12 Algebraic Expressions

Learn and Remember

1. Algebraic expression are formed from constants (fixed numerical value) and variables (various numerical values) connected by the basic operations of addition, subtraction, multiplication and division.
2. Expression can have one or more terms separated by the sign of '+' or '–'.
3. A **term** is a product of factors. Factors containing variables are said to be algebraic factors.
4. A **coefficient** may be either a numerical factor or an algebraic factor or a product of two or more factors. It is said to be the coefficient of the product of the remaining factors.
5. An expression with one or more terms is called **polynomial.** Thus, algebraic expression is called a monomial, a binomial and a trinomial according as it contains one term, two terms and three terms respectively.
6. Terms which have the same algebraic factor are called like term otherwise they are called unlike term.
7. The sum (or difference) of two like terms is a like term with coefficient equal to the sum (or difference) of the coefficient of the two like terms.
8. (*i*) To add two algebraic expressions, the like terms are added as given above (point 7) ; the unlike terms are left as they are.
   (*ii*) To subtract an expression from another, we change the sign of each term of the expression to be subtracted and then add the two expression.
9. The value of variable is the solution of the expression.
10. The general (n^{th}) term of a number pattern (or a sequence) is an expression is n.

TEXTBOOK QUESTIONS SOLVED

Exercise 12.1 *(Page No. 234-235)*

Q1. Get the algebraic expressions in the following cases using variables, constants and arithmetic operations.

(*i*) Subtraction of z from y.

(*ii*) One-half of the sum of numbers x and y.

(*iii*) The number z multiplied by itself.

(*iv*) One-fourth of the product of numbers p and q.

(*v*) Numbers x and y both squared and added.

(*vi*) Number 5 added to three times the product of numbers m and n.

(*vii*) Product of numbers y and z subtracted from 10.

(*viii*) Sum of numbers a and b subtracted from their product.

Sol. (*i*) Subtraction of z from y: $y - z$

(*ii*) One half of the sum of numbers x and y:
Sum of numbers x and $y = x + y$

Therefore, one-half of $x + y = \frac{x+y}{2}$.

(*iii*) The number z multiplied by itself:
$\Rightarrow \quad z \times z = z^2$.

(*iv*) One-fourth of the product of numbers p and q:
Product of numbers p and $q = pq$

Therefore, one-fourth of $pq = \frac{pq}{4}$.

(*v*) Numbers x and y both squared and added:
Square of number $x = x^2$
Square of number $y = y^2$
Therefore, sum of x^2 and $y^2 = x^2 + y^2$.

(*vi*) Number 5 added to three times the product of numbers m and n:
Product of numbers m and $n = mn$
Three times of $mn = 3mn$
Therefore, sum of $3mn$ and $5 = 3mn + 5$.

(*vii*) Product of numbers y and z subtracted from 10:
Product of numbers y and $z = yz$
Therefore, subtract yz from $10 = 10 - yz$.

(*viii*) Sum of numbers a and b subtracted from their product:

Sum of numbers a and $b = a + b$

Product of numbers a and $b = ab$

Therefore, subtract $(a + b)$ from $ab = ab - (a + b)$.

Q2. (*i*) Identify the terms and their factors in the following expressions show the terms and factors by tree diagrams:

(*a*) $x - 3$ (*b*) $1 + x + x^2$ (*c*) $y - y^3$

(*d*) $5xy^2 + 7x^2y$ (*e*) $-ab + 2b^2 - 3a^2$.

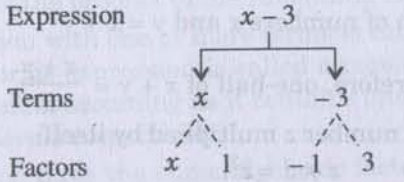
(*ii*) Identify terms and factors in the expressions given below:

(*a*) $-4x + 5$ (*b*) $-4x + 5y$ (*c*) $5y + 3y^2$

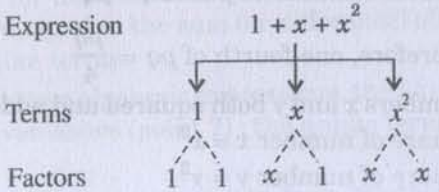
(*d*) $xy + 2x^2y^2$ (*e*) $pq + q$ (*f*) $1.2ab - 2.4b + 3.6a$

(*g*) $\frac{3}{4}x + \frac{1}{4}$ (*h*) $0.1p^2 + 0.2q^2$.

