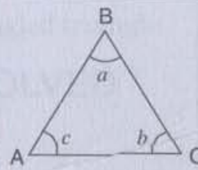
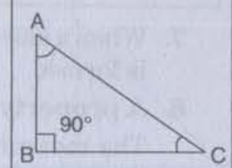
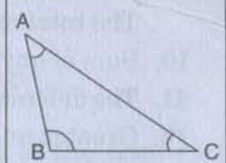
6 The Triangle and its Properties

Learn and Remember

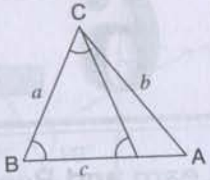
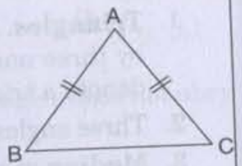
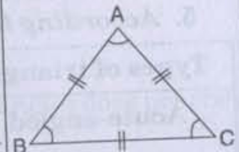
1. **Triangles.** A triangle is a closed, three sided figure obtained by three non-collinear points in a plane and symbol used to denote a triangle is Δ.
2. Three angles and three sides are the six elements of a triangle.
3. **Median** of a triangle is a line-segment joining a vertex to the mid-point of its opposite side. A triangle has three medians.
4. **Altitude** of a triangle is a perpendicular line-segment from a vertex to its opposite side.

Types of triangles:

5. *According to interior angles*

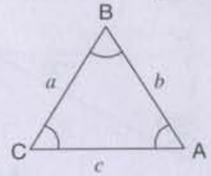
Types of triangles	Property/Definition	Diagram
Acute-angled	Each of the angle of a triangle is less than 90° *i.e.*, $a < 90°$, $b < 90°$ and $c < 90°$.	
Right-angled	One of the angle is equal to 90°, then it is called as right-angled triangle. Rest two angles are complementary to each other.	
Obtuse-angled	One of the angle is obtuse (*i.e.*, greater than 90°) then it is called as obtuse-angled triangle.	

6. *According to the length of sides*

Scalene triangle	A triangle in which none of the three sides are equal, is called a scalene triangle (all the three angles are also different) *i.e.*, $a \neq b \neq c$.	
Isosceles triangle	A triangle in which atleast two sides are equal is called an isosceles triangle. In this triangle, the angle opposite to the congruent sides are also equal. *i.e.*, if AB = AC, then $\angle B = \angle C$.	 AB = BC $\Rightarrow \angle B = \angle C$
Equilateral Triangle	A triangle in which all the three sides are equal, is called as an equilateral triangle. In this triangle, each angle is congruent and equal to 60°. *i.e.*, if AB = BC = CA, then $\angle A = \angle B = \angle C = 60°$.	 AB = BC = AC $\Rightarrow \angle A = \angle B = \angle C = 60°$

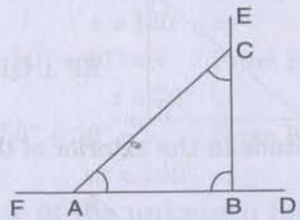
7. When a side of a triangle is produced then an **exterior angle** is formed.
8. **A property of exterior angles:** The measure of any exterior angle of a triangle is equal to the sum of the measure of its interior opposite angles.
9. **The angle sum property of a triangle:** The total measure of the three angles of a triangle is 180°.
10. Sum of any two sides is always greater than the third side.
11. The difference of any two sides is always less than the third side.
12. Greater angle has a greater side opposite to it and smaller angle has a smaller side opposite to it *i.e.*, if two sides of a triangle are not congruent then the angle opposite to the greater side is greater.

13. Let a, b and c be the three sides of ΔABC and c is the largest side then



➢ if $c^2 < a^2 + b^2$, the triangle is acute-angled triangle.
➢ if $c^2 = a^2 + b^2$, the triangle is right-angled triangle.
➢ if $c^2 > a^2 + b^2$, the triangle is obtuse-angled triangle.

14. A triangle has atleast two acute angles.

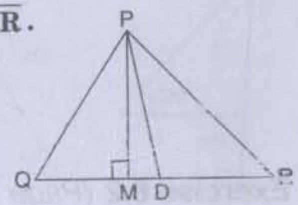


15. **Pythagoras property:** In a right-angled triangle, the square of the hypotenuse equals to the sum of the square of its sides. *i.e.*, $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Altitude})^2$
16. Hypotenuse is the longest side in a right-angled triangle.

