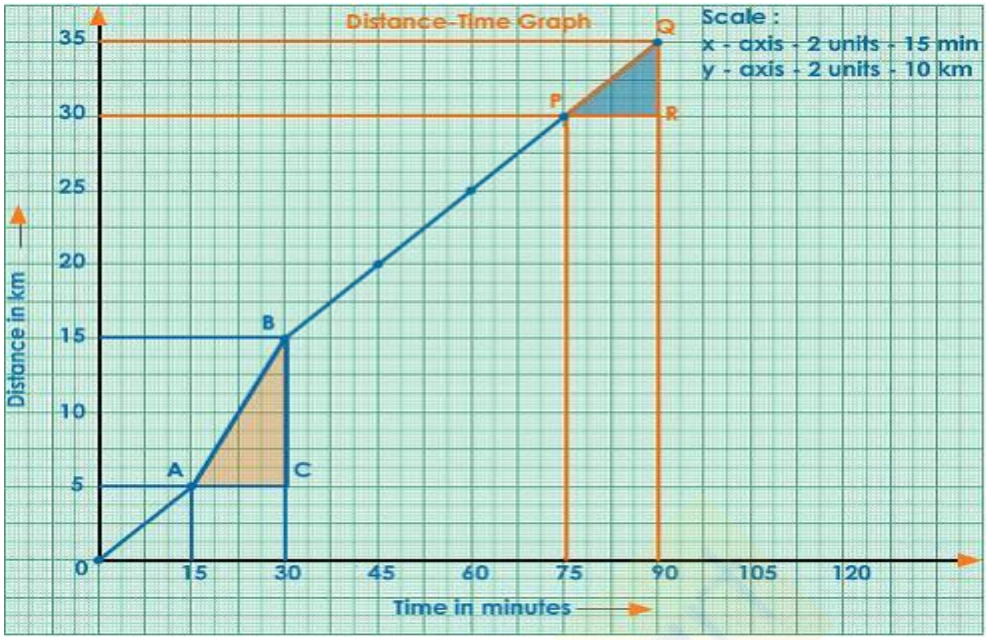


- Write the title and scale chosen for the graph.

$$speed = \frac{BC}{AC} = \frac{15-5}{30-15} = \frac{10}{15} = \frac{2}{3} = 0.666\ km/min.$$

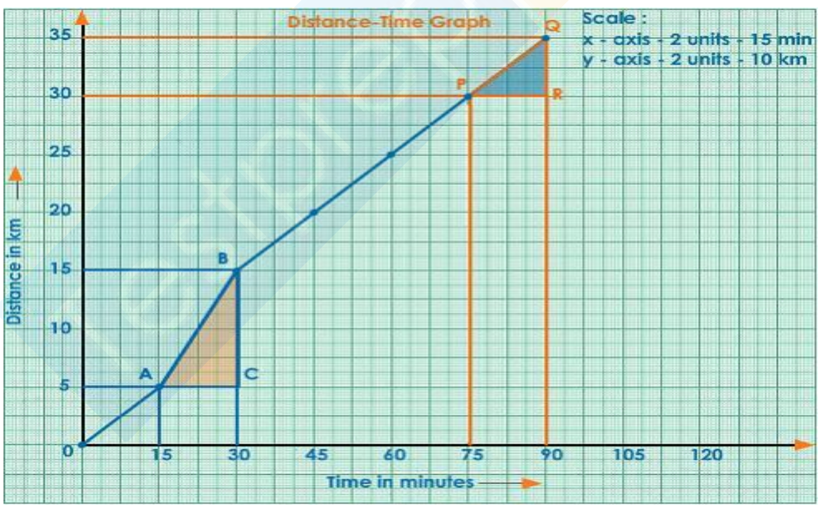


- Consider another two points P and Q on the graph and construct a right angled triangle PRQ.

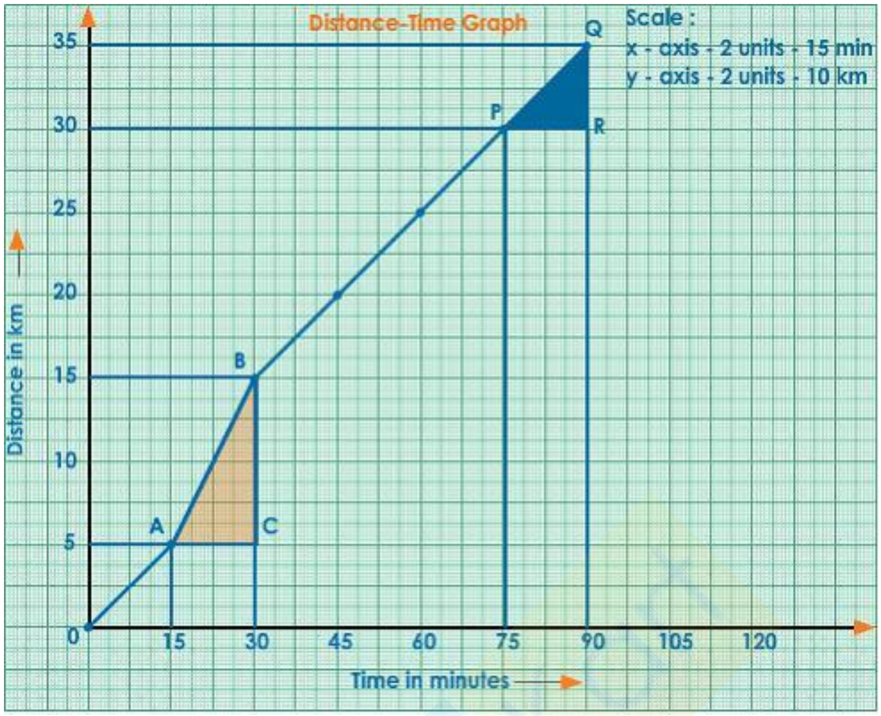


- The slope = speed= $PQ = \frac{QR}{PR} = \frac{35-30}{90-75} = \frac{5}{15} = \frac{1}{3} = 0.333 km/min$

We can infer that speed is not uniform.



- Complete Graph



Nature of S- t Graph for Non- Uniform Motion and Uses of Graphs
Let us now see the nature of S-t graph for non-uniform motion.

Nature of s-t Graph for Non-Uniform Motion
Fig (a) represents the S-t graph when the speed of a moving object increases and Fig (b) represents the S-t graph when the speed of a moving object decreases.

