7 Congruence of Triangles

Learn and Remember

1. Two figures that are exactly of the same shape and size are congruent figures.
2. Two triangles are congruent if their corresponding sides and angles are equal.
3. **SSS congruence of two triangles:** Under a given correspondence, two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.
4. **SAS congruence of two triangles:** Under a given correspondence, two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.
5. **ASA congruence of two triangles:** Under a given correspondence, two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.
6. **RHS congruence of two right-angled triangles:** Under a given correspondence, two right-angled triangles are congruent if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle.

TEXTBOOK QUESTIONS SOLVED

Exercise 7.1 *(Page No. 137)*

Q1. Complete the following statements:

(a) Two line segments are congruent if _____ .

(b) Among two congruent angles, one has a measure of 70°; the measure of the other angle is _____ .

(c) When we write $\angle A = \angle B$, we actually mean _____ .

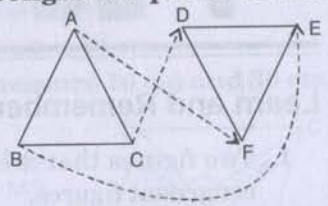
Sol. (a) they have the same length

(b) 70° (c) $m\angle A = m\angle B$.

Q2. Give any two real-life examples for congruent shapes.

Sol. (*i*) Two footballs; (*ii*) Two teacher's tables.

Q3. If $\Delta ABC \cong \Delta FED$ under the correspondence ABC $\leftrightarrow$ FED, write all the corresponding congruent parts of the triangles.



Sol. Given, $\Delta ABC \cong \Delta FED$

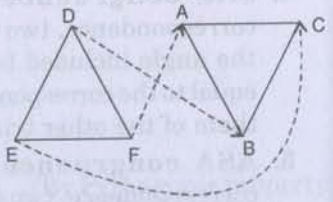
We know that the corresponding congruent parts of the triangles are

1. $\angle A \leftrightarrow \angle F$ 2. $\angle B \leftrightarrow \angle E$ 3. $\angle C \leftrightarrow \angle D$

4. $\overline{AB} \leftrightarrow \overline{FE}$ 5. $\overline{BC} \leftrightarrow \overline{ED}$ 6. $\overline{AC} \leftrightarrow \overline{FD}$.

Q4. If $\Delta DEF \cong \Delta BCA$, write the part(s) of ΔBCA that correspond to

(*i*) $\angle E$ (*ii*) $\overline{EF}$ (*iii*) $\angle F$ (*iv*) $\overline{DF}$



Sol. Given, $\Delta DEF \cong \Delta BCA$

(*i*) $\angle E \leftrightarrow \angle C$ (*ii*) $\overline{EF} \leftrightarrow \overline{CA}$

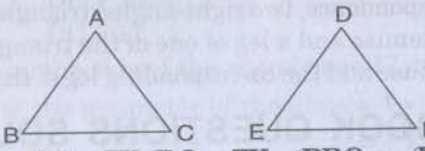
(*iii*) $\angle F \leftrightarrow \angle A$ (*iv*) $\overline{DF} \leftrightarrow \overline{BA}$.

Exercise 7.2 *(Page No. 149-151)*

Q1. Which congruence criterion do you use in the following?

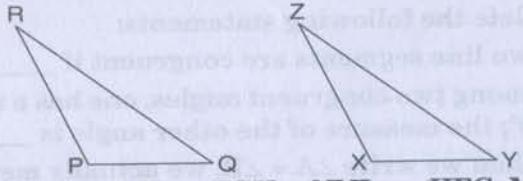
(*a*) Given: AC = DF, AB = DE, BC = EF.

So, $\Delta ABC \cong \Delta DEF$.



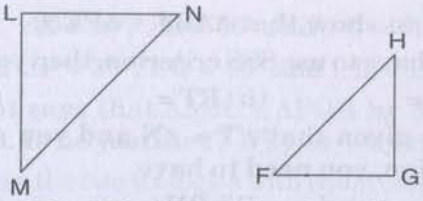
(*b*) Given: RP = ZX, RQ = ZY, $\angle PRQ = \angle XZY$

So, $\Delta PQR \cong \Delta XYZ$



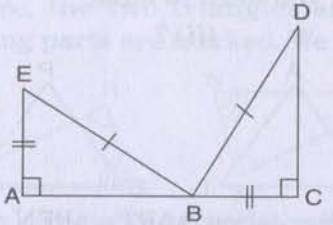
(*c*) Given: $\angle MLN = \angle FGH$, $\angle NML = \angle HFG$, ML = FG.