TEXTBOOK QUESTIONS SOLVED

Exercise 6.1 *(Page No. 116)*

Q1. In ΔPQR, D is the mid-point of $\overline{QR}$.



$\overline{PM}$ is ________

PD is ________

Is QM = MR?

Sol. Given, QD = DR. So, $\overline{PM}$ is altitude.

PD is median.

No, QM $\neq$ MR.

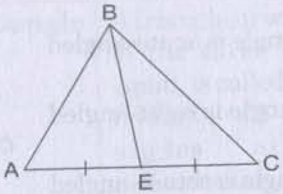
Q2. Draw rough sketches for the following:

(a) In ΔABC, BE is a median.

(b) In ΔPQR, PQ and PR are altitudes of the triangle.

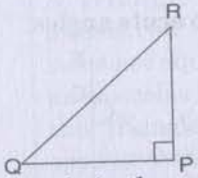
(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

Sol. (*a*) Draw BE is a median in ΔABC,



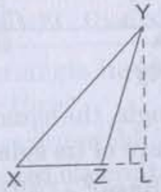
$\because$ AE = EC

(*b*) Draw PQ and PR are the altitudes of the ΔPQR,



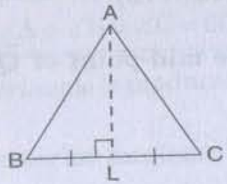
RP $\perp$ QP

(*c*) YL is an altitude in the exterior of the ΔXYZ,



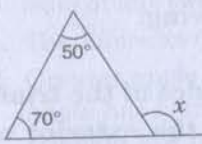
Q3. Verify by drawing a diagram if the median and altitude of a isosceles triangle can be same.

Sol. Isosceles triangle means any two sides are same.
Take ΔABC and draw the median when AB = AC.
AL is the median and altitude of the given triangle.

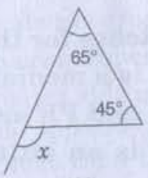


Exercise 6.2 *(Page No. 118-119)*

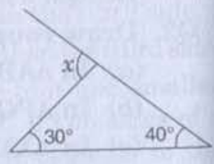
Q1. Find the value of the unknown exterior angle x in the following diagrams:



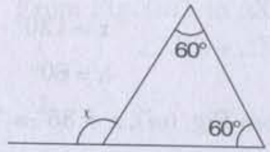
(*i*)



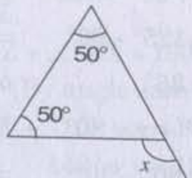
(*ii*)



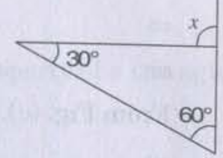
(*iii*)



(*iv*)



(*v*)



(*vi*)

Sol. We know that sum of interior opposite angles = exterior angle

From Fig. (*i*), $50° + 70° = x$ $\Rightarrow$ $x = 120°$

From Fig. (*ii*), $65° + 45° = x$ $\Rightarrow$ $x = 110°$

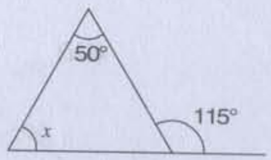
From Fig. (*iii*), $30° + 40° = x$ $\Rightarrow$ $x = 70°$

From Fig. (*iv*), $60° + 60° = x$ $\Rightarrow$ $x = 120°$

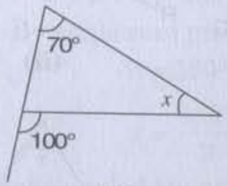
From Fig. (*v*), $50° + 50° = x$ $\Rightarrow$ $x = 100°$

From Fig. (*vi*), $60° + 30° = x$ $\Rightarrow$ $x = 90°$.