From the nature of S-t graph we can conclude whether the object is moving with uniform speed or variable speed.

Uses of Graphical Representation

- Graphical representation is more informative than tables as it gives a visual representation of two quantities (e.g., distance vs. time)

- At a glance a graph gives more information than a table. Both the graphs shown here represent the increasing speed.

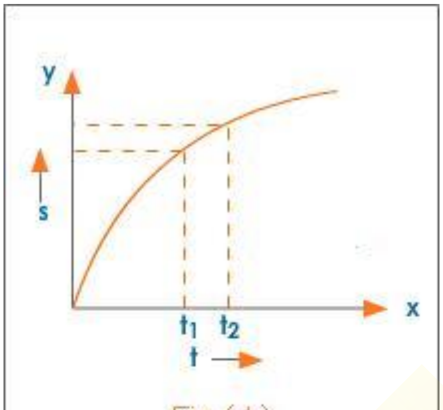


Fig (1)

Fig (1) gives us an idea of nature of variation of speed i.e., increase is greater in the beginning up to time t_1 and relatively lower after t_2.

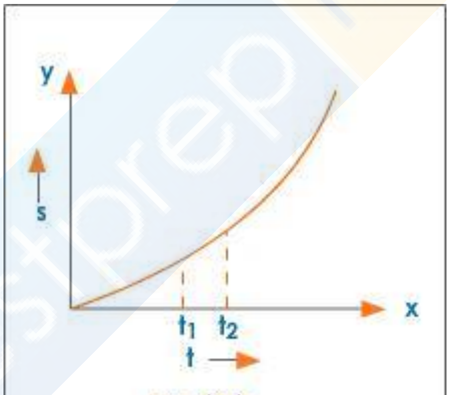


Fig (2)

Similarly, fig (2) gives an idea that the increase in the speed becomes greater after t_1. Similar explanation holds good for the decreasing speed also.

- Graphs can be easily read at a glance.
- Plotting graphs takes less time and is more convenient.
- With graphs, the position of any moving object at any intermediate point of time can be determined.
- Motion of two moving objects can be easily compared.
- Graphs tell us about the nature of motion.

Motion

Velocity-Time Graph

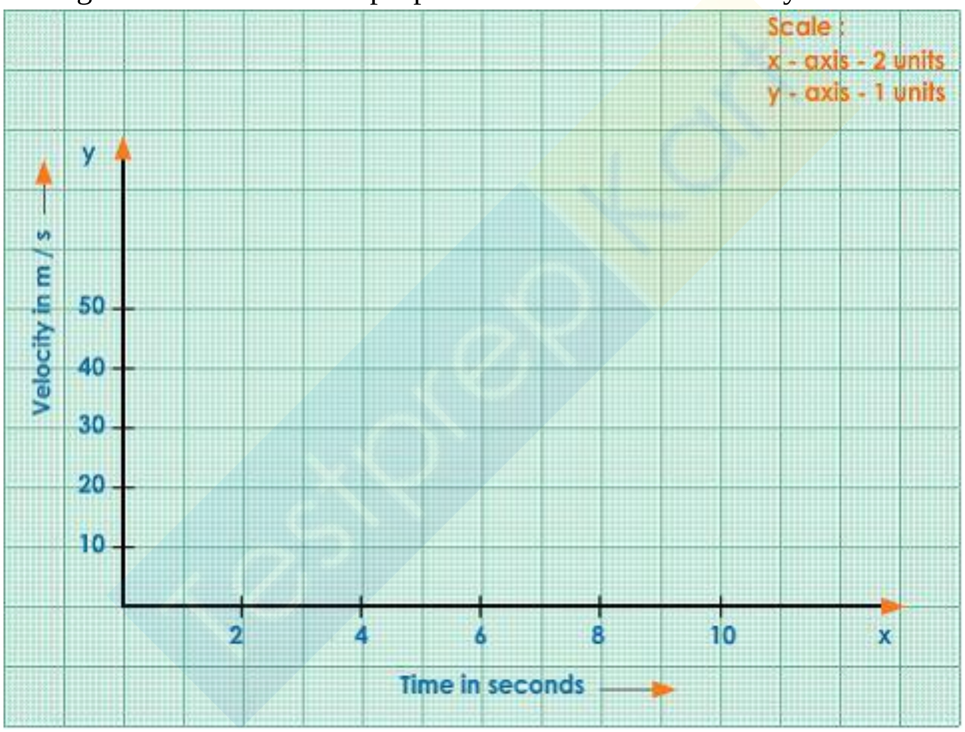
The variation of velocity with time can be represented graphically to calculate acceleration exactly like we calculated speed from distance-time graph.

Let us now plot a velocity-time (v - t) graph for the following data.

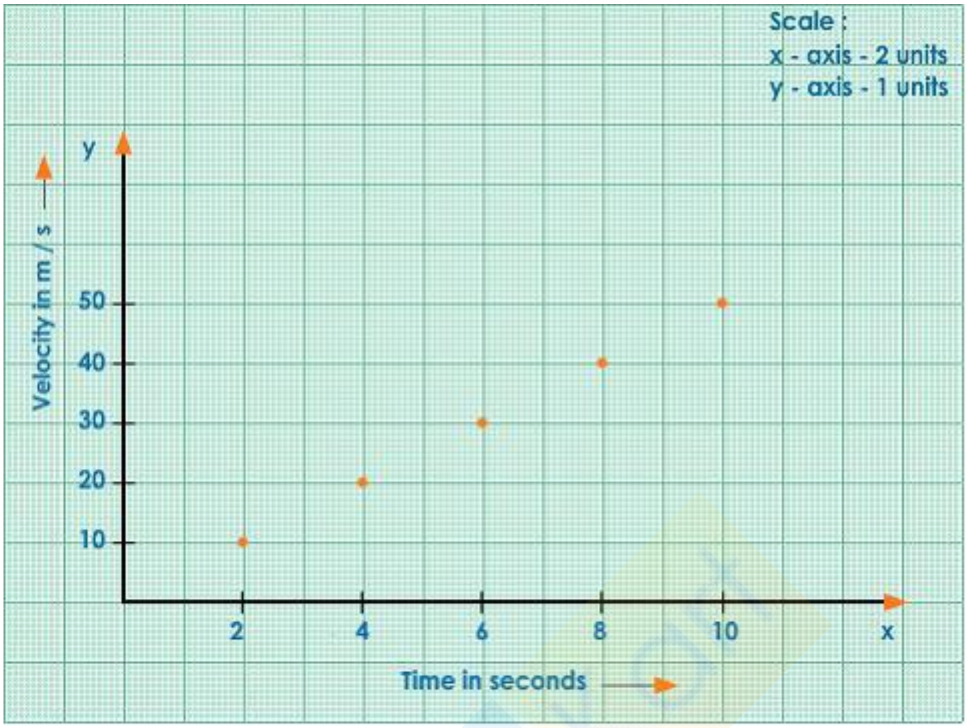
Velocity in m/s	0	10	20	30	40	50
Time in seconds	0	2	4	6	8	10

- Take time along x-axis and velocity along y-axis.