So, $\Delta LMN \cong \Delta GFH$



(*d*) Given: EB = BD, AE = CB, $\angle A = \angle C = 90°$.

So, $\Delta ABE \cong \Delta CDB$



Sol. (*a*) Given, AC = DF, AB = DE, BC = EF

So, $\Delta ABC \cong \Delta DEF$

The three sides of one triangle are equal to the three corresponding sides of another triangle.

Thus, by SSS congruence criterion.

(*b*) Given, RP = ZX, RQ = ZY and $\angle PRQ = \angle XZY$

So, $\Delta PQR \cong \Delta XYZ$

The two sides and the angle included between them in one of the triangle are equal to the corresponding sides and the angle included between them of other triangle.

Thus, by SAS congruence criterion.

(*c*) Given, $\angle MLN = \angle FGH$, $\angle NML = \angle HFG$, ML = FG

So, $\Delta LMN \cong \Delta GFH$

Now side ML is between $\angle L$ and $\angle M$ and side FG is between $\angle F$ and $\angle G$ are equal.

Thus, by ASA congruence criterion.

(*d*) Given, EB = BD, AE = CB, $\angle A = \angle C = 90°$ each.

So, $\Delta ABE \cong \Delta CDB$

Hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle.

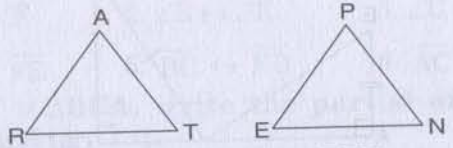
Thus, by RHS congruence criterion.

Q2. You want to show that $\Delta ART \cong \Delta PEN$:

(a) If you have to use SSS criterion, then you need to show
(i) AR = (ii) RT = (iii) AT

(b) If it is given that $\angle T = \angle N$ and you are to use SAS criterion, you need to have
(i) RT = and (ii) PN =

(c) If it is given that AT = PN and you are to use ASA criterion, you need to have
(i) ? (ii) ?



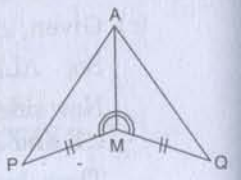
Sol. (a) Using SSS criterion, $\Delta ART \cong \Delta PEN$,
(i) AR = PE (ii) RT = EN (iii) AT = PN.

(b) Given, $\angle T = \angle N$
Using SAS criterion, $\Delta ART \cong \Delta PEN$,
(i) RT = EN (ii) PN = AT.

(c) Given, AT = PN
Using ASA criterion, $\Delta ART \cong \Delta PEN$,
(i) $\angle RAT = \angle EPN$ (ii) $\angle RTA = \angle ENP$.

Q3. You have to show that $\Delta AMP \cong \Delta AMQ$. In the following proof, supply the missing reasons.

Steps	Reasons
(i) PM = QM	(i) _____
(ii) $\angle PMA = \angle QMA$	(ii) _____
(iii) AM = AM	(iii) _____
(iv) $\Delta AMP \cong \Delta AMQ$	(iv) _____



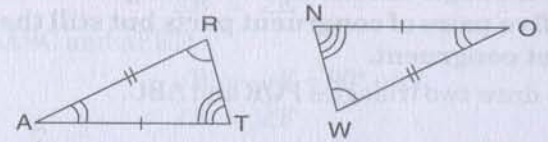
Sol.

Steps	Reasons
(i) PM = QM	(i) Given
(ii) $\angle PMA = \angle QMA$	(ii) Given
(iii) AM = AM	(iii) Common
(iv) $\Delta AMP \cong \Delta AMQ$	(iv) SAS congruence rule

Q4. In ΔABC, $\angle A = 30°$, $\angle B = 40°$ and $\angle C = 110°$
In ΔPQR, $\angle P = 30°$, $\angle Q = 40°$ and $\angle R = 110°$
A student says that $\Delta ABC \cong \Delta PQR$ by AAA congruence criterion. Is he justified? Why or why not?