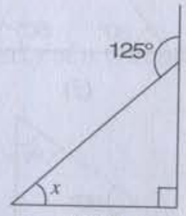
Q2. Find the value of the unknown interior angle x in the following figures:



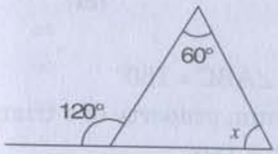
(*i*)



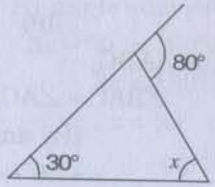
(*ii*)



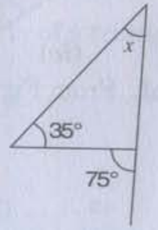
(*iii*)



(*iv*)



(*v*)



(*vi*)

Sol. As we know that sum of interior opposite angles = Exterior angle

From Fig. (*i*), $x + 50° = 115°$
$\Rightarrow$ $x = 115° - 50°$
or $x = 65°$

From Fig. (*ii*), $70° + x = 100°$
$\Rightarrow$ $x = 100° - 70°$
or $x = 30°$

From Fig. (*iii*), $x + 90° = 125°$

$\Rightarrow \quad x = 125° - 90°$

or $\quad x = 35°$

From Fig. (*v*), $30° + x = 80°$

$\Rightarrow \quad x = 80° - 30°$

or $\quad x = 50°$

From Fig. (*iv*), $60° + x = 120°$

$\Rightarrow \quad x = 120° - 60°$

or $\quad x = 60°$

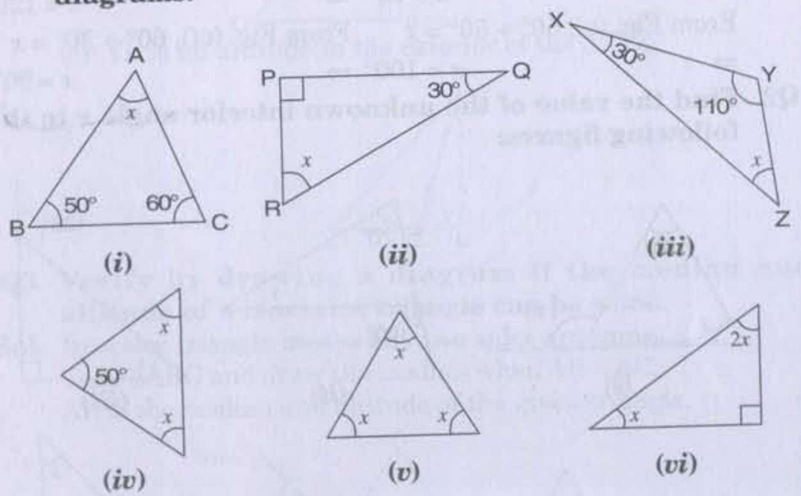
From Fig. (*vi*), $x + 35° = 75°$

$\Rightarrow \quad x = 75° - 35°$

or $\quad x = 40°$.

Exercise 6.3 *(Page No. 121-122)*

Q1. Find the value of the unknown x in the following diagrams:



Sol. From Fig. (*i*), in ΔABC,

$$\angle BAC + \angle ACB + \angle ABC = 180°$$

(By angle sum property of a triangle.)

$\Rightarrow \quad x + 50° + 60° = 180°$

$\Rightarrow \quad x + 110° = 180°$

or $\quad x = 180° - 110° = 70°$.

From Fig. (*ii*), in ΔPQR,

$$\angle RPQ + \angle PQR + \angle RPQ = 180°$$

(By angle sum property of a triangle.)

$\Rightarrow \quad 90° + 30° + x = 180°$

$\Rightarrow \quad 120° + x = 180°$

or $\quad x = 180° - 120° = 60°$.

From Fig. (*iii*), in ΔXYZ,

$$\angle XYZ + \angle XYZ + \angle YZX = 180°$$

(By angle sum property of a triangle.)

$\Rightarrow \quad 30° + 110° + x = 180°$

$\Rightarrow \quad 140° + x = 180°$

or $\quad x = 180° - 140° = 40°$.

From Fig. (*iv*), in given isosceles triangle,

$$x + x + 50° = 180°$$

(By angle sum property of a triangle.)

$\Rightarrow \quad 2x + 50° = 180°$

$\Rightarrow \quad 2x = 180° - 50°$

$\Rightarrow \quad 2x = 130°$

or $\quad x = \dfrac{130°}{2} = 65°$.

From Fig. (*v*), in given equilateral triangle,

$$x + x + x = 180°$$

(By angle sum property of a triangle.)

