14 Practical Geometry

Learn and Remember

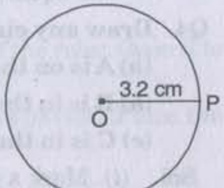
1. A circle can be constructed if its radius is known.
2. A perpendicular bisector divides a given line segment into two equal halves.
3. Geometrical tools like ruler, set squares, protractor and compass are required for geometrical constructions.
4. All angles having multiple of 15° can be constructed with the help of compass.
5. A line parallel to a given line from a point not lying on it can be constructed by making its alternate angles equal.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 14.1

Q1. Draw a circle of radius 3.2 cm.

Sol. Steps of Construction.



(*a*) Open the compass for the required radius of 3.2 cm.

(*b*) Make a point with a sharp pencil where we want the centre of circle to be.

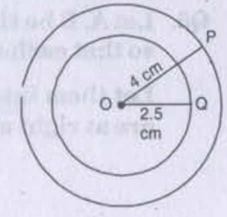
(*c*) Name it O.

(*d*) Place the pointer of compasses on O.

(*e*) Turn the compasses slowly to draw the circle.

Q2. With the same centre O, draw two circles of radii 4 cm and 2.5 cm.

Sol. Steps of Construction.



(*a*) Mark a point 'O' with a sharp pencil where we want the centre of the circle.

(*b*) Open the compasses 4 cm.

(*c*) Place the pointer of the compasses on O.

(*d*) Turn the compasses slowly to draw the circle.

(*e*) Again open the compasses 2.5 cm and place the pointer of the compasses at point O.

Turn the compasses slowly to draw the second circle.

Q3. Draw a circle and any two of its diameters. If you join the ends of these diameters, what is the figure obtained? What figure is obtained if the diameters are perpendicular to each other? How do you check your answer?

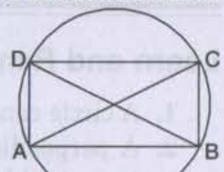
Sol. (*i*) By joining the ends of two diameters, we get a rectangle. By measuring, we find

AB = CD = 3 cm, BC = AD = 2 cm

i.e., pair of opposite sides are equal.

$\angle A = \angle B = \angle C = \angle D = 90°$

i.e., each angle is of 90°. Hence ABCD is a rectangle.



(*ii*) If the diameters are perpendicular to each other, then by joining the ends of two diameters, we get a square.

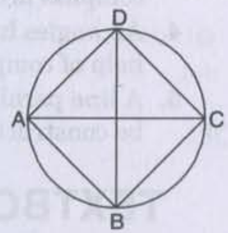
By measuring, we find that

AB = BC = CD = AD = 2.5 cm

i.e., four sides are equal.

$\angle A = \angle B = \angle C = \angle D = 90°$

Thus each angle is of 90°. Thus ABCD is a square.



Q4. Draw any circle and mark points A, B and C such that:

(*a*) A is on the circle.

(*b*) B is in the interior of the circle.

(*c*) C is in the exterior of the circle.

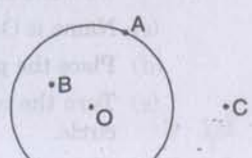
Sol. (*i*) Mark a point 'O' with sharp pencil where we want centre of the circle.

(*ii*) Place the pointer of the compasses at 'O'. Then move the compasses slowly to draw a circle.

(*a*) Point A is on the circle.

(*b*) Point B is in interior of the circle.

(*c*) Point C is in the exterior of the circle.

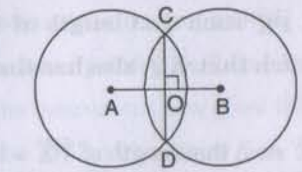


Q5. Let A, B be the centres of two circles of equal radii; draw them so that each one of them passes through the centre of the other.

Let them intersect at C and D. Examine whether $\overline{AB}$ and $\overline{CD}$ are at right angles.

Sol. Draw two circles of equal radii taking A and B as their centre such that each one of them passes through the centre of the other. They intersect at C and D. Join AB and CD.

Yes, AB and CD intersect at right angle as $\angle COB$ is 90°.



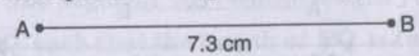
EXERCISE 14.2

Q1. Draw a line segment of length 7.3 cm, using a ruler.