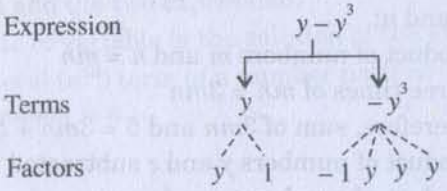
Sol. (*i*) (*a*) $x - 3$



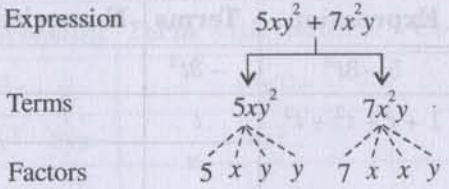
(*b*) $1 + x + x^2$



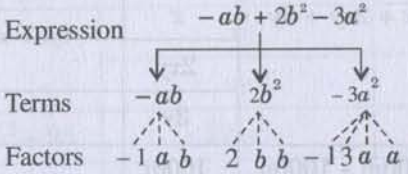
(*c*) $y - y^3$



(*d*) $5xy^2 + 7x^2y$



(*e*) $-ab + 2b^2 - 3a^2$



(*ii*) (*a*) $-4x + 5$

Terms: $-4x$, 5

Factors: $-4, x$; 5

(*b*) $-4x + 5y$

Terms: $-4x$, $5y$

Factors: $-4, x$; $5, y$

(*c*) $5y + 3y^2$

Terms: $5y$, $3y^2$

Factors: $5, y$; $3, y, y$

(*d*) $xy + 2x^2y^2$

Terms: xy, $2x^2y^2$

Factors: x, y; $2, x, x, y, y$

(*e*) $pq + q$

Terms: pq, q

Factors: p, q; q

(*f*) $1.2ab - 2.4b + 3.6a$

Terms: $1.2ab$, $-2.4b$, $3.6a$

Factors: $1.2, a, b$; $-2.4, b$; $3.6, a$

(*g*) $\frac{3}{4}x + \frac{1}{4}$

Terms: $\frac{3}{4}x$, $\frac{1}{4}$

Factors: $\frac{3}{4}, x$; $\frac{1}{4}$

(*h*) $0.1p^2 + 0.2q^2$

Terms: $0.1p^2$, $0.2q^2$

Factors: $0.1, p, p$; $0.2, q, q$.

Q3. Identify the numerical coefficients of terms (other than constants) in the following expressions:

(*i*) $5 - 3t^2$ (*ii*) $1 + t + t^2 + t^3$ (*iii*) $x + 2xy + 3y$

(*iv*) $100m + 1000n$ (*v*) $-p^2q^2 + 7pq$ (*vi*) $1.2a + 0.8b$

(*vii*) $3.14r^2$ (*viii*) $2(l + b)$ (*ix*) $0.1y + 0.01y^2$.

Sol.

S.No.	Expression	Terms	Numerical coefficient
(i)	$5 - 3t^2$	$-3t^2$	-3
(ii)	$1 + t + t^2 + t^3$	t	1
		t^2	1
		t^3	1
(iii)	$x + 2xy + 3y$	x	1
		$2xy$	2
		$3y$	3
(iv)	$100m + 1000n$	$100m$	100
		$1000n$	1000
(v)	$-p^2q^2 + 7pq$	$-p^2q^2$	-1
		$7pq$	7
(vi)	$1.2a + 0.8b$	$1.2a$	1.2
		$0.8b$	0.8
(vii)	$3.14r^2$	$3.14r^2$	3.14
(viii)	$2(l + b) = 2l + 2b$	$2l$	2
		$2b$	2
(ix)	$0.1y + 0.01y^2$	$0.1y$	0.1
		$0.01y^2$	0.01

Q4. (a) Identify terms which contain x and give the coefficient of x.

(i) $y^2x + y$ (ii) $13y^2 - 8yx$ (iii) $x + y + 2$
(iv) $5 + z + zx$ (v) $1 + x + xy$ (vi) $12xy^2 + 25$
(vii) $7x + xy^2$.

(b) Identify terms which contain y^2 and give the coefficient of y^2.

(i) $8 - xy^2$ (ii) $5y^2 + 7x$ (iii) $2x^2y - 15xy^2 + 7y^2$.

Sol. (a)

S.No.	Expression	Term with factor x	Coefficient of x
(i)	$y^2x + y$	y^2x	y^2
(ii)	$13y^2 - 8yx$	$-8yx$	$-8y$
(iii)	$x + y + 2$	x	1
(iv)	$5 + z + zx$	zx	z
(v)	$1 + x + xy$	x	1
		xy	y
(vi)	$12xy^2 + 25$	$12xy^2$	$12y^2$
(vii)	$7x + xy^2$	xy^2	y^2
		$7x$	7

(b)

S.No.	Expression	Term contains y^2	Coefficient of y^2
(i)	$8 - xy^2$	$-xy^2$	$-x$
(ii)	$5y^2 + 7x$	$5y^2$	5
(iii)	$2x^2y - 15xy^2 + 7y^2$	$-15xy^2$	$-15x$
		$7y^2$	7

Q5. Classify into monomials, binomials and trinomials.

(i) $4y - 7z$ (ii) y^2 (iii) $x + y - xy$
(iv) 100 (v) $ab - a - b$ (vi) $5 - 3t$
(vii) $4p^2q - 4pq^2$ (viii) $7mn$ (ix) $z^2 - 3z + 8$
(x) $a^2 + b^2$ (xi) $z^2 + z$ (xii) $1 + x + x^2$.

Sol.

S.No.	Expression	Type of polynomial
(i)	$4y - 7z$	Binomial
(ii)	y^2	Monomial
(iii)	$x + y - xy$	Trinomial

(iv)	100	Monomial
(v)	$ab - a - b$	Trinomial
(vi)	$5 - 3t$	Binomial
(vii)	$4p^2q - 4pq^2$	Binomial
(viii)	$7mn$	Monomial
(ix)	$z^2 - 3z + 8$	Trinomial
(x)	$a^2 + b^2$	Binomial
(xi)	$z^2 + z$	Binomial
(xii)	$1 + x + x^2$	Trinomial

Q6. State whether a given pair of terms is of like or unlike terms.

