3 Playing with Numbers

Learn and Remember

1. **Factor.** A factor of a number is a number which completely divides the first number without leaving a remainder.
   (*a*) 1 is a factor of every number.
   (*b*) 1 is the only number which has exactly one factor, namely itself.
2. **Multiple.** A multiple of a number is a number obtained by multiplying it by a natural number.
3. **Even numbers.** All multiples of 2 are called even numbers.
4. **Odd numbers.** Numbers which are not multiples of 2 are called odd numbers.
5. **Prime numbers.** The number which has only two factors (the number itself and one) is called prime number.
6. **Composite number.** The number which has more than two factors is called composite number.
7. **A number is divisible by —**
   (*a*) 2, if the unit's digit of the number is 0, 2, 4, 6 or 8.
   (*b*) 3, if the sum of digits is divisible by 3.
   (*c*) 4, if the number formed by its digits in ten's and unit's place is divisible by 4.
   (*d*) 5, if unit digit is 0 or 5.
   (*e*) 6, if it is divisible by both 2 and 3.
   (*f*) 8, if the number formed by its digit is hundred's, ten's and unit's place is divisible by 8.
   (*g*) 9, if the sum of digits is divisible by 9.
   (*h*) 10, if the unit's digit is 0.
   (*i*) 11, if the difference of the sum of its digits in odd places and the sum of its digits in even places is either 0 or divisible by 11.
8. The H.C.F. of two or more numbers is the largest number which is divisible by all the given numbers.

9. The L.C.M. of two or more numbers is the smallest number which is divisible by all the given numbers.
10. **Co-Prime Numbers.** If the two numbers have only one common factor *i.e.,* 1, then the pair of such number is known as co-prime number.
11. The H.C.F. of two prime or co-prime numbers are equal.
12. The L.C.M. of two or more numbers is a factor of their L.C.M.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 3.1

Q1. Write all the factors of the following numbers:

(*a*) 24 (*b*) 15 (*c*) 21 (*d*) 27 (*e*) 12
(*f*) 20 (*g*) 18 (*h*) 23 (*i*) 36

Sol. (*a*) $24 = 1 \times 24$
$= 2 \times 12$
$= 3 \times 8$
$= 4 \times 6$
$= 6 \times 4$
$\therefore$ factors of 24 are : 1, 2, 3, 4, 6, 8, 12, 24.

(*b*) $15 = 1 \times 15$
$= 3 \times 5$
$= 5 \times 3$
$\therefore$ factors of 15 are : 1, 3, 5, 15.

(*c*) $21 = 1 \times 21$
$= 3 \times 7$
$= 7 \times 3$
$\therefore$ factors of 21 are : 1, 3, 7, 21.

(*d*) $27 = 1 \times 27$
$= 3 \times 9$
$= 9 \times 3$
$\therefore$ factors of 27 are : 1, 3, 9, 27.

(*e*) $12 = 1 \times 12$
$= 2 \times 6$
$= 3 \times 4$
$= 4 \times 3$
$\therefore$ factors of 12 are : 1, 2, 3, 4, 6, 12.

(*f*) $20 = 1 \times 20$
$= 2 \times 10$
$= 4 \times 5$
$= 5 \times 4$
$\therefore$ factors of 20 are : 1, 2, 4, 5, 10, 20.

(*g*) $18 = 1 \times 18$
$= 2 \times 9$
$= 3 \times 6$
$= 6 \times 3$
$\therefore$ factors of 18 are : 1, 2, 3, 6, 9, 18.

(*h*) $23 = 1 \times 23$
$\therefore$ factors of 23 are : 1, 23.

(*i*) $36 = 1 \times 36$
$= 2 \times 18$
$= 3 \times 12$
$= 4 \times 9$
$= 6 \times 6$
$\therefore$ Prime factors are : 1, 2, 3, 4, 6, 9, 12, 18, 36.

Q2. Write first five multiples of:

(*a*) 5 (*b*) 8 (*c*) 9

Sol. (*a*) $5 \times 1 = 5$ $5 \times 2 = 10$ $5 \times 3 = 15$
$5 \times 4 = 20$ $5 \times 5 = 25$
$\therefore$ first five multiples of 5 are 5, 10, 15, 20, 25.

(*b*) $8 \times 1 = 8$ $8 \times 2 = 16$ $8 \times 3 = 24$
$8 \times 4 = 32$ $8 \times 5 = 40$
$\therefore$ first five multiples of 8 are 8, 16, 24, 32, 40.

(*c*) $9 \times 1 = 9$ $9 \times 2 = 18$ $9 \times 3 = 27$
$9 \times 4 = 36$ $9 \times 5 = 45$
$\therefore$ first five multiples of 9 are 9, 18, 27, 36, 45.