Sol. (*i*) Place the zero mark of the ruler at a point A.

(*ii*) Mark a point B at a distance of 7.3 cm from A and join AB.

(*iii*) $\overline{AB}$ is the required line segment of length 7.3 cm.



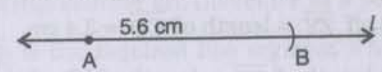
Q2. Construct a line segment of length 5.6 cm using ruler and compasses.

Sol. (*i*) Draw a line '*l*'. Mark a point A on this line.

(*ii*) Place the compasses pointer on zero mark of the ruler. Open it to place the pencil point upto 5.6 cm mark.

(*iii*) Without changing the opening of the compasses. Place the pointer on A and cut an arc '*l*' at B.

(*iv*) $\overline{AB}$ is the line segment of the required length.



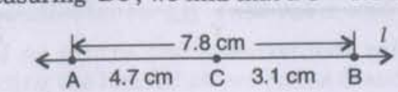
Q3. Construct $\overline{AB}$ of length 7.8 cm. From this cut off $\overline{AC}$ of length 4.7 cm. Measure $\overline{BC}$.

Sol. (*i*) Place the zero mark of the ruler at A.

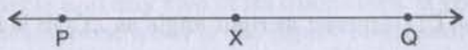
(*ii*) Mark a point B at a distance 7.8 cm from A.

(*iii*) Again, mark a point C at a distance 4.7 cm from A.

(*iv*) By measuring $\overline{BC}$, we find that BC = 3.1 cm.



Q4. Given $\overline{AB}$ of length 3.9 cm, construct $\overline{PQ}$ such that the length $\overline{PQ}$ is twice that of $\overline{AB}$. Verify by measurement.



(Hint : Construct $\overline{PX}$ such that length of $\overline{PX}$ = length of $\overline{AB}$; then cut off $\overline{XQ}$ such that $\overline{XQ}$ also has the length of $\overline{AB}$.)

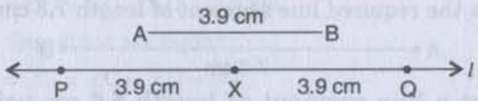
Sol. (*i*) Draw a line '*l*'.

(*ii*) Construct $\overline{PX}$ such that length of $\overline{PX}$ = length of $\overline{AB}$.

(*iii*) Then cut of $\overline{XQ}$ such that $\overline{XQ}$ also has the length of $\overline{AB}$.

(*iv*) Thus the length of $\overline{PX}$ and length of $\overline{XQ}$ added together make twice the length of $\overline{AB}$.

(*v*) Verification : By measurement we find that PQ = 7.8 cm = 3.9 cm + 3.9 cm = $\overline{AB}$ + $\overline{AB}$ = 2 × $\overline{AB}$.



Q5. Given $\overline{AB}$ of length 7.3 cm and $\overline{CD}$ of length 3.4 cm, construct a line segment $\overline{XY}$ such that the length of $\overline{XY}$ is equal to the difference between the lengths of $\overline{AB}$ and $\overline{CD}$. Verify by measurement.

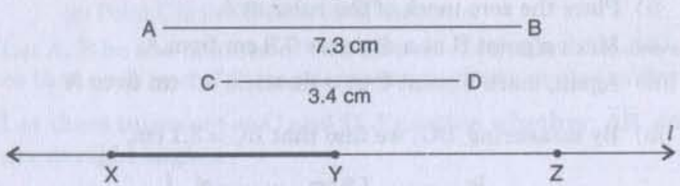
Sol. (*i*) Draw a line '*l*' and take a point X on it.

(*ii*) Construct $\overline{XZ}$ such that length of $\overline{XZ}$ = length of $\overline{AB}$ = 7.3 cm.

(*iii*) Then cut off $\overline{ZY}$ = length of $\overline{CD}$ = 3.4 cm.

(*iv*) Thus the length of $\overline{XY}$ = length of AB – length of $\overline{CD}$.

(*v*) Verification : By measurement, we find that of $\overline{XY}$ = 3.9 cm = 7.3 – 3.4 cm = $\overline{AB}$ – $\overline{CD}$.



EXERCISE 14.3

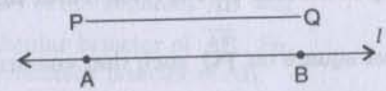
Q1. Draw any line segment $\overline{PQ}$. Without measuring $\overline{PQ}$, construct a copy of $\overline{PQ}$.