(i) 1, 100 (ii) $-7x, \frac{5}{2}x$ (iii) $-29x, -29y$

(iv) $14xy, 42yx$ (v) $4m^2p, 4mp^2$ (vi) $12xz, 12x^2z^2$.

Sol.

S.No.	Pair of terms	Like/Unlike terms
(i)	1, 100	Like terms
(ii)	$-7x, \frac{5}{2}x$	Like terms
(iii)	$-29x, -29y$	Unlike terms
(iv)	$14xy, 42yx$	Like terms
(v)	$4m^2p, 4mp^2$	Unlike terms
(vi)	$12xz, 12x^2z^2$	Unlike terms

Q7. Identify like terms in the following:

(a) $-xy^2, -4yx^2, 8x^2, 2xy^2, 7y, -11x^2, -100x, -11yx, 20x^2y, -6x^2, y, 2xy, 3x$.

(b) $10pq, 7p, 8q, -p^2q^2, -7qp, -100q, -23, 12q^2p^2, -5p^2, 41, 2405p, 78qp, 13p^2q, qp^2, 701p^2$.

Sol. (a) Like terms are:

(i) $-xy^2, 2xy^2$ (ii) $-4yx^2, 20x^2y$

(iii) $8x^2, -11x^2, -6x^2$ (iv) $7y, y$

(v) $-100x, 3x$ (vi) $-11yx, 2xy$.

(b) Like terms are:

(i) $10pq, -7qp, 78qp$ (ii) $7p, 2405p$

(iii) $8q, -100q$ (iv) $-p^2q^2, 12q^2p^2$

(v) $-23, 41$ (vi) $-5p^2, 701p^2$

(vii) $13p^2q, qp^2$.

Exercise 12.2 *(Page No. 239-240)*

Q1. Simplify combining like terms:

(i) $21b - 32 + 7b - 20b$

(ii) $-z^2 + 13z^2 - 5z + 7z^3 - 15z$

(iii) $p - (p - q) - q - (q - p)$

(iv) $3a - 2b - ab - (a - b + ab) + 3ab + b - a$

(v) $5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$

(vi) $(3y^2 + 5y - 4) - (8y - y^2 - 4)$.

Sol. (i) $21b - 32 + 7b - 20b$

$= 21b + 7b - 20b - 32$ (Rearranging like terms)

$= (21 + 7 - 20)b - 32$

$= 8b - 32$.

(ii) $-z^2 + 13z^2 - 5z + 7z^3 - 15z$

$= 7z^3 - z^2 + 13z^2 - 5z - 15z$

(Rearranging like terms)

$= 7z^3 + (-1 + 13)z^2 - (5 + 15)z$

$= 7z^3 + 12z^2 - 20z$.

(iii) $p - (p - q) - q - (q - p)$

$= p - p + q - q - q + p$

$= p - p + p + q - q - q$ (Rearranging like terms)

$= (1 - 1 + 1)p + (1 - 1 - 1)q$

$= (1)p + (-1)q$

$= p - q$.

(iv) $3a - 2b - ab - (a - b + ab) + 3ab + b - a$

$= 3a - 2b - ab - a + b - ab + 3ab + b - a$

$= 3a - a - a - 2b + b + b - ab - ab + 3ab$

(Rearranging like terms)

$= (3 - 1 - 1)a + (-2 + 1 + 1)b + (-1 - 1 + 3)ab$

$= (1)a + (0)b + (1)ab$

$= a + 0 + ab$

$= a + ab.$

(v) $5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$

$= 5x^2y + 3yx^2 + 8xy^2 - 5x^2 + x^2 - 3y^2 - y^2 - 3y^2$

(Rearranging like terms)

$= (5 + 3)x^2y + 8xy^2 + (-5 + 1)x^2 - (3 + 1 + 3)y^2$

$= 8x^2y + 8xy^2 - 4x^2 - 7y^2.$

(vi) $(3y^2 + 5y - 4) - (8y - y^2 - 4)$

$= 3y^2 + 5y - 4 - 8y + y^2 + 4$

$= 3y^2 + y^2 + 5y - 8y - 4 + 4$

(Rearranging like terms)

$= (3 + 1)y^2 + (5 - 8)y + 0$

$= (4)y^2 + (-3)y + 0$

$= 4y^2 - 3y.$

Q2. Add:

(i) $3mn, -5mn, 8mn, -4mn$

(ii) $t - 8tz, 3tz - z, z - t$

(iii) $-7mn + 5, 12mn + 2, 9mn - 8, -2mn - 3$

(iv) $a + b - 3, b - a + 3, a - b + 3$

(v) $14x + 10y - 12xy - 13, 18 - 7x - 10y + 8xy, 4xy$

(vi) $5m - 7n, 3n - 4m + 2, 2m - 3mn - 5$

(vii) $4x^2y, -3xy^2, -5xy^2, 5x^2y$

(viii) $3p^2q^2 - 4pq + 5, -10p^2q^2, 15 + 9pq + 7p^2q^2$

(ix) $ab - 4a, 4b - ab, 4a - 4b$

(x) $x^2 - y^2 - 1, y^2 - 1 - x^2, 1 - x^2 - y^2.$

Sol. (i) $3mn, -5mn, 8mn, -4mn$

$= 3mn + (-5mn) + 8mn + (-4mn)$ (Adding)