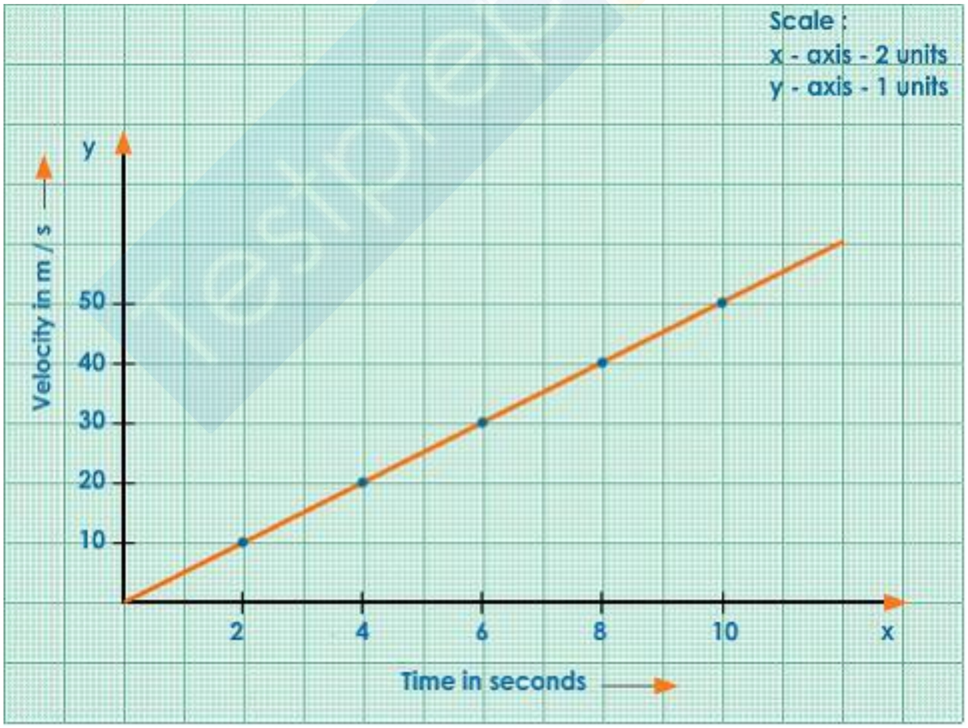
- Analyse the given data and make a proper choice of scale for x and y axes.



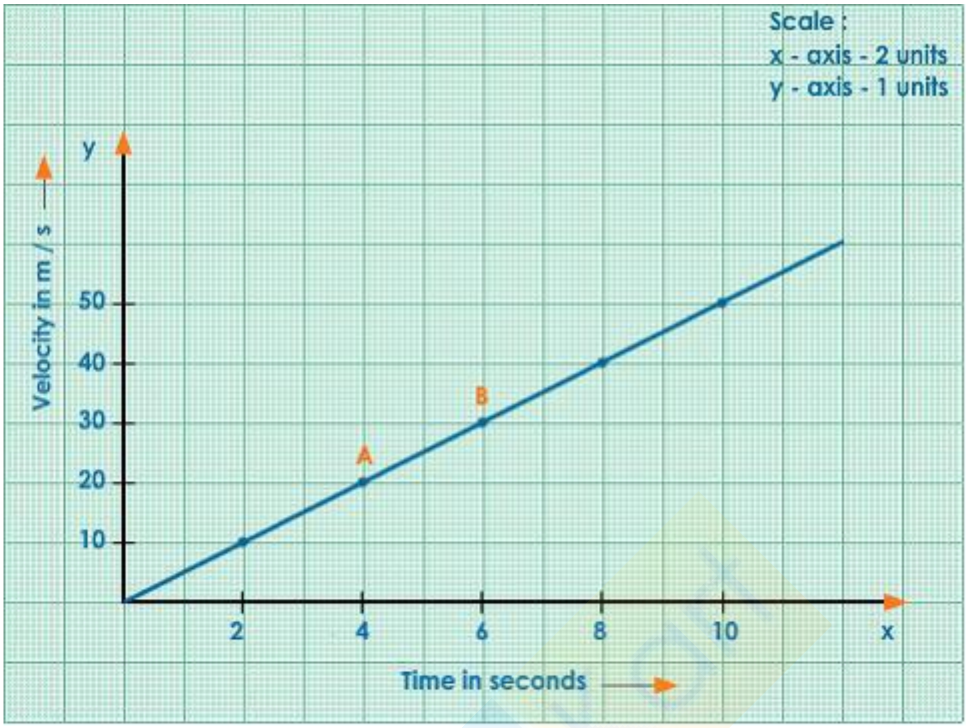
- Plot the given points.