Sol. (*i*) Given $\overline{PQ}$ whose length is not known.

(*ii*) Fix the compasses pointer on P and the pencil end on Q. The opening of the instrument now gives the length of $\overline{PQ}$.

(*iii*) Draw any line '*l*'. Choose a point A on '*l*'. Without changing the compasses setting. Place the pointer on A.

(*iv*) Strike an arc that cut '*l*' at a point. Say 'B'. Now $\overline{AB}$ is a copy of $\overline{PQ}$.



Q2. Given some line segment $\overline{AB}$, whose length you do not know, construct $\overline{PQ}$ such that the length of $\overline{PQ}$ is twice that of $\overline{AB}$.

Sol. (*i*) Given $\overline{AB}$ whose length is not known.

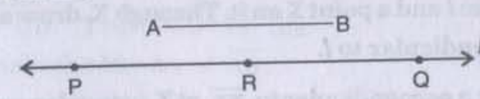
(*ii*) Fix the compasses pointer on A and the pencil end on B. The opening instrument now gives the length $\overline{AB}$.

(*iii*) Draw any line '*l*'. Choose a point P on *l*. Without changing the compasses setting. Place the pointer on P.

(*iv*) Strike on arc that cut '*l*' at a point R.

(*v*) Now place the pointer on R and without changing the compasses setting, strike another arc that cut '*l*' at a point Q.

(*vi*) Thus $\overline{PQ}$ is the required line segment whose length is twice that of AB.



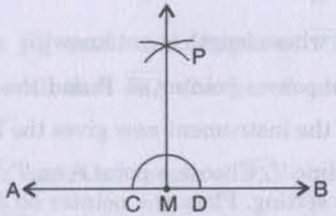
EXERCISE 14.4

Q1. Draw any line segment $\overline{AB}$. Mark any point M on it. Through M, draw a perpendicular to $\overline{AB}$. (use ruler and compasses)

Sol. (*i*) With M as centre and a convenient radius, draw an arc intersecting the line AB at two points C and B.

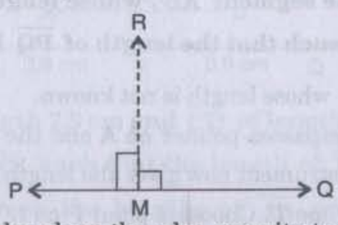
(*ii*) With C and D as centre and a radius greater than MC, draw two arcs, which cut each other at P.

(*iii*) Join PM. Then PM is perpendicular to AB through the point M.



Q2. Draw any line segment $\overline{PQ}$. Take any point R not on it. Through R, draw a perpendicular to $\overline{PQ}$. (use ruler and set-square).

Sol. (*i*) Place a set-square on $\overleftrightarrow{PQ}$ such that one arm of its right angle aligns along $\overline{PQ}$.



(*ii*) Place a ruler along the edge opposite to the right angle of the set-square.

(*iii*) Hold the ruler fixed. Slide the set square along the ruler till the point R touches the other arm of the set-square.

(*iv*) Join RM along the edge through R meeting $\overleftrightarrow{PQ}$ at M. Then RM ⊥ PQ.

Q3. Draw a line *l* and a point X on it. Through X, draw a line segment $\overline{XY}$ perpendicular to *l*.

Now draw a perpendicular to $\overline{XY}$ at Y. (use ruler and compasses)

Sol. (*i*) Draw a line '*l*' and take point X on it.

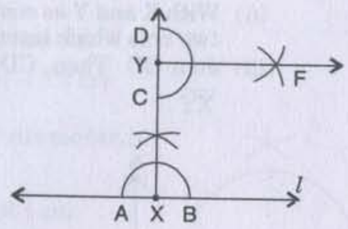
(*ii*) With 'X' as centre and a convenient radius, draw an arc intersecting the line '*l*' at two points A and B.

(*iii*) With A and B as centre and a radius greater than XA, draw two arcs, which cut each other at C.

(*iv*) Join XC and produce it to Y. Then XY is perpendicular to *l*.

(*v*) With D as centre and a convenient radius, draw an arc intersecting XY at two points C and D.

(*vi*) With C and D as centre and radius greater than YD, draw two arcs which cut each other at F.