$= (3 - 5 + 8 - 4)mn$

$= (11 - 9)mn$

$= 2mn.$

(ii) $t - 8tz, 3tz - z, z - t$

$= (t - 8tz) + (3tz - z) + (z - t)$ (Adding)

$= t - 8tz + 3tz - z + z - t$

$= t - t - 8tz + 3tz - z + z$ (Rearranging like terms)

$= (1 - 1)t + (-8 + 3)tz + (-1 + 1)z$

$= 0(t) + (-5)tz + (0)z$

$= 0 - 5tz + 0$

$= -5tz.$

(iii) $-7mn + 5, 12mn + 2, 9mn - 8, -2mn - 3$

$= (-7mn + 5) + (12mn + 2) + (9mn - 8) + (-2mn - 3)$

(Adding)

$= -7mn + 5 + 12mn + 2 + 9mn - 8 - 2mn - 3$

$= -7mn + 12mn + 9mn - 2mn + 5 + 2 - 8 - 3$

(Rearranging like terms)

$= (-7 + 12 + 9 - 2)mn + 7 - 11$

$= 12mn - 4.$

(iv) $a + b - 3, b - a + 3, a - b + 3$

$= (a + b - 3) + (b - a + 3) + (a - b + 3)$ (Adding)

$= a + b - 3 + b - a + 3 + a - b + 3$

$= (a - a + a) + (b + b - b) - 3 + 3 + 3$

(Rearranging like terms)

$= a + b + 3.$

(v) $14x + 10y - 12xy - 13, 18 - 7x - 10y + 8xy, 4xy$

$= (14x + 10y - 12xy - 13) + (18 - 7x - 10y + 8xy) + 4xy$

(Adding)

$= 14x + 10y - 12xy - 13 + 18 - 7x - 10y + 8xy + 4xy$

$= 14x - 7x + 10y - 10y - 12xy + 8xy + 4xy - 13 + 18$

(Rearranging like terms)

$= (14 - 7)x + (10 - 10)y + (-12 + 8 + 4)xy - 13 + 18$

$= 7x + (0)y + (0)xy + 5$

$= 7x + 5.$

(vi) $5m - 7n, 3n - 4m + 2, 2m - 3mn - 5$

$= (5m - 7n) + (3n - 4m + 2) + (2m - 3mn - 5)$ (Adding)

$= 5m - 7n + 3n - 4m + 2 + 2m - 3mn - 5$

$= 5m - 4m + 2m - 7n + 3n - 3mn + 2 - 5$

(Rearranging like terms)

$= (5 - 4 + 2)m + (-7 + 3)n - 3mn - 3$

$= 3m - 4n - 3mn - 3.$

(vii) $4x^2y, -3xy^2, -5xy^2, 5x^2y$

$= 4x^2y + (-3xy^2) + (-5xy^2) + (5x^2y)$ (Adding)

$= 4x^2y + 5x^2y - 3xy^2 - 5xy^2$

(Rearranging like terms)

$= (4 + 5)x^2y - (3 + 5)xy^2$

$= 9x^2y - 8xy^2.$

(viii) $3p^2q^2 - 4pq + 5, -10p^2q^2, 15 + 9pq + 7p^2q^2$

$= (3p^2q^2 - 4pq + 5) + (-10p^2q^2) + (15 + 9pq + 7p^2q^2)$

(Adding)

$= 3p^2q^2 - 4pq + 5 - 10p^2q^2 + 15 + 9pq + 7p^2q^2$

$= 3p^2q^2 - 10p^2q^2 + 7p^2q^2 - 4pq + 9pq + 5 + 15$

(Rearranging like terms)

$= (3 - 10 + 7)p^2q^2 + (-4 + 9)pq + 5 + 15$

$= (0)\,p^2q^2 + (5)pq + 20$

$= 5pq + 20.$

(ix) $ab - 4a, 4b - ab, 4a - 4b$

$= (ab - 4a) + (4b - ab) + (4a - 4b)$ (Adding)

$= ab - 4a + 4b - ab + 4a - 4b$

$= -4a + 4a + 4b - 4b + ab - ab$

(Rearranging like terms)

$= (-4 + 4)a + (4 - 4)b + (1 - 1)ab$

$= (0)a + (0)b + (0)ab$

$= 0.$

(x) $x^2 - y^2 - 1, y^2 - 1 - x^2, 1 - x^2 - y^2$

$= (x^2 - y^2 - 1) + (y^2 - 1 - x^2) + (1 - x^2 - y^2)$ (Adding)

$= x^2 - y^2 - 1 + y^2 - 1 - x^2 + 1 - x^2 - y^2$

$= x^2 - x^2 - x^2 - y^2 + y^2 - y^2 - 1 - 1 + 1$

(Rearranging like terms)

$= (1 - 1 - 1)x^2 + (-1 + 1 - 1)y^2 - 1 - 1 + 1$

$= -x^2 - y^2 - 1.$

Q3. Subtract:

(i) $-5y^2$ from y^2 **(ii) $6xy$ from $-12xy$**

(iii) $(a - b)$ from $(a + b)$ **(iv) $a(b - 5)$ from $b(5 - a)$**

(v) $-m^2 + 5mn$ from $4m^2 - 3mn + 8$

(vi) $-x^2 + 10x - 5$ from $5x - 10$

(vii) $5a^2 - 7ab + 5b^2$ from $3ab - 2a^2 - 2b^2$

(viii) $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$

Sol. (i) $-5y^2$ from y^2

$= y^2 - (-5y^2)$ (Subtracting)