Q3. Match the items in column 1 with the items in column 2:

Column 1	Column 2
(*i*) 35	**(*a*) Multiple of 8**
(*ii*) 15	**(*b*) Multiple of 7**
(*iii*) 16	**(*c*) Multiple of 70**
(*iv*) 20	**(*d*) Factor of 30**
(*v*) 25	**(*e*) Factor of 50**
	(*f*) Factor of 20

Sol. (*i*) (*b*) (*ii*) (*d*) (*iii*) (*a*) (*iv*) (*f*) (*v*) (*e*).

Q4. Find all the multiples of 9 upto 100.

Sol. Multiples of '9' are :

$9 \times 1 = 9$	$9 \times 4 = 36$	$9 \times 7 = 63$	$9 \times 10 = 90$
$9 \times 2 = 18$	$9 \times 5 = 45$	$9 \times 8 = 72$	$9 \times 11 = 99$
$9 \times 3 = 27$	$9 \times 6 = 54$	$9 \times 9 = 81$	

$\therefore$ multiples of 9 are : 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99.

EXERCISE 3.2

Q1. What is the sum of any two:

(*a*) Odd numbers. (*b*) Even numbers.

Sol. (*a*) Odd numbers, example : (1 + 3 = 4, 3 + 5 = 8)

(*b*) Even numbers, example : (2 + 4 = 6, 6 + 4 = 10).

Q2. State whether the following statements are true or false:

(*a*) The sum of three odd numbers is even.

(*b*) The sum of two odd numbers and one even number is even.

(*c*) The product of three odd numbers is odd.

(*d*) If an even number is divided by 2, the quotient is always odd.

(*e*) All prime numbers are odd.

(*f*) Prime numbers do not have any factors.

(*g*) Sum of two prime numbers is always even.

(*h*) 2 is the only even prime number.

(*i*) All even numbers are composite numbers.

(*j*) The product of two even numbers is always even.

Sol. (*a*) False (*b*) True (*c*) True (*d*) False (*e*) False
(*f*) False (*g*) False (*h*) True (*i*) False (*j*) True.

Q3. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers upto 100.

Sol. 17 and 71; 37 and 73 ; 79 and 97.

Q4. Write down separately the prime and composite numbers less than 20.

Sol. Prime numbers : 2, 3, 5, 7, 11, 13, 17, 19.

Composite numbers : 4, 6, 8, 9, 10, 12, 14, 15, 16, 18.

Q5. What is the greatest prime number between 1 and 10?

Sol. '7' is the greatest prime number between 1 and 10.

Q6. Express the following as the sum of two odd primes?

(*a*) 44 (*b*) 36 (*c*) 24 (*d*) 18

Sol. (*a*) 3 + 41 = 44 (3 and 41 are both odd primes.)

(*b*) 5 + 31 = 36 (5 and 31 are both odd primes.)

(*c*) 7 + 17 = 24 (7 and 17 are both odd primes.)

(*d*) 7 + 11 = 18 (7 and 11 are both odd primes.)

Q7. Give three pairs of prime numbers whose difference is 2.

[Remark : Two prime numbers whose difference is 2 are called twin primes].

Sol. 5 and 3; 7 and 5; 13 and 11.

Q8. Which of the following numbers are prime?

(*a*) 23 (*b*) 51 (*c*) 37 (*d*) 26.

Sol. (*a*) 23 and (*c*) 37 are prime numbers.

Q9. Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

Sol. 90, 91, 92, 93, 94, 95, 96.

Q10. Express each of the following numbers as the sum of three odd primes:

(*a*) 21 (*b*) 31 (*c*) 53 (*d*) 61.

Sol. (*a*) 21 = 3 + 7 + 11 (*b*) 31 = 3 + 11 + 17

(*c*) 53 = 13 + 17 + 23 (*d*) 61 = 19 + 29 + 13.

Q11. Write five pairs of prime numbers less than 20 whose sum is divisible by 5. (Hint : 3 + 7 = 10)

Sol. 2 + 3 = 5 (5 is divisible by 5)

7 + 13 = 20 (20 is divisible by 5)

3 + 17 = 20 (20 is divisible by 5)

2 + 13 = 15 (15 is divisible by 5)

5 + 5 = 10 (10 is divisible by 5)

Q12. Fill in the blanks:

(*a*) A number which has only two factors is called a ..... .

(*b*) A number which has more than two factors is called a ..... .

(*c*) 1 is neither ..... nor ..... .

(*d*) The smallest prime number is ..... .

(*e*) The smallest composite number is ..... .

(*f*) The smallest even number is ..... .

Sol. (*a*) Prime number (*b*) Composite number

(*c*) Prime number and composite number (*d*) 2

(*e*) 4 (*f*) 2.

EXERCISE 3.3

Q1. Using divisibility tests, determine which of the following numbers are divisible by 2; by 3; by 4 ; by 5 ; by 6; by 8; by 9; by 10; by 11. (say yes or no).