(*vii*) Join YF, then YF is perpendicular to XY at Y.

EXERCISE 14.5

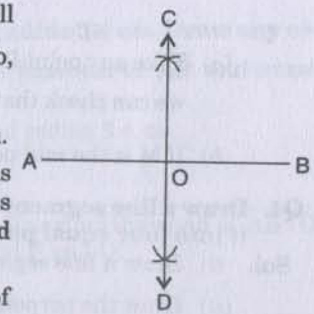
Q1. Draw $\overline{AB}$ of length 7.3 cm and find its axis of symmetry.

Sol. Axis of symmetry of line segment $\overline{AB}$ will be the perpendicular bisector of $\overline{AB}$. So, draw the perpendicular bisector of AB.

(*i*) Draw a line segment $\overline{AB}$ = 7.3 cm.

(*ii*) With A and B as centre and radius more than half of AB, draw two arcs which intersect each other at C and D.

(*iii*) Join CD. Then CD is the axis of symmetry of the line segment AB.

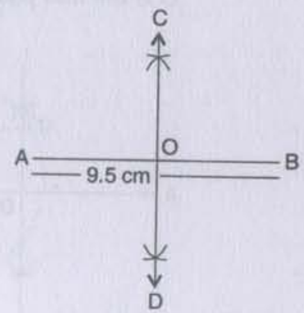


Q2. Draw a line segment of length 9.5 cm and construct its perpendicular bisector.

Sol. (*i*) Draw a line segment $\overline{AB}$ = 9.5 cm.

(*ii*) With A and B as centre and radius more than half of $\overline{AB}$, draw two arcs which intersect each other at C and D.

(*iii*) Join CD. Then CD is the perpendicular bisector of $\overline{AB}$.



Q3. Draw the perpendicular bisector of $\overline{XY}$ whose length is 10.3 cm.

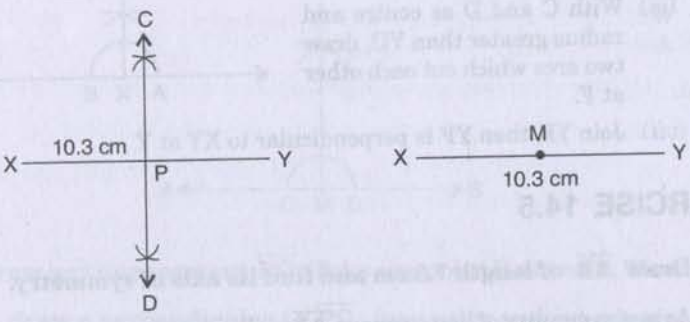
(*a*) Take any point P on the bisector drawn. Examine whether PX = PY.

(*b*) If M is the mid point of $\overline{XY}$, what can you say about the lengths MX and XY?

Sol. (*i*) Draw a line segment $\overline{XY}$ = 10.3 cm.

(*ii*) With X and Y as centre and radius more than half of $\overline{XY}$, draw two arcs which intersect each other at C and D.

(*iii*) Join CD. Then, CD is the required perpendicular bisector of $\overline{XY}$.



(*i*) (*ii*)

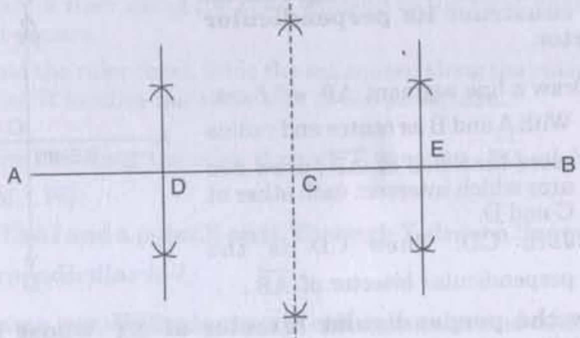
(*a*) Take any point P on the bisector drawn. With the help of divider we can check that $\overline{PX} = \overline{PY}$.

(*b*) If M is the mid point of $\overline{XY}$. Then $\overline{MX} = \frac{1}{2}\ \overline{XY}$.

Q4. Draw a line segment of length 12.8 cm. Using compasses, divide it into four equal parts. Verify by actual measurement.

Sol. (*i*) Draw a line segment AB = 12.8 cm.