$= y^2 + 5y^2$

$= (1 + 5)y^2 = 6y^2.$

(ii) $6xy$ from $-12xy$

$= -12xy - (6xy)$ (Subtracting)

$= (-12 - 6)xy$

$= -18xy.$

(iii) $(a - b)$ from $(a + b)$

$= (a + b) - (a - b)$ (Subtracting)

$= a + b - a + b$

$= a - a + b + b$

(Rearranging like terms together)

$= (1 - 1)a + (1 + 1)b$

$= (0)a + 2b = 0 + 2b$

$= 2b.$

(iv) $a(b - 5)$ from $b(5 - a)$

$= b(5 - a) - a(b - 5)$ (Subtracting)

$= 5b - ab - ab + 5a$

$= 5b - (1 + 1)ab + 5a$

$= 5a + 5b - 2ab.$

(v) $-m^2 + 5mn$ from $4m^2 - 3mn + 8$

Subtracting expressions by column method:

$4m^2 - 3mn + 8$ (Subtracting and arranging like terms)
$-\ m^2 + 5mn$
$-\ (+)\quad (-)$
$5m^2 - 8mn + 8$ **Ans.**

(*vi*) $-x^2 + 10x - 5$ from $5x - 10$

Subtracting expressions by column method:

$5x - 10$ (Subtracting and arranging like terms)
$-x^2 + 10x - 5$
$-\ (+)\quad (-)\quad (+)$
$x^2 - 5x - 5$ **Ans.**

(*vii*) $5a^2 - 7ab + 5b^2$ from $3ab - 2a^2 - 2b^2$

$3ab - 2a^2 - 2b^2$ (Subtracting and arranging like terms)
$-7ab + 5a^2 + 5b^2$
$-\ (+)\quad (-)\quad (-)$
$10ab - 7a^2 - 7b^2$ **Ans.**

(*viii*) $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$

$5p^2 + 3q^2 - pq$ (Subtracting and arranging like terms)
$-3p^2 - 5q^2 + 4pq$
$-\ (+)\quad (+)\quad (-)$
$8p^2 + 8q^2 - 5pq$ **Ans.**

Q4. (*a*) What should be added to $x^2 + xy + y^2$ to obtain $2x^2 + 3xy$?

(*b*) What should be subtracted from $2a + 8b + 10$ to get $-3a + 7b + 16$?

Sol. (*a*) Let X polynomial should be added.

Then, $x^2 + xy + y^2 + X = 2x^2 + 3xy$

$\Rightarrow$ $X = 2x^2 + 3xy - (x^2 + xy + y^2)$

(transposing polynomial)

We write the expressions one below the other with the like terms appearing exactly below like terms as:

$2x^2 + 3xy$
$x^2 + xy + y^2$
$-\ (-)\quad (-)\quad (-)$
$x^2 + 2xy - y^2$

Thus, $x^2 + 2xy - y^2$ is the polynomial should be added.

(*b*) Let Y polynomial should be subtracted.

Then, $2a + 8b + 10 - Y = -3a + 7b + 16$

$\Rightarrow$ $Y = 2a + 8b + 10 - (-3a + 7b + 16)$

(transposing polynomial)

We write the expressions one below the other with the like terms appearing exactly below like terms as:

$2a + 8b + 10$
$-3a + 7b + 16$
$-\ (+)\quad (-)\quad (-)$
$5a + b - 6$

Thus, $5a + b - 6$ is the polynomial should be subtracted.

Q5. What should be taken away from $3x^2 - 4y^2 + 5xy + 20$ to obtain $-x^2 - y^2 + 6xy + 20$?

Sol. Let R polynomial should be subtracted.

Then,

$3x^2 - 4y^2 + 5xy + 20 - R = -x^2 - y^2 + 6xy + 20$

$\Rightarrow$ $R = 3x^2 - 4y^2 + 5xy + 20 - (-x^2 - y^2 + 6xy + 20)$

Subtracting and arranging the like terms in column,

$3x^2 - 4y^2 + 5xy + 20$
$-x^2 - y^2 + 6xy + 20$
$-\ (+)\quad (+)\quad (-)\quad (-)$
$4x^2 - 3y^2 - xy$

Thus, $4x^2 - 3y^2 - xy$ expression should be taken away.

Q6. (*a*) From the sum of $3x - y + 11$ and $-y - 11$, subtract $3x - y - 11$.

(*b*) From the sum of $4 + 3x$ and $5 - 4x + 2x^2$, subtract the sum of $3x^2 - 5x$ and $-x^2 + 2x + 5$.

Sol. (*a*) The sum of $3x - y + 11$ and $-y - 11$ is:

Arranging the like terms in columns,

$$\begin{array}{r} 3x - y + 11 \\ + \quad -y - 11 \\ \hline 3x - 2y \\ \hline \end{array}$$

Now, subtracting $3x - y - 11$ from the sum $3x - 2y$

$= 3x - 2y - (3x - y - 11)$

Arranging the like terms in columns,

$$\begin{array}{r} 3x - 2y \\ 3x - y - 11 \\ - \ (-) \quad (+) \quad (+) \\ \hline -y + 11 \\ \hline \end{array}$$

Ans.