Number	Divisible by								
	2	3	4	5	6	8	9	10	11
128	Yes	No	Yes	No	No	Yes	No	No	No
990	.....	.....	.....	.....	.....	.....	.....	.....	.....
1586	.....	.....	.....	.....	.....	.....	.....	.....	.....
275	.....	.....	.....	.....	.....	.....	.....	.....	.....
6686	.....	.....	.....	.....	.....	.....	.....	.....	.....
639210	.....	.....	.....	.....	.....	.....	.....	.....	.....
429714	.....	.....	.....	.....	.....	.....	.....	.....	.....
2856	.....	.....	.....	.....	.....	.....	.....	.....	.....
3060	.....	.....	.....	.....	.....	.....	.....	.....	.....
406839	.....	.....	.....	.....	.....	.....	.....	.....	.....

Sol.

Number	Divisible by								
	2	3	4	5	6	8	9	10	11
128	Yes	No	Yes	No	No	Yes	No	No	No
990	Yes	Yes	No	Yes	Yes	No	Yes	Yes	Yes
1586	Yes	No	No	No	No	No	No	No	No
275	No	No	No	Yes	No	No	No	No	Yes
6686	Yes	No	No	No	No	No	No	No	No
639210	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes
429714	Yes	Yes	No	No	Yes	No	Yes	No	No
2856	Yes	Yes	Yes	No	Yes	Yes	No	No	No
3060	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No
406839	No	Yes	No	No	No	No	No	No	No

Q2. Using divisibility tests, determine which of the following numbers are divisible by 4; by 8.

(*a*) 572 (*b*) 726352 (*c*) 5500 (*d*) 6000 (*e*) 12159

(*f*) 14560 (*g*) 21084 (*h*) 31795072 (*i*) 1700 (*j*) 2150

Sol. (*a*) 572 → Divisible by 4 (last two digits are divisible by 4)
→ Not divisible by 8 (last three digits are not divisible by 8)

(*b*) 726352 → Divisible by 4 (last two digits are divisible by 4)
→ Divisible by 8 (last three digits are divisible by 8)

(*c*) 5500 → Divisible by 4 (last two digits are zero)
→ Not divisible by 8 (last three digits are not divisible by 8)

(*d*) 6000 → Divisible by 4 (last two digits are 0)
→ Divisible by 8 (last three digits are zero)

(*e*) 12159 → Not divisible by 4 and 8 because it is an odd number.

(*f*) 14560 → Divisible by 4 (last two digits are divisible by 4)
→ Divisible by 8 (last three digits are divisible by 8)

(*g*) 21084 → Divisible by 4 (last two digits are divisible by 4)
→ Not divisible by 8 (last three digits are not divisible by 8)

(*h*) 31795072 → Divisible by 4 (last two digits are divisible by 4)
→ Divisible by 8 (last three digits are divisible by 8)

(*i*) 1700 → Divisible by 4 (last two digits are zero)
→ Not divisible by 8 (last three digits are not divisible by 8)

(*j*) 2150 → Not divisible by 4 (last two digits are not divisible by 4)
→ Not divisible by 8 (last three digits are not divisible by 8).

Q3. Using divisibility tests, determine which of following numbers are divisible by 6 :

(*a*) 297144 (*b*) 1258 (*c*) 4335 (*d*) 61233
(*e*) 901352 (*f*) 438750 (*g*) 1790184 (*h*) 12583
(*i*) 639210 (*j*) 17852

Sol. (*a*) **Divisible by 6 :** Divisible by 2 as '4' is at unit place. Sum of digits is 27 which is divisible by 3. Therefore number is also divisible by 3. Since the number is divisible by both 2 and 3 therefore it is also divisible by 6.

(*b*) **Not divisible by 6 :** Divisible by 2 but not by 3
Divisible by 2 as '8' appears at unit place.
Not divisible by 3 as sum of digits is 16 which is not divisible by 3.

(*c*) **Not divisible by 6 :** Divisible by 3 but not by 2.
Not divisible by 2 as '5' appears at unit place.

Divisible by 3 as sum of digits is '15' which is divisible by 3.

(*d*) **Not divisible by 6 :** Divisible by 3 but not by 2.
Not divisible by 2 as '3' appears at unit place.
Divisible by 3 as sum of digits is '15' which is divisible by 3.

(*e*) **Not divisible by 6 :** Divisible by 2 but not by 3.
Divisible by 2 as '2' appears at unit place.
Not divisible by 3 as sum of digits is 20 which is not divisible by 3.

(*f*) **Divisible by 6 :** Divisible by both 2 and 3.
Divisible by 2 as '0' appears at unit place.
Divisible by 3 as sum of digits is 27 which is divisible by 3.

(*g*) **Divisible by 6 :** Divisible by both 2 and 3.
Divisible by 2 as '4' appears at unit place.
Divisible by 3 as sum of digits is 30 which is divisible by 3.

(*h*) **Not divisible by 6 :** Not divisible by 2 and 3.
Not divisible by 2 as '3' is at unit place.
Not divisible by 3 as sum of digits is 19 which is not divisible by 3.