(*ii*) Draw the perpendicular bisector of $\overline{AB}$ which cut it at C. Thus, C is the mid point of $\overline{AB}$.



(*iii*) Draw the perpendicular bisector of $\overline{AC}$ which cut it at D. Thus D is the mid point of $\overline{AC}$.

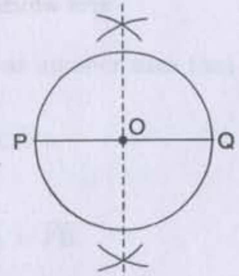
(*iv*) Again, draw the perpendicular bisector of $\overline{CB}$ which it at E. Thus E is the mid point of $\overline{CB}$.

(*v*) Now, point C, D and E divide the line segment $\overline{AB}$ in four equal parts.

(*vi*) By actual measurement, we find that

$$\overline{AD} = \overline{DC} = \overline{CE} = \overline{EB} = 3.2 \text{ cm.}$$

Q5. With $\overline{PQ}$ of length 6.1 cm as diameter, draw a circle.



Sol. (*i*) Draw a line segment $\overline{PQ}$ **= 6.1** cm.

(*ii*) Draw the perpendicular bisector of PQ which cuts, it at O. Thus O is the mid point of $\overline{PQ}$.

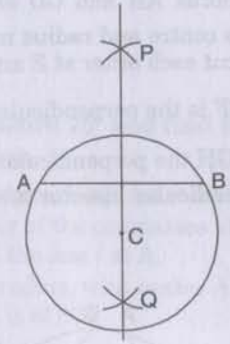
(*iii*) With O as centre and OP or OQ as radius draw a circle where diameter is the line segment $\overline{PQ}$.

Q6. Draw a circle with centre C and radius 3.4 cm. Draw any chord $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$ and examine if it passes through C.

Sol. (*i*) Draw a circle with centre C and radius 3.4 cm.

(*ii*) Draw any chord $\overline{AB}$.

(*iii*) With A and B as centres and radius more than half of $\overline{AB}$. Draw two arcs which cut each other at P and Q.



(*iv*) Join PQ. Then, PQ is the perpendicular bisector of $\overline{AB}$.

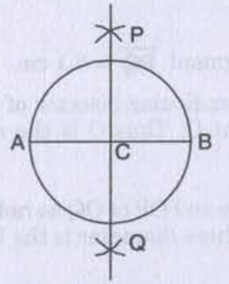
(*v*) This perpendicular bisector of $\overline{AB}$ passes through the centre C of the circle.

Q7. Repeat Question 6, if $\overline{AB}$ happens to be a diameter.

Sol. (*i*) Draw a circle with centre C and radius 3.4 cm.

(*ii*) Draw its diameter $\overline{AB}$.

(*iii*) With A and B as centre and radius more than half of it, draw two arcs which cut each other at P and Q.



(*iv*) Join PQ. Then PQ is the perpendicular bisector of $\overline{AB}$.

(*v*) Clearly, this perpendicular bisector of $\overline{AB}$, passes through the centre C of the circle.

Q8. Draw a circle of radius 4 cm. Draw any two of its chords. Construct the perpendicular bisectors of these chords. Where do they meet?.

Sol. (*i*) Draw circle with centre O and radius 4 cm.

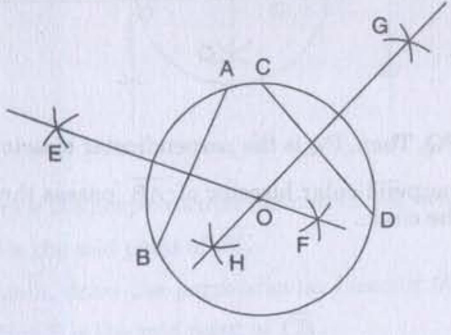
(*ii*) Draw any two chords $\overline{AB}$ and $\overline{CD}$ of this circle.

(*iii*) With A and B as centre and radius more than half of AB draw two arcs which cut each other at E and F.

(*iv*) Join EF. Thus EF is the perpendicular bisector of chord $\overline{AB}$.

(*v*) Similarly draw GH the perpendicular bisector of chord $\overline{CD}$.

(*vi*) These two perpendicular bisector meet at O, the centre of the circle.