(*b*) The sum of $4 + 3x$ and $5 - 4x + 2x^2$ is:

Arranging the like terms in columns,

$$\begin{array}{r} 4 + 3x \\ + \ 5 - 4x + 2x^2 \\ \hline 9 - x + 2x^2 \\ \hline \end{array}$$

and the sum of $3x^2 - 5x$ and $-x^2 + 2x + 5$ is (By column method):

$$\begin{array}{r} 3x^2 - 5x \\ + \ -x^2 + 2x + 5 \\ \hline 2x^2 - 3x + 5 \\ \hline \end{array}$$

Subtracting the sum $2x^2 - 3x + 5$ from the sum $9 - x + 2x^2$

i.e., $9 - x + 2x^2 - (2x^2 - 3x + 5)$

Arranging the like terms in columns:

$$\begin{array}{r} 9 - x + 2x^2 \\ 5 - 3x + 2x^2 \\ - \ (-) \quad (+) \quad (-) \\ \hline 4 + 2x \\ \hline \end{array}$$

Thus, the answer is $2x + 4$.

Exercise 12.3 *(Page No. 242)*

Q1. If $m = 2$, find the value of:

(*i*) $m - 2$ (*ii*) $3m - 5$ (*iii*) $9 - 5m$

(*iv*) $3m^2 - 2m - 7$ (*v*) $\frac{5m}{2} - 4$.

Sol. (*i*) $m - 2$

Putting $m = 2$ in $m - 2$ *i.e.*,

$= 2 - 2 = 0$.

(*ii*) $3m - 5$

Putting $m = 2$ in $3m - 5$ *i.e.*,

$= 3(2) - 5 = 6 - 5 = 1$.

(*iii*) $9 - 5m$

Putting $m = 2$ in $9 - 5m$ *i.e.*,

$= 9 - 5(2) = 9 - 10 = -1$.

(*iv*) $3m^2 - 2m - 7$

Putting $m = 2$ in $3m^2 - 2m - 7$ *i.e.*,

$= 3(2)^2 - 2(2) - 7 = 3(4) - 4 - 7$

$= 12 - 4 - 7 = 1$.

(*v*) $\frac{5m}{2} - 4$

Putting $m = 2$ in $\frac{5m}{2} - 4$ *i.e.*,

$= \frac{5}{2}(2) - 4 = 5 - 4 = 1$.

Q2. If $p = -2$, find the value of:

(*i*) $4p + 7$ (*ii*) $-3p^2 + 4p + 7$ (*iii*) $-2p^3 - 3p^2 + 4p + 7$

Sol. (*i*) $4p + 7$

Putting $p = -2$ in $4p + 7$ *i.e.*,

$= 4(-2) + 7 = -8 + 7 = -1$.

(*ii*) $-3p^2 + 4p + 7$

Putting $p = -2$ in $-3p^2 + 4p + 7$ *i.e.*,

$= -3(-2)^2 + 4(-2) + 7$

$= -3(4) - 8 + 7 = -12 - 8 + 7 = -13$.

(*iii*) $-2p^3 - 3p^2 + 4p + 7$

Putting $p = -2$ in $-2p^3 - 3p^2 + 4p + 7$ *i.e.*,

$= -2(-2)^3 - 3(-2)^2 + 4(-2) + 7$

$= -2(-8) - 3(4) - 8 + 7 = 16 - 12 - 1$

$= 3$.

Q3. Find the value of the following expressions, when $x = -1$:

(i) $2x-7$ (ii) $-x+2$ (iii) x^2+2x+1 (iv) $2x^2-x-2$

Sol. (i) $2x-7$

Putting $x=-1$ in $2x-7$ *i.e.*,

$= 2(-1)-7 = -2-7 = -9.$

(ii) $-x+2$

Putting $x=-1$ in $-x+2$ *i.e.*,

$= -(-1)+2 = 1+2 = 3.$

(iii) x^2+2x+1

Putting $x=-1$ in x^2+2x+1 *i.e.*,

$= (-1)^2+2(-1)+1 = 1-2+1 = 0.$

(iv) $2x^2-x-2$

Putting $x=-1$ in $2x^2-x-2$ *i.e.*,

$= 2(-1)^2-(-1)-2 = 2(1)+1-2$

$= 2+1-2 = 1.$

Q4. If $a = 2, b = -2$, find the value of:

(i) a^2+b^2 (ii) a^2+ab+b^2 (iii) a^2-b^2

Sol. On putting $a = 2$ and $b = -2$ in

(i) a^2+b^2, we get

$a^2+b^2 = (2)^2+(-2)^2 = 4+4 = 8.$

(ii) a^2+ab+b^2, we get

$a^2+ab+b^2 = (2)^2+(2)(-2)+(-2)^2$

$= 4-4+4 = 4.$

(iii) a^2-b^2, we get

$a^2-b^2 = (2)^2-(-2)^2 = 4-4 = 0.$

Q5. When $a = 0, b = -1$, find the value of the given expressions:

(i) $2a+2b$ (ii) $2a^2+b^2+1$

(iii) $2a^2b+2ab^2+ab$ (iv) $a^2+ab+2.$

Sol. On putting $a = 0$ and $b = -1$ in

(i) $2a+2b$, we get

$2a+2b = 2(0)+2(-1) = 0-2 = -2.$

(ii) $2a^2+b^2+1$, we get

$2a^2+b^2+1 = 2(0)+(-1)^2+1 = 0+1+1 = 2.$

(iii) $2a^2b+2ab^2+ab$, we get

$2a^2b+2ab^2+ab = 2(0)^2(-1)+2(0)(-1)^2+(0)(-1)$

$= 0+0+0 = 0.$

(iv) a^2+ab+2, we get

$a^2+ab+2 = (0)^2+(0)(-1)+2 = 0+0+2 = 2.$

Q6. Simplify the expressions and find the value if x is equal to 2:

(i) $x+7+4(x-5)$ (ii) $3(x+2)+5x-7$

(iii) $6x+5(x-2)$ (iv) $4(2x-1)+3x+11.$

Sol. (i) $x+7+4(x-5)$

$= x+7+4x-20$ (Solving)

$= x+4x+7-20$ (Rearranging like terms)

$= (1+4)x+(-13)$

$= 5x-13.$ ...(i)

Putting $x = 2$ in equation (i), we get

$5x-13 = 5(2)-13 = 10-13 = -3.$

(ii) $3(x+2)+5x-7$

$= 3x+6+5x-7$ (Solving)

$= 3x+5x+6-7$ (Rearranging like terms)

$= (3+5)x+(-1)$

$= 8x-1.$ ...(i)

Putting $x = 2$ in equation (i), we get

$8x-1 = 8(2)-1 = 16-1 = 15.$

(iii) $6x+5(x-2)$

$= 6x+5x-10$ (Solving)

$= (6+5)x-10$

$= 11x-10.$ ...(i)

Putting $x = 2$ in equation (i), we get

$11x-10 = 11(2)-10 = 22-10 = 12.$

(iv) $4(2x-1)+3x+11$

$= 8x-4+3x+11$ (Solving)

$= 8x+3x-4+11$ (Rearranging like terms)

$= (8+3)x-4+11$

$= 11x+7$ ...(i)

Putting $x = 2$ in equation (i), we get

$11x+7 = 11(2)+7 = 22+7 = 29.$

Q7. Simplify these expressions and find their values if $x = 3$, $a = -1$, $b = -2$.

(i) $3x - 5 - x + 9$ **(ii)** $2 - 8x + 4x + 4$

(iii) $3a + 5 - 8a + 1$ **(iv)** $10 - 3b - 4 - 5b$

(v) $2a - 2b - 4 - 5 + a$.

Sol. (i) $3x - 5 - x + 9$

$= 3x - x - 5 + 9$ (Rearranging like terms)

$= (3 - 1)x + 4$

$= 2x + 4$ ...(i)

Putting $x = 3$ in equation (i), we get

$2x + 4 = 2(3) + 4) = 6 + 4 = 10$.

(ii) $2 - 8x + 4x + 4$

$= -8x + 4x + 2 + 4$ (Rearranging like terms)

$= (-8 + 4)x + 6$

$= -4x + 6$ ...(i)

Putting $x = 3$ in equation (i), we get

$-4x + 6 = -4(3) + 6 = -12 + 6 = -6$.

(iii) $3a + 5 - 8a + 1$

$= 3a - 8a + 5 + 1$ (Rearranging like terms)

$= (3 - 8)a + 5 + 1$

$= -5a + 6$ ...(i)

Putting $a = -1$ in equation (i), we get

$-5a + 6 = -5(-1) + 6 = 5 + 6 = 11$.

(iv) $10 - 3b - 4 - 5b$

$= -3b - 5b + 10 - 4$ (Rearranging like terms)

$= (-3 - 5)b + 10 - 4$

$= -8b + 6$ ...(i)

Putting $b = -2$ in equation (i), we get

$-8b + 6 = -8(-2) + 6 = 16 + 6 = 22$.

(v) $2a - 2b - 4 - 5 + a$

$= 2a + a - 2b - 4 - 5$ (Rearranging like terms)

$= (2 + 1)a - 2b - 9$

$= 3a - 2b - 9$ ...(i)

Putting $a = -1$ and $b = -2$ in equation (i), we get

$3a - 2b - 9 = 3(-1) - 2(-2) - 9 = -3 + 4 - 9$

$= 4 - 12 = -8$.

Q8. (i) If $z = 10$, find the value of $z^3 - 3(z - 10)$.

(ii) If $p = -10$, find the value of $p^2 - 2p - 100$.

Sol. (i) Given, $z = 10$, then putting the value in $z^3 - 3(z - 10)$, we get

$z^3 - 3(z - 10) = z^3 - 3z + 30$

$= (10)^3 - 3(10) + 30$

$= 1000 - 30 + 30 = 1000$.

(ii) Given, $p = -10$, then putting the value in $p^2 - 2p - 100$, we get

$p^2 - 2p - 100 = (-10)^2 - 2(-10) - 100$

$= 100 + 20 - 100 = 20$.

Q9. What should be the value of a if the value of $2x^2 + x - a$ equals to 5, when $x = 0$?

Sol. Given that

$2x^2 + x - a = 5$

Putting $x = 0$ in given equation,

$2(0)^2 + 0 - a = 5$

$\Rightarrow \quad 0 + 0 - a = 5$

$\Rightarrow \quad -a = 5$

or $\quad a = -5$

Thus, the value of $a = -5$.

Q10. Simplify the expression and find its value when $a = 5$ and $b = -3$, $2(a^2 + ab) + 3 - ab$.