(*i*) **Divisible by 6 :** Divisible by both 2 and 3.
Divisible by 2 as '0' appears at unit place.
Divisible by 3 as sum of digits be '21' which is divisible by 3.

(*j*) **Not divisible by 6 :** Divisible by 2 but not divisible by 3.
Divisible by 2 as '2' appears at unit place.
Not divisible by 3 as sum of digits be 23 which is not divisible by 3.

Q4. Using divisibility tests, determine which of the following numbers are divisible by 11:

(*a*) 5445 (*b*) 10824 (*c*) 7138965 (*d*) 70169308
(*e*) 10000001 (*f*) 901153

Sol. (*a*) Sum of the digits at odd places : 4 + 5 = 9
Sum of the digits at even places : 4 + 5 = 9
Difference : 9 – 9 = 0
Since the difference is zero. ∴ number is divisible by 11.

(*b*) Sum of digits at odd places : 4 + 8 + 1 = 13
Sum of digits at even places : 2 + 0 = 2
Difference : 13 – 2 = 11
Since the difference is divisible by 11. ∴ number is divisible by 11.

(*c*) Sum of digits at odd places : 5 + 9 + 3 + 7 = 24
Sum of digits at even places : 6 + 8 + 1 = 15
Difference : 24 – 15 = 9
Since the difference is neither zero nor multiple of 11 ∴ number is not divisible by 11.

(*d*) Sum of digits at odd places : 8 + 3 + 6 + 0 = 17
Sum of digits at even places : 0 + 9 + 1 + 7 = 17
Difference = 17 – 17 = 0
Since the difference is zero ∴ the number is divisible by 11.

(*e*) Sum of digits at odd places : 1 + 0 + 0 + 0 = 1
Sum of digits at even place : 0 + 0 + 0 + 1 = 1
Difference : 1 – 1 = 0
Since the difference is zero, ∴ the number is divisible by 11.

(*f*) Sum of digits at odd places : 3 + 1 + 0 = 4
Sum of digits at even places : 5 + 1 + 9 = 15
Difference : 15 – 4 = 11
Since the difference is divisible by 11. ∴ the number is divisible by 11.

Q5. Write the smallest digit and the largest digit in the blank space of each of the following numbers so that the number formed is divisible by 3:

(*a*) _____ 6724 **(*b*) 4765 _____ 2**

Sol. (*a*) _____ 6724
Smallest digit : 2
Largest digit : 8

(*b*) 4765 _____ 2
Smallest digit : 0
Largest digit : 9.

Q6. Write a digit in the blank space of each of the following numbers so that the number formed is divisible by 11:

(*a*) 92 _____ 389 **(*b*) 8 _____ 9484**

Sol. (*a*) 8
(*b*) 6

EXERCISE 3.4

Q1. Find the common factors of:

(*a*) 20 and 28 (*b*) 15 and 25 (*c*) 35 and 50 (*d*) 56 and 120

Sol. (*a*) Factors of 20 = 1, 2, 4, 5, 10, 20
Factors of 28 = 1, 2, 4, 7, 14, 28
Common factors = 1, 2, 4.

(*b*) Factors of 15 = 1, 3, 5, 15
Factors of 25 = 1, 5, 25
Common factors are 1 and 5.

(*c*) Factors of 35 = 1, 5, 7, 35
Factors of 50 = 1, 2, 5, 10, 25, 50
Common factors are 1 and 5.

(*d*) Factors of 56 = 1, 2, 4, 7, 8, 14, 28, 56
Factors of 120 = 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 60, 120
Common factors are 1, 2, 4, 8.

Q2. Find the common factors of:

(*a*) 4, 8 and 12 **(*b*) 5, 15 and 25**

Sol. (*a*) Factors of 4 = 1, 2, 4
Factors of 8 = 1, 2, 4, 8
Factors of 12 = 1, 2, 3, 4, 6, 12
Common factors of 4, 8 and 12 are 1, 2 and 4.

(*b*) Factors of 5 = 1, 5
Factors of 15 = 1, 3, 5, 15
Factors of 25 = 1, 5, 25
Common factors of 5, 15 and 25 are 1 and 5.

Q3. Find first three common multiples of:

(*a*) 6 and 8 **(*b*) 12 and 18**

Sol. (*a*) Multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 72, ...
Multiples of 8 = 8, 16, 24, 32, 40, 48, 56, 64, 72, ...
Common multiples = 24, 48, 72.

(*b*) Multiples of 12 = 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 .....
Multiples of 18 = 18, 36, 54, 72, 90, 108 .....
Common multiples = 36, 72, 108.

Q4. Write all the numbers less than 100 which are common multiples of 3 and 4.

Sol. Multiples of 3 = 3, 6, 9, 12, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90, 93, 96, 99.
Multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100.
Common multiples = 12, 24, 36, 48, 60, 72, 84, 96.

Q5. Which of the following numbers are co-prime?