Q9. Draw any angle with vertex O. Take a point A on one of its arms and B on another such that OA = OB. Draw the perpendicular bisectors of $\overline{OA}$ and $\overline{OB}$. Let them meet at P. Is PA = PB?

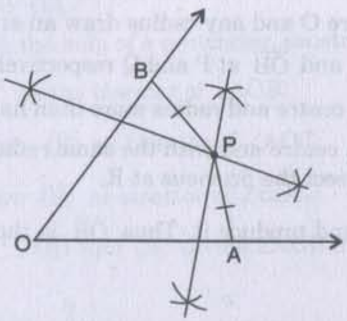
Sol. (*i*) Draw any angle with vertex O.

(*ii*) Take a point A on one of its arms and B on another such that $\widehat{OA} = \widehat{OB}$.

(*iii*) Draw perpendicular bisector of $\overline{OA}$ and $\overline{OB}$

(*iv*) Let them meet at P. Join PA and PB.

(*v*) With the help of divider we check that $\overline{PA} = \overline{PB}$.



EXERCISE 14.6

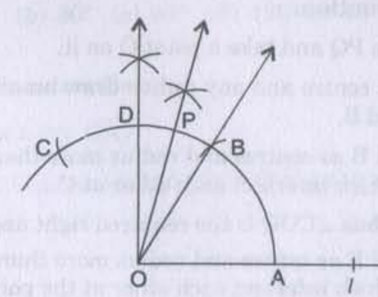
Q1. Draw ∠POQ of measure 75° and find its line of symmetry.

Sol. Steps of Construction.

(*i*) Draw a line *l* and mark a point O on it.

(*ii*) Place the pointer of the compasses at O and draw an arc of any radius with cut the line *l* at A.

(*iii*) With the same radius, with centre A, cut the previous arc at B. Join OB, which is of 60°.



(*iv*) With the same radius, with centre B, cut the previous arc at C. And then make bisector of ∠BOC. The angle is of 90°, mark it at D.

(*v*) Thus ∠DOA = 90°.

(*vi*) Draw $\overline{OP}$ as bisector of ∠DOB.

Thus, ∠POA is of 75°.

Q2. Draw an angle of measure 147° and construct its bisector.

Sol. Steps of Construction.

(*i*) Draw a ray $\overline{OA}$.

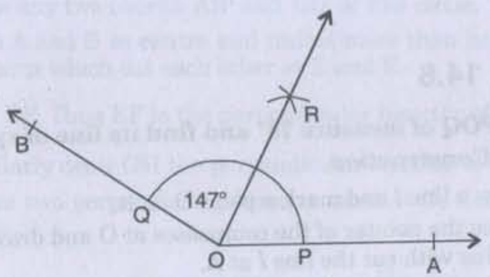
(*ii*) With the help of protractor construct ∠AOB = 147°.

(*iii*) With centre O and any radius draw an arc which intersect the arms $\overline{OA}$ and $\overline{OB}$ at P and Q respectively.

(*iv*) With P as centre and radius more than half of PQ, draw an arc.

(*v*) With Q as centre and with the same radius, draw an other arc with intersect the previous at R.

(*vi*) Join OR and produce it. Thus $\overline{OR}$ is the required bisector of ∠AOB.



Q3. Draw a right angle and construct its bisector.

Sol. Steps of Construction.

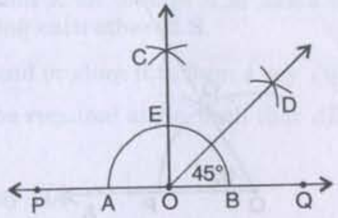
(*i*) Draw a line PQ and take a point O on it.

(*ii*) With 'O' as centre and any radius draw an arc which intersect PQ at A and B.

(*iii*) With A and B as centres and radius more than half of AB draw two arcs which intersect each other at C.

(*iv*) Join OC. Thus ∠COQ is the required right angle.

(*v*) With B and E as centre and radius more than half of BE, draw two arcs which intersect each other at the point D.



(*vi*) Join OD. Thus $\overline{OD}$ is the required bisector of ∠COQ.

Q4. Draw an angle of measure 153° and divide it into four equal parts.

Sol. Steps of Construction.

(*i*) Draw a ray $\overline{OA}$.