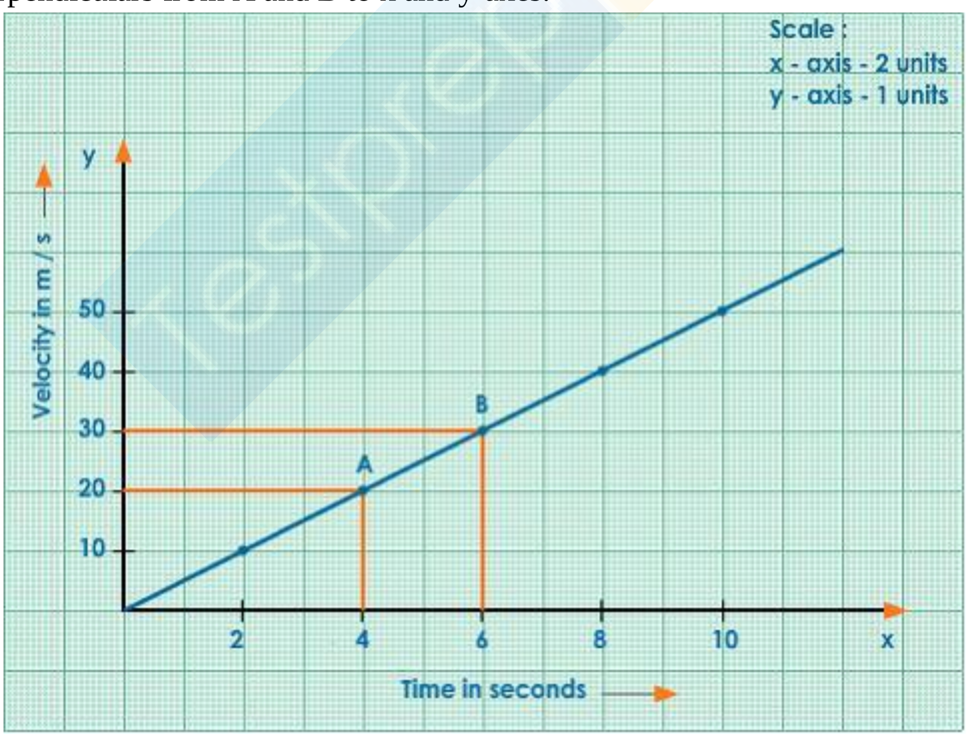
- Join the points



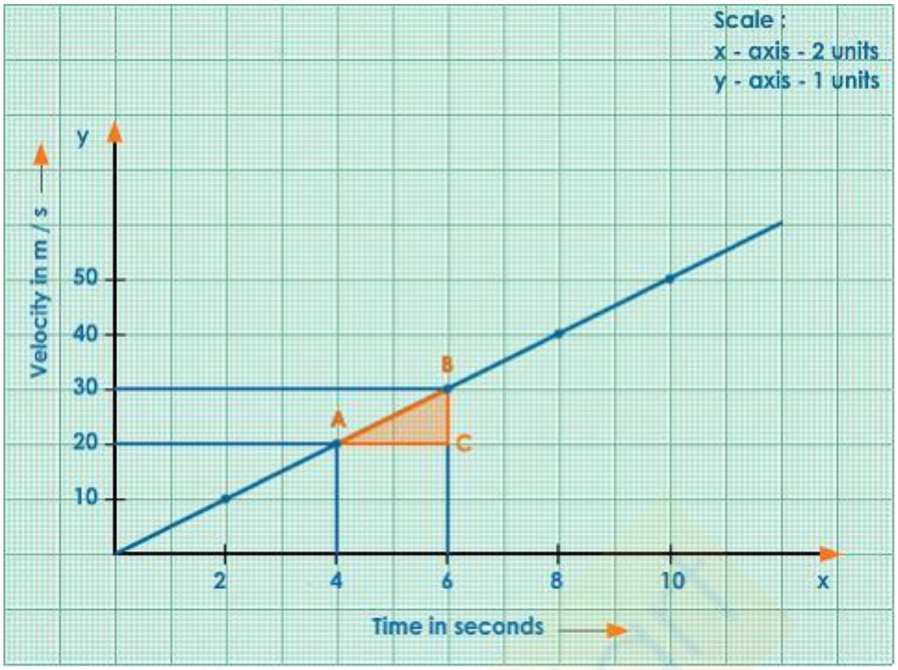
- Consider any two points A and B on the straight-line graph.



- Draw perpendiculars from A and B to x and y-axes.



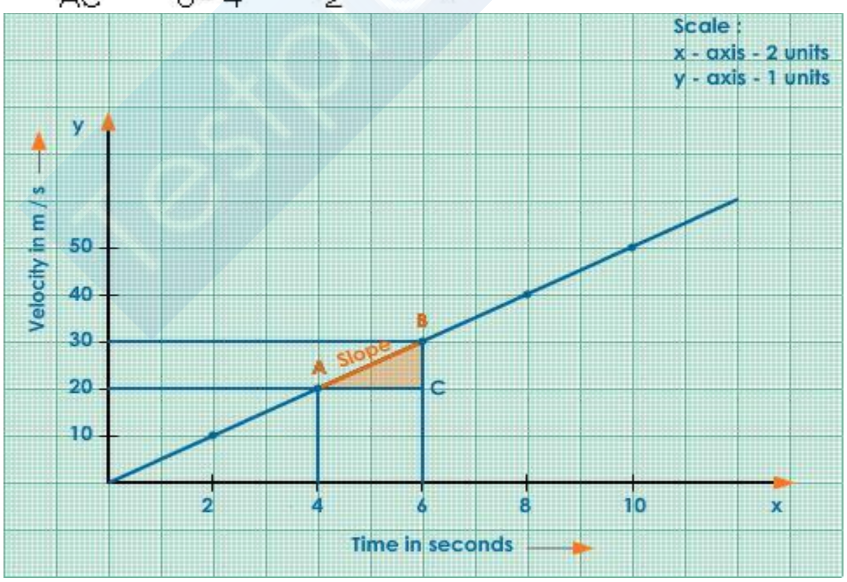
- Join A to C, ACB forms a right-angled triangle.



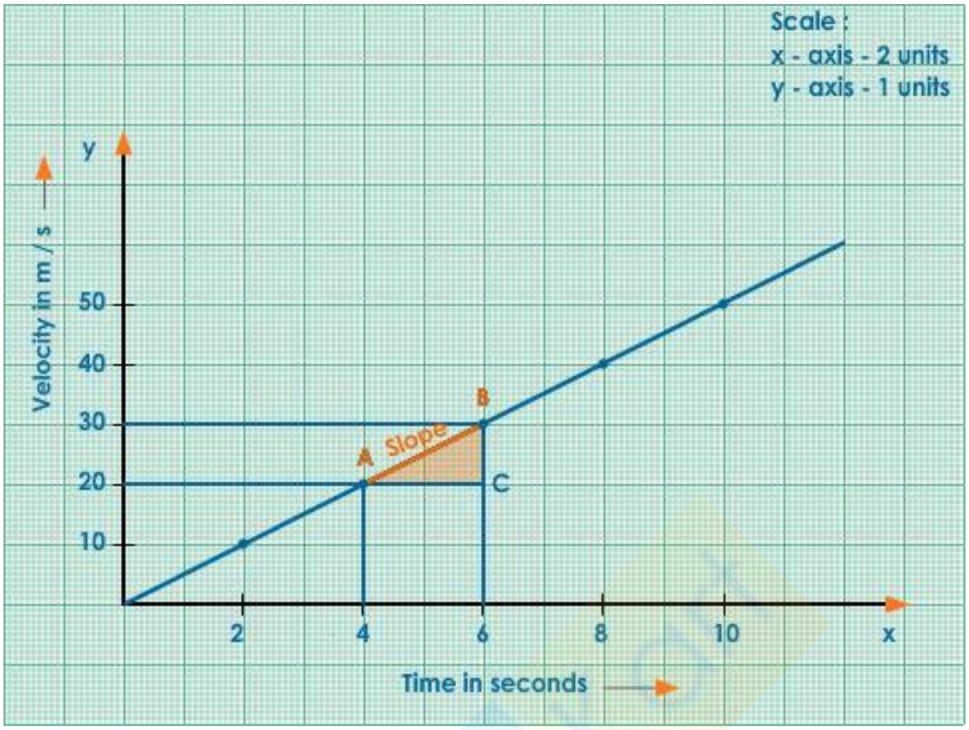
- Slope of the graph (AB) $= \frac{BC}{AC} = \frac{Change\ in\ velocity}{time} = acceleration\, f$