(*a*) 18 and 35 (*b*) 15 and 37 (*c*) 30 and 415
(*d*) 17 and 68 (*e*) 216 and 215 (*f*) 81 and 16

Sol. (*a*) Factors of 18 = 1, 2, 3, 6, 9, 18
Factors of 35 = 1, 5, 7, 35
Common factor = 1
Since both have only one common factor *i.e.*, 1, therefore, they are co-prime numbers.

(*b*) Factors of 15 = 1, 3, 5, 15
Factors of 37 = 1, 37

Common factor = 1

Since both have only one common factor *i.e.*, 1, therefore, they are co-prime numbers.

(*c*) Factors of 30 = 1, 2, 5, 6, 15, 30

Factors of 415 = 1, 5, ..., 83, 415

Common factor = 1, 5

Since they have more than one common factor, therefore, they are not co-prime numbers.

(*d*) Factors of 17 = 1, 17

Factors of 68 = 1, 2, 4, 17, 34, 68

Common factor = 1, 17

Since they have more than one common factor, therefore, they are not co-prime numbers.

(*e*) Factors of 216 = 1, 2, 3, 4, 6, 8, 36, 72, 108, 216

Factors of 215 = 1, 5, 43, 215

Common factor = 1

Since they have only one common factor *i.e.*, 1, therefore, they are co-prime numbers.

(*f*) Factors of 81 = 1, 3, 9, 27, 81

Factors of 16 = 1, 2, 4, 8, 16

Common factor = 1

Since they have only one common factor *i.e.*, 1, therefore, they are co-prime numbers.

Q.6. A number is divisible by both 5 and 12. By which other number will that number be always divisible?

Sol. 5 × 12 = 60. The number must be divisible by 60.

Q.7. A number is divisible by 12. By what other numbers will that number be divisible?

Sol. Factor of 12 are 1, 2, 3, 4, 6, 12. Therefore the number also be divisible by 1, 2, 3, 4 and 6.

EXERCISE 3.5

Q1. Which of the following statements are true?

(*a*) If a number is divisible by 3, it must be divisible by 9.

(*b*) If a number is divisible by 9, it must be divisible by 3.

(*c*) A number is divisible by 18, if it is divisible by both 3 and 6.

(*d*) If a number is divisible by 9 and 10 both, then it must be divisible by 90.

(*e*) If two numbers are co-primes, at least one of them must be prime.

(*f*) All numbers which are divisible by 4 must also be divisible by 8.

(*g*) All numbers which are divisible by 8 must also be divisible by 4.

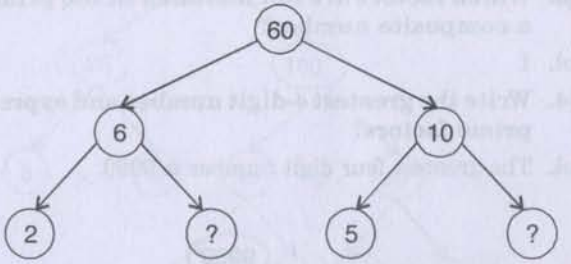
(*h*) If a number exactly divides two numbers separately, it must exactly divide their sum.

(*i*) If a number exactly divides the sum of two numbers, it must exactly divide the two numbers separately?

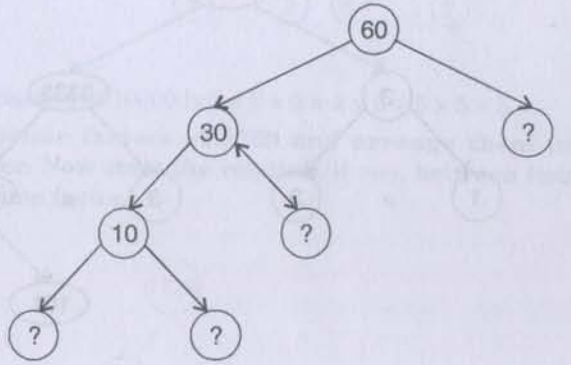
Sol. (*b*), (*c*), (*d*), (*g*), (*h*).

Q2. Here are two different factor trees for 60. Write the missing numbers.

(*a*)

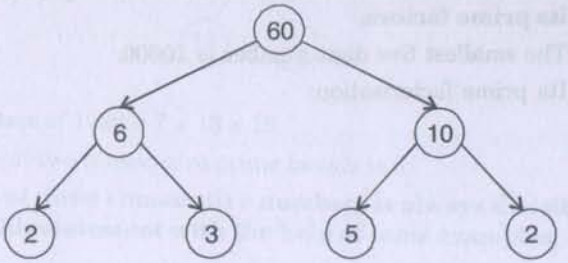


(*b*)

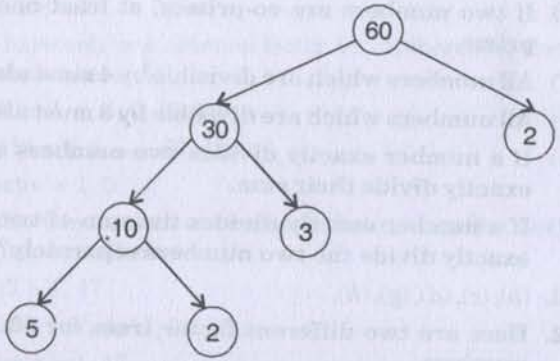


Sol.