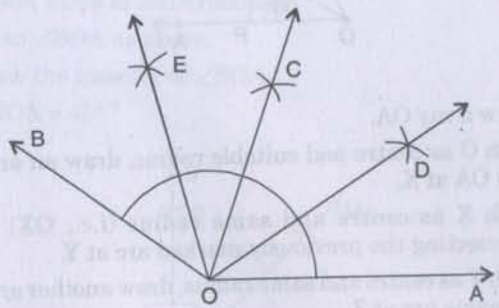
(*ii*) At O, with the help of a protractor, construct ∠AOB = 153°.

(*iii*) Draw $\overline{OC}$ as the bisector of ∠AOB.

(*iv*) Again, draw $\overline{OD}$ as bisector of ∠AOC.

(*v*) Again draw $\overline{OE}$ as bisector of ∠BOC.

(*vi*) Thus, $\overline{OC}$, $\overline{OD}$ and $\overline{OE}$ divide ∠AOB in four equal parts.



Q5. Construct with ruler and compasses, angles of following measures:

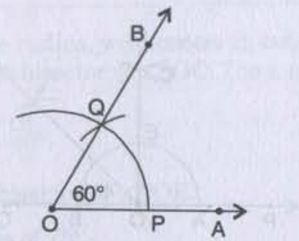
(*a*) 60° (*b*) 30° (*c*) 90° (*d*) 120° (*e*) 45° (*f*) 135°

Sol. (*a*) 60°

Steps of Construction.

(*i*) Draw a ray $\overline{OA}$.

(*ii*) With O as centre and any radius make an arc, that cut $\overline{OA}$ at B.



(*iii*) With P as centre and same arc, cut previous arc at Q.

(*iv*) Join OQ. ∠BOA = 60°.

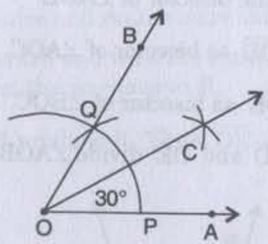
(*b*) 30°

(*i*) Make the ∠AOB of 60° as above.

(*ii*) Put the pointer on P and make an arc.

(*iii*) Put the pointer on Q and with same radius cut the previous at C.

(*iv*) ∠COA is of 30°.



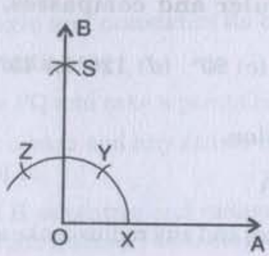
(*c*) 90°

(*i*) Draw a ray OA.

(*ii*) With O as centre and suitable radius, draw an arc intersecting arm OA at X.

(*iii*) With X as centre and same radius (*i.e.,* OX) draw an arc intersecting the previously marked arc at Y.

(*iv*) With Y as centre and same radius, draw another arc intersecting the same arc at Z.



(*v*) With Y and Z as centres and same radius, draw two arcs intersecting each other at S.

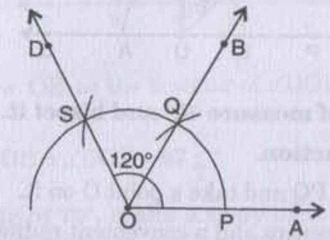
(*vi*) Join OS and produce it to form a ray $\overline{OB}$.

(*vii*) BOA is the required angle, such that ∠BOA = 90°.

(*d*) 120°

(*i*) Draw a ray $\overline{OA}$.

(*ii*) With O as centre and any radius. Make an arc, that cut $\overline{OB}$ at B.



(*iii*) With P as centre and same arc cut previous arc at Q.

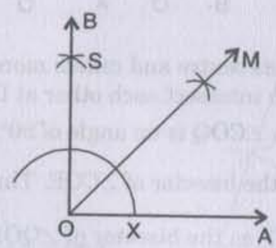
(*iv*) With Q as centre and same radius cut the arc at S. Join OS. Which is of 120°. ∠AOD is 120°.

(*e*) 45° (see 90°, steps of construction)

(*i*) Make ∠BOA as above.

(*ii*) Draw the bisector of ∠BOA.

(*iii*) ∠MOA = 45°.



(*f*) 135°

(*i*) Draw a line PQ and take a point O on it.

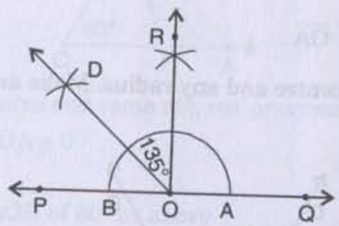
(*ii*) With 'O' as centre and convenient radius, draw an arc which intersect PQ at A and B.