Calculations

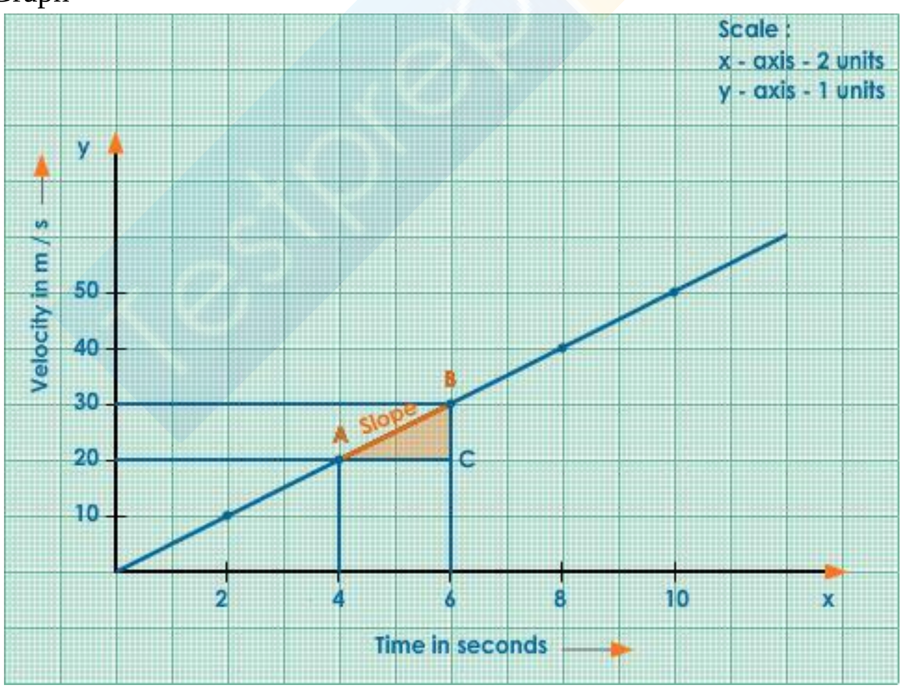
$$\text{Acceleration} = \frac{BC}{AC} = \frac{30-20}{6-4} = \frac{10}{2} = 5\,m/s^2$$



- Write the title for the graph.



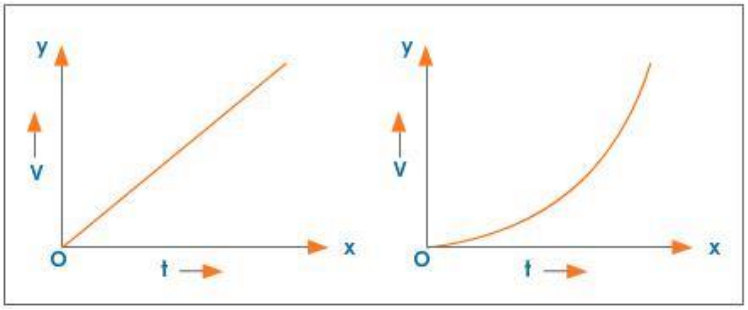
- Complete Graph



V - T Graph

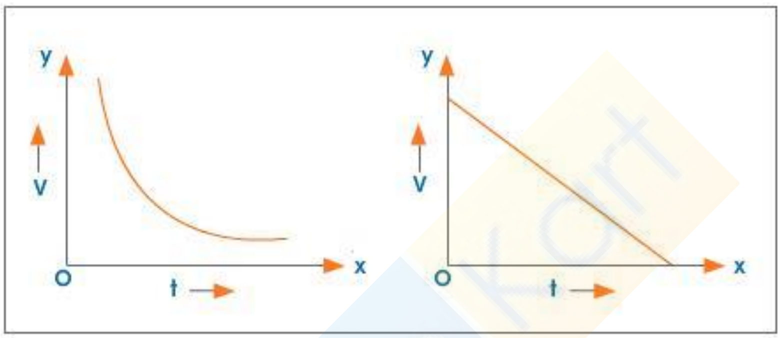
Let us now see the nature of the v - t graph for the different types of motion.

a) Increasing acceleration

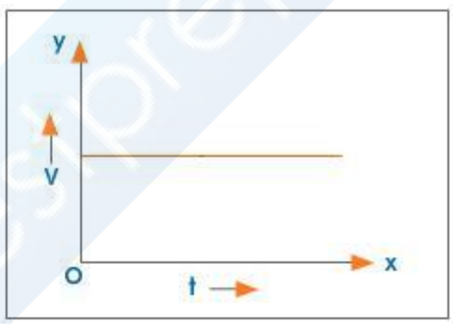


Uniform Acceleration **Non-uniform Acceleration**

(b) Decreasing acceleration



Non-uniform Retardation **Uniform Retardation**



Zero Acceleration

Uses of Velocity-time Graphs

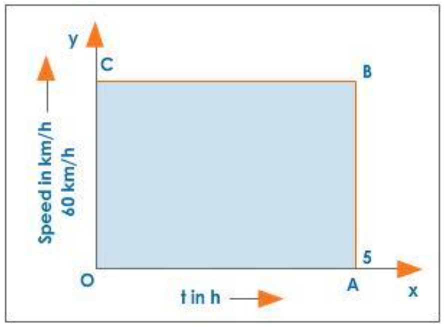
The following results can be deduced from velocity-time graph.