(*a*)



(*b*)

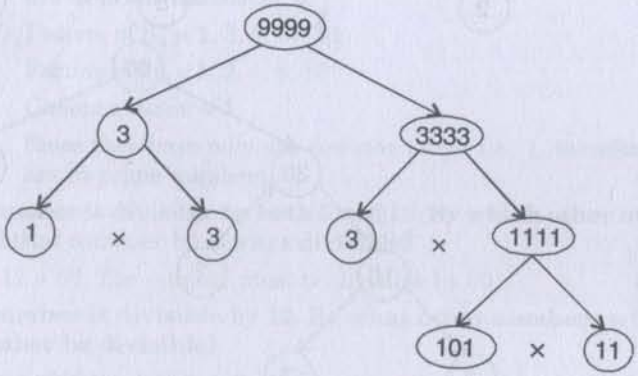


Q3. Which factors are not included in the prime factorisation of a composite number?

Sol. 1

Q4. Write the greatest 4-digit number and express it in terms of its prime factors?

Sol. The greatest four digit number is 9999.

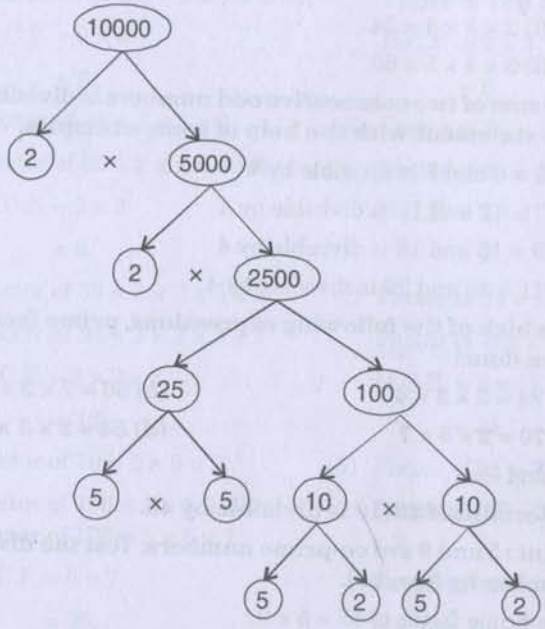


The prime factorisation of 9999 is $3 \times 3 \times 11 \times 101$.

Q5. Write the smallest 5-digit number and express it in the form of its prime factors.

Sol. The smallest five digit number is 10000.

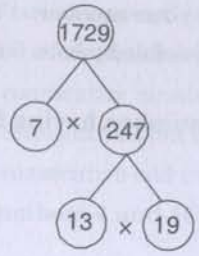
Its prime factorisation:



The prime factorisation of 10000 is $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$.

Q6. Find all the prime factors of 1729 and arrange them in ascending order. Now state the relation, if any, between two consecutive prime factors.

Sol.



$\therefore$ Prime factors of $1729 = 7 \times 13 \times 19$

The difference of two consecutive prime factors is 6.

Q7. The product of three consecutive numbers is always divisible by 6. Verify this statement with the help of some examples.

Sol. Among the three consecutive numbers, there must be one even number and one multiple of 3. Thus the product must be multiple of 6.

Eq. (*i*) $2 \times 3 \times 4 = 24$

(*ii*) $3 \times 4 \times 5 = 60$.

Q8. The sum of two consecutive odd numbers is divisible by 4. Verify this statement with the help of some examples.

Sol. $3 + 5 = 8$ and 8 is divisible by 4

$5 + 7 = 12$ and 12 is divisible by 4

$7 + 9 = 16$ and 16 is divisible by 4

$9 + 11 = 20$ and 20 is divisible by 4.

Q9. In which of the following expressions, prime factorisation has been done:

(*a*) $24 = 2 \times 3 \times 4$ **(*b*) $56 = 7 \times 2 \times 2 \times 2$**

(*c*) $70 = 2 \times 5 \times 7$ **(*d*) $54 = 2 \times 3 \times 9$**

Sol. (*b*) and (*c*).

Q10. Determine if 25110 is divisible by 45.

[Hint : 5 and 9 are co-prime numbers. Test the divisibility of the number by 5 and 9].

Sol. The prime factor of $45 = 5 \times 9$.

25110 is divisible by 5 as '0' is at unit place.

25110 is divisible by 9 as sum of digits it 9.

$\therefore$ The number must be divisible by $5 \times 9 = 45$.

Q11. 18 is divisible by both 2 and 3. It is also divisible by $2 \times 3 = 6$. Similarly, a number is divisible by both 4 and 6. Can we say that the number must also be divisible by $4 \times 6 = 24$? If not, give an example to justify your answer.