(*iii*) With A and B as centres and radius more than half of AB. Draw two arcs which intersect each other at R.

(*iv*) Join OR. Then ∠QOR = ∠POR = 90°.

(*v*) Draw $\overline{OD}$ as the bisector of ∠POR.

Then ∠QOD is the required angle of 135°.

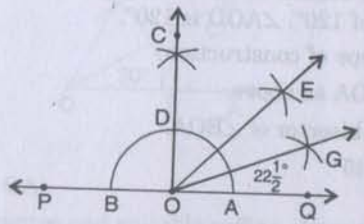


Q6. Draw an angle of measure 45° and bisect it.

Sol. Steps of Construction.

(*i*) Draw a line PQ and take a point O on it.

(*ii*) With 'O' as centre and a convenient radius, draw an arc which intersect PQ at two points A and B.



(*iii*) With A and B as centre and radius more than half of AB. Draw two arcs which intersect each other at C.

(*iv*) Join OC. Then ∠COQ is an angle of 90°.

(*v*) Draw $\overline{OE}$ as the bisector of ∠COE. Thus ∠QOE = 45°.

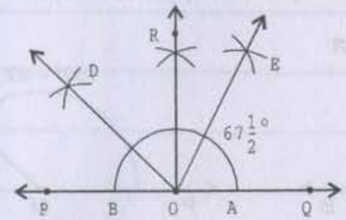
(*vi*) Now draw $\overline{OG}$ as the bisector of ∠QOE.

Thus ∠QOG = ∠EOG = $22\frac{1}{2}$°.

Q7. Draw an angle of measure 135° and bisect it.

Sol. Steps of Construction.

(First 5 same as 135°)



(*vi*) Now, draw $\overline{OE}$ as the bisector of ∠QOD.

Thus ∠QOE = ∠DOE = $67\frac{1}{2}$°.

Q8. Draw an angle of 70°. Make a copy of it using only a straight edge and compasses.

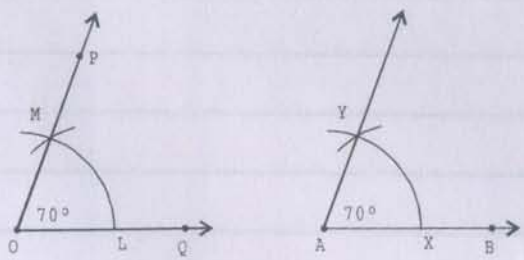
Sol. Steps of Construction.

(*i*) Draw an angle 70° with protractor, name is ∠POQ.

(*ii*) Draw a ray $\overline{AB}$.

(*iii*) Place the compasses at O and draw an arc to cut the rays of ∠POQ at L and M.

(*iv*) Use the same compasses setting to draw an arc with A as centre, cutting AB at X.



(*v*) Set your compasses to the length LM with the same radius.

(*vi*) Place the compasses pointer at X and draw the arc to cut the arc drawn earlier at Y.

(*vii*) Join AY. This gives ∠YAX,

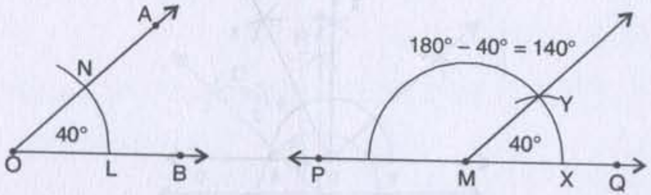
∠YAX = 70°.

Q9. Draw an angle of 40°. Copy its supplementary angle.

Sol. Steps of Construction.

(*i*) Draw an angle 40° with protractor. Name is ∠AOB.

(*ii*) Draw a line PQ. Take any point M on it.



(*iii*) Place the compasses at O and draw an arc to cut the rays of ∠AOB at L and N.

(*iv*) Use the same compasses setting to draw an arc O as centre, cutting MQ at X.

(*v*) Set your compasses to length LN with the same radius.

(*vi*) Place the compasses at X and draw the arc to cut the arc drawn earlier Y.

(*vii*) Join MY.

(*viii*) ∠QMY = 40° and ∠PMY is supplementary of it.

□□