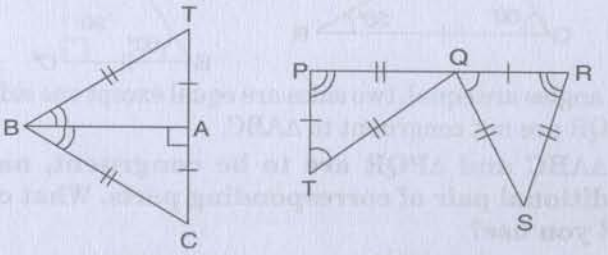
Sol. No, because the two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be an enlarged copy of the other.

Q5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write $\Delta RAT \cong$?



Sol. In the figure, given two triangles are congruent. So, the corresponding parts are $A \leftrightarrow O$, $R \leftrightarrow W$, $T \leftrightarrow N$.
We can write, $\Delta RAT \cong \Delta WON$. (By SAS congruence rule)

Q6. Complete the congruence statement:



$\Delta BCA \cong$? **$\Delta QRS \cong$?**

Sol. In ΔBAT and ΔBAC, given triangles are congruent so the corresponding parts are $B \leftrightarrow B$, $A \leftrightarrow A$, $T \leftrightarrow C$
Thus, $\Delta BCA \cong \Delta BTA$ (By SSS congruence rule)
In ΔQRS and ΔTPQ, given triangles are congruent so the corresponding parts are $P \leftrightarrow R$, $T \leftrightarrow Q$, $Q \leftrightarrow S$
Thus, $\Delta QRS \cong \Delta TPQ$ (By SSS congruence rule)

Q7. In a squared sheet, draw two triangles of equal areas such that
(i) the triangles are congruent.
(ii) the triangles are not congruent.
What can you say about their perimeters?

Sol. In a squared sheet, draw ΔABC and ΔPQR.
When two triangles have equal areas and

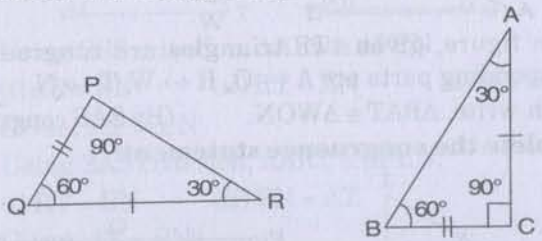
(*i*) these triangles are congruent, *i.e.*, $\Delta ABC \cong \Delta PQR$ (By SSS rule)

Then, their perimeters are same because length of sides of first triangle are equal to the length of sides of another triangle by SSS congruence rule.

(*ii*) But, if the triangles are not congruent *i.e.*, $\Delta ABC \ncong \Delta PQR$. Then their perimeters are not same because length of sides of first triangle are not equal to the length of corresponding sides of another triangle.

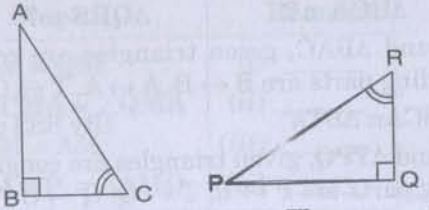
Q8. Draw a rough sketch of two triangles such that they have five pairs of congruent parts but still the triangles are not congruent.

Sol. Let us draw two triangles PQR and ABC.



All angles are equal, two sides are equal except one side. Hence, ΔPQR are not congruent to ΔABC.

Q9. If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



Sol. ΔABC and ΔPQR are congruent. Then one additional pair is

$\overline{BC} = \overline{QR}$

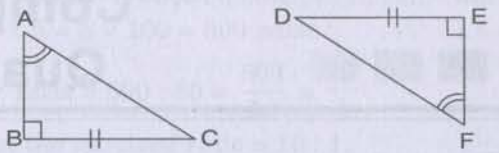
Given, $\angle B = \angle Q = 90°$ each

$\angle C = \angle R$

$\overline{BC} = \overline{QR}$ (Proved)

So, $\Delta ABC \cong \Delta PQR$ (By ASA congruence rule)

Q10. Explain, why $\Delta ABC \cong \Delta FED$.



Sol. Given, $\angle A = \angle F$

$BC = ED$

$\angle B = \angle E$

In ΔABC and ΔFED,

$\angle B = \angle E = 90°$ each (Given)

$\angle A = \angle F$ (Given)

side BC = side ED

Thus, $\Delta ABC \cong \Delta FED$. (By RHS criterion)

□□