Sol. Simplifying the given equation

$= 2(a^2 + ab) + 3 - ab$

$= 2a^2 + 2ab + 3 - ab$

$= 2a^2 + 2ab - ab + 3$

(Rearranging like terms together)

$= 2a^2 + (2 - 1)ab + 3$

$= 2a^2 + ab + 3$. ...(i)

Putting $a = 5$ and $b = -3$ in equation (i), we get

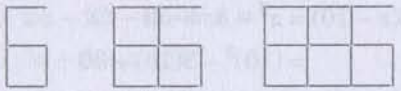
$2a^2 + ab + 3 = 2(5)^2 + (5)(-3) + 3$

$= 2(25) - 15 + 3$

$= 50 - 15 + 3 = 38$.

Exercise 12.4 *(Page No. 246-247)*

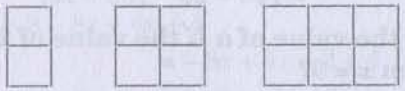
Q1. Observe the patterns of digits made from line segments of equal length. You will find such segmented digits on the display of electronic watches or calculators.

(*a*) 

6 11 16 21... (5*n* + 1)...

(*b*) 

4 7 10 13... (3*n* + 1)...

(*c*) 

7 12 17 22... (5*n* + 2)...

If the number of digits formed is taken to be *n*, the number of segments required to form *n* digits is given by the algebraic expression appearing on the right of each pattern.

How many segments are required to form 5, 10, 100 digits of the kind **?**

Sol.

S.No.	Symbol	Digit's number	Pattern's formulae	Number of segments
(*i*)		5	5*n* + 1	26
		10		51
		100		501
(*ii*)		5	(3*n* + 1)	16
		10		31
		100		301
(*iii*)		5	(5*n* + 2)	27
		10		52
		100		502

(*i*) 5*n* + 1

Putting $n = 5$, $5 \times 5 + 1 = 25 + 1 = 26$

$n = 10$, $5 \times 10 + 1 = 50 + 1 = 51$

$n = 100$, $5 \times 100 + 1 = 500 + 1 = 501$.

(*ii*) 3*n* + 1

Putting $n = 5$, $3 \times 5 + 1 = 15 + 1 = 16$

$n = 10$, $3 \times 10 + 1 = 30 + 1 = 31$

$n = 100$, $3 \times 100 + 1 = 300 + 1 = 301$.

(*iii*) 5*n* + 2

Putting $n = 5$, $5 \times 5 + 2 = 25 + 2 = 27$

$n = 10$, $5 \times 10 + 2 = 50 + 2 = 52$

$n = 100$, $5 \times 100 + 2 = 500 + 2 = 502$.

Q2. Use the given algebraic expression to complete the table of number patterns:

S.No.	Expression	Terms									
		1st	2nd	3rd	4th	5th	...	10th	...	100th	...
(*i*)	2*n* – 1	1	3	5	7	9	—	19	—	—	—
(*ii*)	3*n* + 2	2	5	8	11	—	—	—	—	—	—
(*iii*)	4*n* + 1	5	9	13	17	—	—	—	—	—	—
(*iv*)	7*n* + 20	27	34	41	48	—	—	—	—	—	—
(*v*)	$n^2 + 1$	2	5	10	17	—	—	—	—	10,001	—

Sol. (*i*) 2*n* – 1

Putting $n = 100$, $2n - 1 = 2 \times 100 - 1 = 200 - 1 = 199$.

(*ii*) 3*n* + 2

Putting $n = 5$, $3n + 2 = 3 \times 5 + 2 = 15 + 2 = 17$

$n = 10$, $3n + 2 = 3 \times 10 + 2 = 30 + 2 = 32$

$n = 100$, $3n + 2 = 3 \times 100 + 2 = 300 + 2 = 302$.

(*iii*) 4*n* + 1

Putting $n = 5$, $4n + 1 = 4 \times 5 + 1 = 20 + 1 = 21$

$n = 10$, $4n + 1 = 4 \times 10 + 1 = 40 + 1 = 41$

$n = 100$, $4n + 1 = 4 \times 100 + 1 = 400 + 1 = 401$.

(*iv*) $7n + 20$

Putting $n = 5$, $7n + 20 = 7 \times 5 + 20 = 35 + 20 = 55$

$n = 10$, $7n + 20 = 7 \times 10 + 20 = 70 + 20 = 90$

$n = 100$, $7n + 20 = 7 \times 100 + 20 = 700 + 20 = 720$.

(*v*) $n^2 + 1$

Putting $n = 5$, $n^2 + 1 = (5)^2 + 1 = 25 + 1 = 26$

$n = 10$, $n^2 + 1 = (10)^2 + 1 = 100 + 1 = 101$.

Now, complete table is

S.No.	Expression	Terms									
		1st	2nd	3rd	4th	5th	...	10th	...	100th	...
(*i*)	$2n - 1$	1	3	5	7	9	—	19	—	**199**	—
(*ii*)	$3n + 2$	2	5	8	11	**17**	—	**32**	—	**302**	—
(*iii*)	$4n + 1$	5	9	13	17	**21**	—	**41**	—	**401**	—
(*iv*)	$7n + 20$	27	34	41	48	**55**	—	**90**	—	**720**	—
(*v*)	$n^2 + 1$	2	5	10	17	**26**	—	**101**	—	10,001	—

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