- The acceleration produced in a body
- The distance covered by a moving object
- We can derive the equations of motion

Speed - Time Graph

To calculate the distance covered by a moving object from speed-time graph

The figure below gives the speed-time graph of a car moving with a uniform speed of 60 km/h for 5 hours.



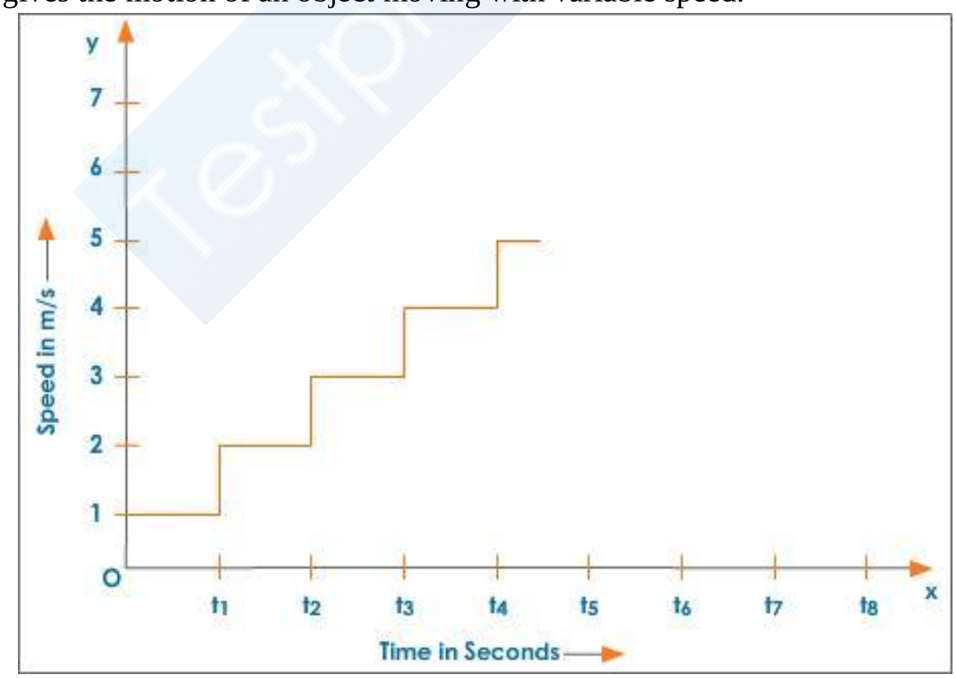
Speed-Time Graph of a Car Moving with Uniform Speed
Distance travelled by the car (S) = v x t

$= 60 \times 5$

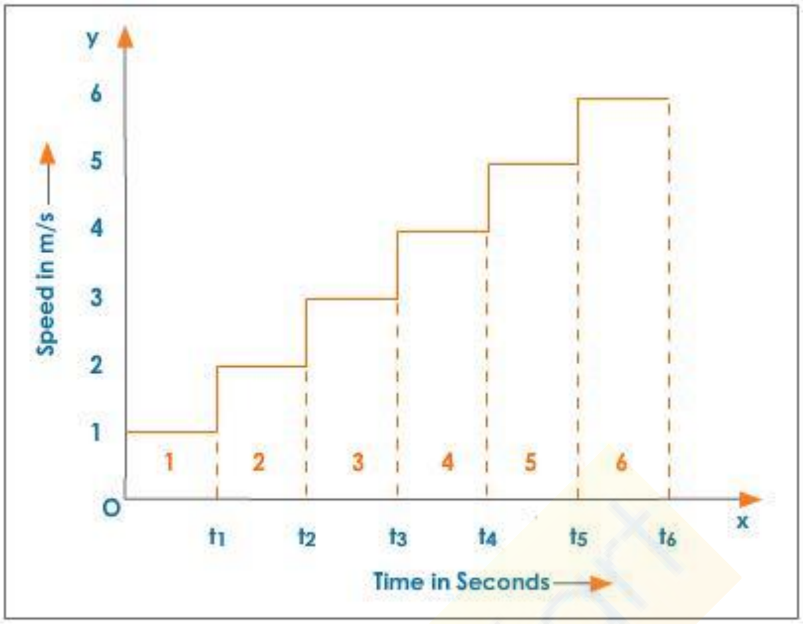
$= 300 \text{ km}$

But 60 km/h = OC = breadth of the rectangle OABC
5h = OA = length of the rectangle OABC
i.e., the distance covered by the car = length × breadth = 300 km.

To calculate the distance covered by a moving object from a speed-time graph we just have to find the area enclosed by the speed-time graph and the time axis.
In case of non-uniform motion, the distance covered by the object the speed of the object is increasing in steps. The speed remains constant during the time interval 0 - t_1, t_1 - t_2, t_2 - t_3 ......
Figure below gives the motion of an object moving with variable speed.



Speed - Time Graph for an Object Moving with Variable Speed



Calculation of Distance

∴ The total distance covered by the object during the time interval

$0\text{-}t_6$ = Area of rectangle 1+ area of rectangle 2+……+ area of rectangle 6.

Motion

Equations of Motion

The variable quantities in a uniformly accelerated rectilinear motion are time, speed, distance covered and acceleration. Simple relations exist between these quantities. These relations are expressed in terms of equations called equations of motion

The equations of motion are:

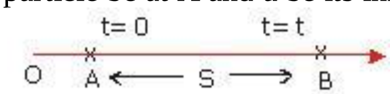
(1) $v = u + at$

(2) $S = ut + \frac{1}{2} at^2$

(3) $v^2 - u^2 = 2aS$

Derivation of the First Equation of Motion

Consider a particle moving along a straight line with uniform acceleration 'a'. At t = 0, let the particle be at A and u be its initial velocity and when t = t, v be its final velocity.



$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}} = \frac{v-u}{t}$$

$$a = \frac{v - u}{t}$$

$$v - u = at$$

$v = u + at$ **I equation of motion**

Second Equation of Motion

$$\text{Average velocity} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{Average velocity} = \frac{S}{t} \quad .... (1)$$

Average velocity can also be written as $\frac{u + v}{2}$

$$\text{Average velocity} = \frac{u + v}{2} \quad .... (2)$$

From equations (1) and (2)

$$\frac{S}{t} = \frac{u + v}{2} \quad .... (3)$$

The first equation of motion is v = u + at.