Sol. No. Number 12 is divisible by both 6 and 4 but 12 is not divisible by 24.

Q12. I am the smallest number, having four different prime factors. Can you find me?

Sol. $2 \times 3 \times 5 \times 7 = 210$.

EXERCISE 3.6

Q1. Find the H.C.F. of the following numbers.

(*a*) 18, 48 **(*b*) 30, 42** **(*c*) 18, 60** **(*d*) 27, 63**

(*e*) 36, 84 **(*f*) 34, 102** **(*g*) 70, 105, 175** **(*h*) 91, 112, 49**

(*i*) 18, 54, 81 **(*j*) 12, 45, 75**

Sol. (*a*) Factor of $18 = 2 \times 3 \times 3$

Factor of $48 = 2 \times 2 \times 2 \times 2 \times 3$

H.C.F. $= 2 \times 3$

$= 6$.

(*b*) Factor of $30 = 2 \times 3 \times 5$

Factor of $42 = 2 \times 3 \times 7$

H.C.F. $= 2 \times 3$

$= 6$.

(*c*) Factor of $18 = 2 \times 3 \times 3$

Factor of $60 = 2 \times 2 \times 3 \times 5$

H.C.F. $= 2 \times 3$

$= 6$.

(*d*) Factor of $27 = 3 \times 3 \times 3$

Factor of $63 = 3 \times 3 \times 7$

H.C.F. $= 3 \times 3$

$= 9$.

(*e*) Factor of $36 = 2 \times 2 \times 3 \times 3$

Factor of $84 = 2 \times 2 \times 3 \times 7$

H.C.F. $= 2 \times 2 \times 3$

$= 12$.

(*f*) Factor of $34 = 2 \times 17$

Factor of $102 = 2 \times 3 \times 17$

H.C.F. $= 2 \times 17$

$= 34$.

(*g*) Factor of $70 = 2 \times 5 \times 7$

Factor of $105 = 3 \times 5 \times 7$

Factor of $175 = 5 \times 5 \times 7$

H.C.F. $= 5 \times 7$

$= 35$.

(*h*) Factor of $91 = 7 \times 13$

Factor of $112 = 7 \times 2 \times 2 \times 2 \times 2$

Factor of $49 = 7 \times 7$

H.C.F. $= 7$.

(*i*) Factor of $18 = 2 \times 3 \times 3$

Factor of $54 = 2 \times 3 \times 3 \times 3$

Factor of $81 = 3 \times 3 \times 3 \times 3$

H.C.F. $= 3 \times 3$

$= 9$.

(*j*) Factor of $12 = 2 \times 2 \times 3$

Factor of $45 = 5 \times 3 \times 3$

Factor of $75 = 3 \times 5 \times 5$

H.C.F. $= 3$.

Q2. What is the H.C.F. of two consecutive

(*a*) numbers? **(*b*) even numbers?** **(*c*) odd numbers?**

Sol. (*a*) The H.C.F. of two consecutive numbers be 1.

(*b*) The H.C.F. of two consecutive even numbers be 2.

(*c*) The H.C.F. of two consecutive odd numbers be 1.

Q3. H.C.F. of co-prime numbers 4 and 15 was found as follows by factorisation:

$4 = 2 \times 2$ and $15 = 3 \times 5$ since there is no common prime factor, so H.C.F. of 4 and 15 is 0. Is the answer correct? If not, what is the correct H.C.F?

Sol. No, the correct H.C.F. is 1.

EXERCISE 3.7

Q1. Renu purchases two bags of fertiliser of weights 75 kg and 69 kg. Find the maximum value of weight which can measure the weight of the fertiliser exact number of times.

Sol. Since we have to find out the maximum value of weight to measure 75 kg and 69 kg. We have to find out 'Highest common factor' of 75 and 69.

Factor of $75 = 3 \times 5 \times 5$

Factor of $69 = 3 \times 23$

H.C.F. = 3

Therefore 3 kg weight is required.

Q2. Three boys step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm, respectively. What is the minimum distance each should cover so that all can cover the distance in complete steps?

Sol. Since minimum distance is required. Therefore the least common multiple of 63, 70 and 77 are to be calculated.

L.C.M. of 63, 70, 77

7	63, 70, 77
9	9, 10, 11
10	1, 10, 11
11	1, 1, 11
	1, 1, 1

L.C.M. of 63, 70, 77 = $7 \times 9 \times 10 \times 11 = 6930$ cm.

Q3. The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find the longest tape which can measure the three dimensions of the room exactly.

Sol. The measurement of longest tape requires highest common factor of 825 cm, 675 cm and 450 cm.

Factor of $825 = 3 \times 5 \times 5 \times 11$

Factor of $675 = 3 \times 5 \times 5 \times 3 \times 3$

Factor of $450 = 3 \times 3 \times 5 \times 5 \times 2$

H.C.F. = $3 \times 5 \times 5 = 75$ cm.