Substituting the value of v in equation (3), we get

$$\frac{S}{t} = \frac{u + u + at}{2} = \frac{(u + u + at)t}{2}$$

$$= \frac{(2u + at)t}{2} = \frac{2ut + at^2}{2} = \frac{2ut}{2} + \frac{at^2}{2}$$

$$S = ut + \frac{1}{2}at^2$$

$$S = ut + \frac{1}{2}at^2 \qquad \textbf{II equation of motion.}$$

Third Equation of Motion

The first equation of motion is v = u + at.

v - u = at ... (1)

$$\text{Average velocity} = \frac{S}{t} \quad ... (2)$$

$$\text{Average velocity} = \frac{u + v}{2} \quad .... (3)$$

From equation (2) and equation (3) we get,

$$\frac{u + v}{2} = \frac{S}{t} \quad .... (4)$$

Multiplying equation (1) and equation (4) we get,

$$(v - u)(v + u) = at \times \frac{2S}{t}$$

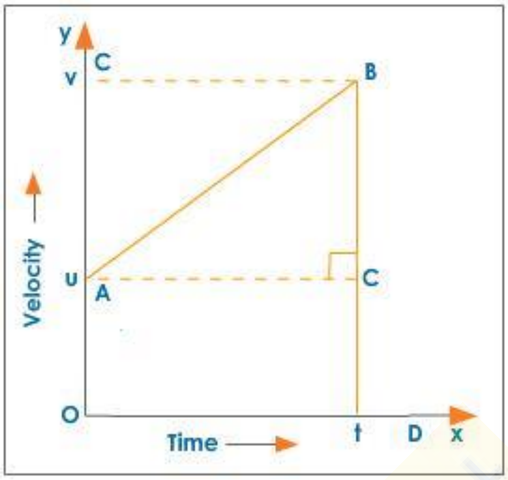
(v - u) (v + u) = 2aS

[We make use of the identity $a^2 - b^2 = (a + b)(a - b)$]

$v^2 - u^2 = 2aS$ **III equation of motion**

Derivations of Equations of Motion (Graphically)

First Equation of Motion



Graphical Derivation of First Equation

Consider an object moving with a uniform velocity u in a straight line. Let it be given a uniform acceleration a at time t = 0 when its initial velocity is u. As a result of the acceleration, its velocity increases to v (final velocity) in time t and S is the distance covered by the object in time t.

The figure shows the velocity-time graph of the motion of the object.

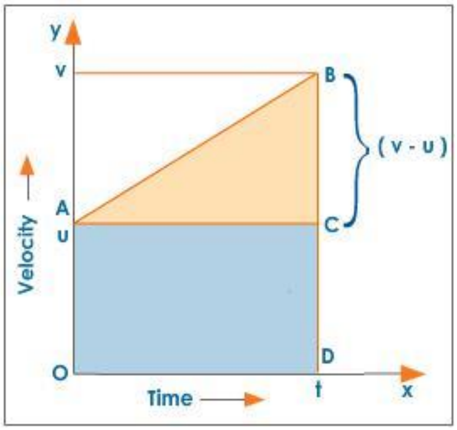
Slope of the v - t graph gives the acceleration of the moving object.

Thus, acceleration = slope = AB = $\frac{BC}{AC} = \frac{v - u}{t - 0}$

$a = \frac{v-u}{t}$ ⟹v - u = at ⟹v = u + at **I equation of motion**

Second Equation of Motion

Let u be the initial velocity of an object and 'a' the acceleration produced in the body. The distance travelled S in time t is given by the area enclosed by the velocity-time graph for the time interval 0 to t.



Graphical Derivation of Second Equation

Distance travelled S = area of the trapezium ABDO

= area of rectangle ACDO + area of ΔABC

$= OD \times OA + \frac{1}{2} BC \times AC$

$= t \times u + \frac{1}{2}(v - u) \times t$

$= ut + \frac{1}{2}(v - u) \times t$

$= t \times u + \frac{1}{2}(v - u) \times t$

$= ut + \frac{1}{2}(v - u) \times t$

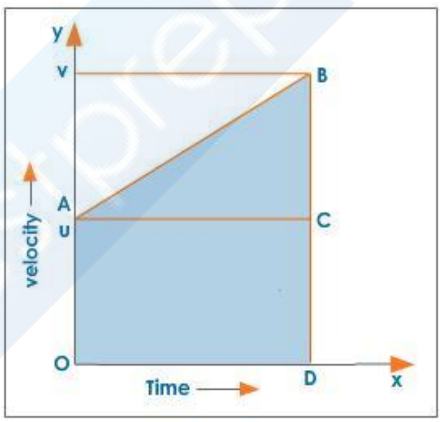
(v = u + at I eqn of motion; v - u = at)

$S = ut + \frac{1}{2} at \times t$

$S = ut + \frac{1}{2} at^2$ **II equation of motion**

Third Equation of Motion

Let 'u' be the initial velocity of an object and a be the acceleration produced in the body. The distance travelled 'S' in time 't' is given by the area enclosed by the v - t graph.



Graphical Derivation of Third Equation

S = area of the trapezium OABD.

$= \frac{1}{2}(b_1 + b_2)h$

$= \frac{1}{2}(OA + BD)AC$

$= \frac{1}{2}(u + v)t \quad .... (1)$

But we know that $a = \frac{v - u}{t}$

$$\text{Or } t = \frac{v - u}{a}$$

Substituting the value of t in equation (1) we get,

$$S = \frac{1}{2}\frac{(u + v)(v - u)}{a} = \frac{1}{2}\frac{(v + u)(v - u)}{a}$$

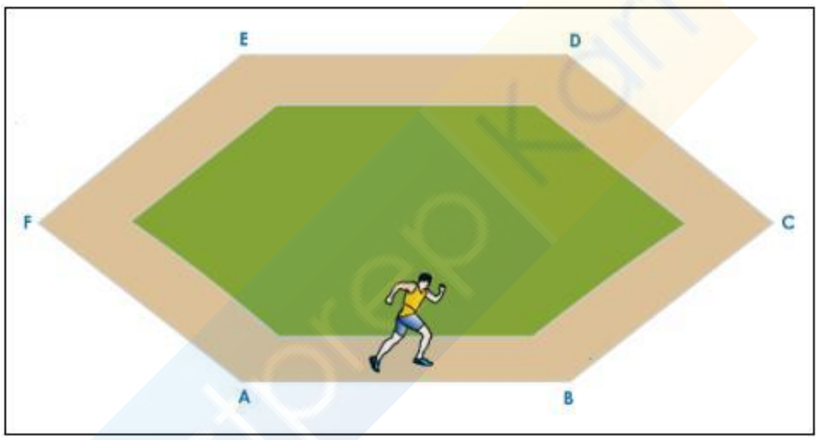
2aS = (v + u) (v - u)