The length of longest tape is 75 cm.

Q4. Determine the smallest 3-digit number which is exactly divisible by 6, 8 and 12.

Sol. L.C.M. of 6, 8, 12

2	6, 8, 12
2	3, 4, 6
2	3, 2, 3
3	3, 1, 3
	1, 1, 1

$= 2 \times 2 \times 2 \times 3 = 24$

The smallest 3-digit number is 100.

To find the number, 100 is divided by 24.

$$24 \overline{)100} (4$$
$$\underline{96}$$
$$4$$

The number is $100 + (24 - 4) = 120$.

Q5. Determine the largest 3-digit number exactly divisible by 8, 10 and 12.

Sol. The L.C.M. of 8, 10, 12

2	8, 10, 12
2	4, 5, 6
2	2, 5, 3
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

L.C.M. = $2 \times 2 \times 2 \times 3 \times 5 = 120$

The largest three digit number is 999.

To find out the number, 999 is divided by 120.

$$120 \overline{)999} (8$$
$$\underline{960}$$
$$39$$

The number is $999 - 39 = 960$.

Q6. The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m. at what time will they change simultaneously again?

Sol. The L.C.M. of 48, 72 and 108 are

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

L.C.M. of 48, 72 and 108 is

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$= 432$ sec.

After 432 sec. the three lights change simultaneously.

Now 432 sec = 7 m 12 sec.

Therefore the time is 7 a.m. + 7 m 12 sec

= 7 : 07 : 12 a.m.

Q7. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of three containers exact number of times.

Sol. To find the maximum capacity of container, the H.C.F. of 403, 434 and 465 is required.

Factor of $403 = 13 \times 31$

Factor of $434 = 2 \times 7 \times 31$

Factor of $465 = 5 \times 3 \times 31$

H.C.F. = 31

31 litres of container is required to measure the quantity.

Q8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.

Sol. The L.C.M. of 6, 15 and 18 is

3	6, 15, 18
2	2, 5, 6
3	1, 5, 3
5	1, 5, 1
	1, 1, 1

$= 3 \times 3 \times 2 \times 5 = 90$

The number is $90 + 5 = 95$.

Q9. Find the smallest 4-digit number which is divisible by 18, 24 and 32.

Sol. The number which is exactly divisible by 18, 24 and 32.

The L.C.M. of 18, 24 and 32

2	18, 24, 32
2	9, 12, 16
2	9, 6, 8
3	9, 3, 4
3	3, 1, 4
4	1, 1, 4
	1, 1, 1

$= 2 \times 2 \times 2 \times 3 \times 3 \times 4 = 288$

The smallest four digit number is 1000

$$\begin{array}{r} 288 \overline{)\,1000\,(\,3} \\ \underline{864} \\ 136 \end{array}$$

The required number is $1000 + (288 - 136) = 1152$.

Q10. Find the L.C.M. of the following numbers:

(a) 9 and 4 (b) 12 and 5 (c) 6 and 5 (d) 15 and 4

Observe a common property in the obtained L.C.Ms. Is L.C.M. the product of two numbers in each case?

Sol. (a) L.C.M. of 9 and 4

2	9, 4
2	9, 2
3	9, 1
3	3, 1
	1, 1

$= 2 \times 2 \times 3 \times 3 = 36$

(b) L.C.M. of 12 and 5

2	12, 5
2	6, 5
3	3, 5
5	1, 5
	1, 1

$= 2 \times 2 \times 3 \times 5 = 60$

(c) L.C.M. of 6 and 5

2	6, 5
3	3, 5
5	1, 5
	1, 1

$= 2 \times 3 \times 5 = 30$

(d) L.C.M. of 15 and 4

2	15, 4
2	15, 2
3	15, 1
5	5, 1
	1, 1

$= 2 \times 2 \times 3 \times 5 = 60$

Yes, the L.C.M. is equal to the product of two numbers in each case. And L.C.M. is also the multiple of 3.

Q11. Find the L.C.M. of the following numbers in which one number is the factor of the other.

(a) 5, 20 **(b) 6, 18**

(c) 12, 48 **(d) 9, 45**

What do you observe in the results obtained?

Sol. (a) L.C.M. of 5, 20

2	5, 20
2	5, 10
5	5, 5
	1, 1

$= 2 \times 2 \times 5 = 20.$

(b) L.C.M. of 6, 18

2	6, 18
3	3, 9
3	1, 3
	1, 1

$= 2 \times 3 \times 3 = 18.$

(c) L.C.M. of 12 and 48

2	12, 48
2	6, 24
2	3, 12
2	3, 6
3	3, 3
	1, 1

$= 2 \times 2 \times 2 \times 2 \times 3 = 48$

(d) L.C.M. of 9, 45

3	9, 45
3	3, 15
5	1, 5
	1, 1

$= 3 \times 3 \times 5 = 45.$

This is all case where the smallest number is the factor of largest number, then the L.C.M. of two numbers is equal to that of largest number.

□□