(v + u)(v - u) = 2aS [using the identity $a^2 - b^2 = (a + b)(a - b)$]

$v^2 - u^2 = 2aS$ **III Equation of Motion**

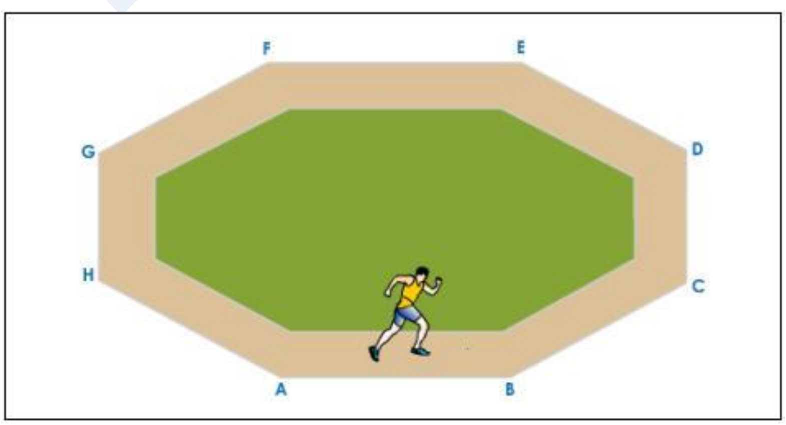
Circular Motion

In the example discussed under the topic uniform and non-uniform motion we have classified motion along circular track as an example of non-uniform motion. Let us now see why circular motion is considered as non-uniform motion. The figure shows an athlete running with uniform speed on a hexagonal track.



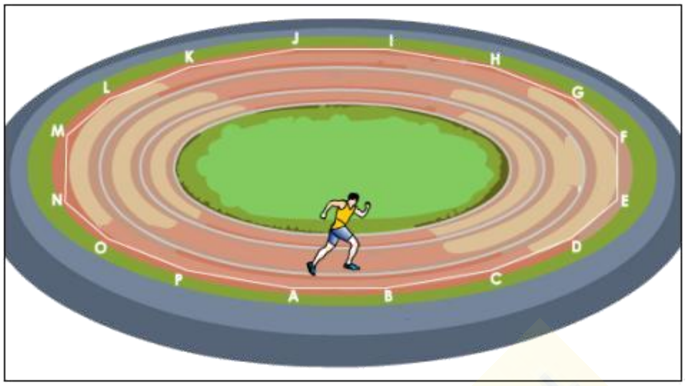
Athlete Running on a Regular Hexagonal Track

The athlete runs with uniform speed along the segments AB, BC, CD, DE, EF and FA of the track and at the turns he quickly changes his direction of motion to stay on the track, without changing his speed. Similarly if the track was a regular octagon the athlete would have changed the direction of motion eight times to remain on the track.



Athlete Running on a Regular Octagonal Track

As the number of sides of the track increases, the athlete has to turn more often. Now if we increase the number of sides indefinitely the shape of the track will approach the shape of a circle. Therefore, a circle can be considered as a polygon with infinite sides and hence motion along a circular path is classified as non-uniform motion.

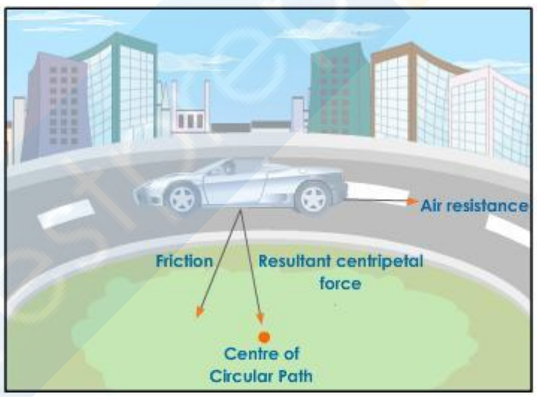


Athlete Running on a Circular Track

Thus, an object moving along a circular track with uniform speed is an example for a non - uniform motion because the direction of motion of the object goes on changing at every instant of time.

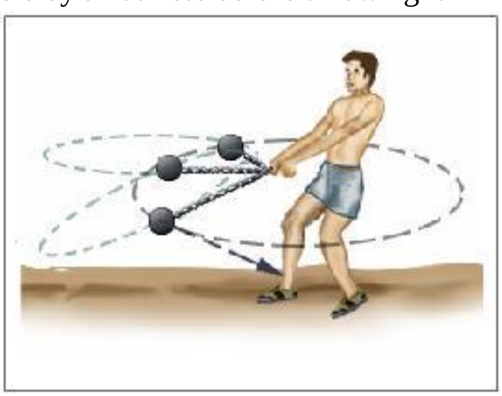
Examples of Uniform Circular Motion

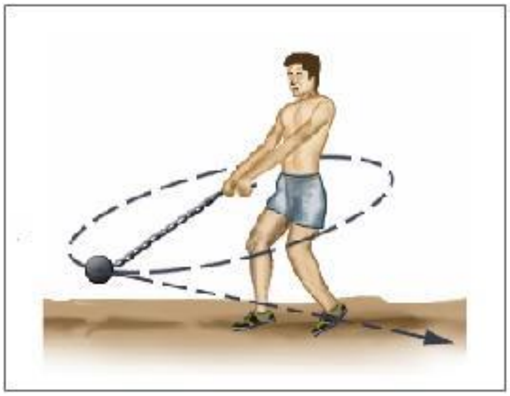
(1) A car negotiating a curve with uniform speed



Car Negotiating a Curve

(2) Whirling a hammer in a circle by an athlete before throwing it





Whirling a hammer in a circle

(3) An aircraft looping the loop.

Aircraft Looping the Loop

Expression for Linear Velocity

Suppose an athlete takes t seconds to go once around the circular path of radius r, then the velocity v is given by the relation

$v = \frac{\text{distance travelled}}{\text{time}}$

Distance travelled = circumference of the circle

$= 2\pi r$

Linear velocity $= \frac{2\pi r}{t}$