$\Rightarrow \quad 3x = 180°$

or $\quad x = \dfrac{180°}{3} = 60°$.

From Fig.(*vi*), in given right-angled triangle,

$$x + 2x + 90° = 180°$$

(By angle sum property of a triangle.)

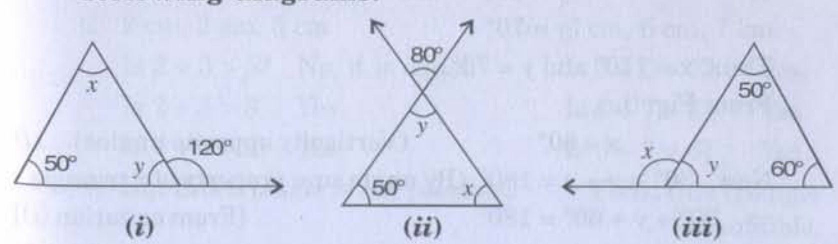
$\Rightarrow \quad 3x + 90° = 180°$

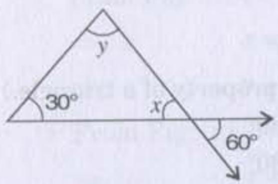
$\Rightarrow \quad 3x = 180° - 90°$

$\Rightarrow \quad 3x = 90°$

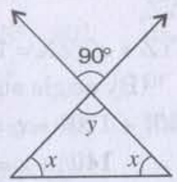
or $\quad x = \dfrac{90°}{3} = 30°$.

Q2. Find the values of the unknowns x and y in the following diagrams:

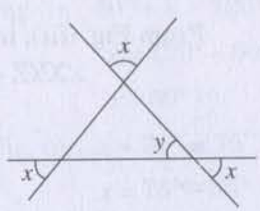




(iv)



(v)



(vi)

Sol. From Fig. (*i*),

$50° + x = 120°$ (Exterior angle property of a triangle.)

$\Rightarrow \quad x = 120° - 50°$

or $\quad x = 70°$ ...(*i*)

Now, $50° + x + y = 180°$ (By angle sum property of a triangle)

$50° + 70° + y = 180°$ [From equation (*i*)]

$\Rightarrow \quad 120° + y = 180°$

$\Rightarrow \quad y = 180° - 120°$

or $\quad y = 60°$

Thus, $x = 70°$ and $y = 60°$.

From Fig. (*ii*),

$y = 80°$ ...(*i*) (Vertically opposite angles)

Now, $50° + y + x = 180°$ (By angle sum property of a triangle.)

$\Rightarrow 50° + 80° + x = 180°$ [From equation (*i*)]

$\Rightarrow \quad 130° + x = 180°$

$\Rightarrow \quad x = 180° - 130°$

or $\quad x = 50°$

Thus, $x = 50°$ and $y = 80°$.

From Fig. (*iii*),

$50° + 60° = x$ (Exterior angle property of a triangle.)

or $\quad x = 110°$

Now, $50° + 60° + y = 180°$ (By angle sum property of a triangle.)

$\Rightarrow \quad 110° + y = 180°$

$\Rightarrow \quad y = 180° - 110°$

or $\quad y = 70°$

Thus, $x = 110°$ and $y = 70°$.

From Fig. (*iv*),

$x = 60°$ (Vertically opposite angles) ...(*i*)

Now, $30° + y + x = 180°$ (By angle sum property of a triangle.)

$\Rightarrow \quad 30° + y + 60° = 180°$ [From equation (*i*)]

$\Rightarrow \quad y + 90° = 180°$

$\Rightarrow \quad y = 180° - 90°$

or $\quad y = 90°$

Thus, $x = 60°$ and $y = 90°$.

From Fig. (*v*),

$y = 90°$ (Vertically opposite angles) ...(*i*)

Now, $y + x + x = 180°$ (By angle sum property of a triangle.)

$\Rightarrow \quad 90° + 2x = 180°$ [From equation (*i*)]

$\Rightarrow \quad 2x = 180° - 90°$

$\Rightarrow \quad 2x = 90°$

or $\quad x = \frac{90°}{2} = 45°$

Thus, $x = 45°$ and $y = 90°$.

From Fig. (*vi*),

$x = y$ (Vertically opposite angles) ...(*i*)

Now, $x + x + y = 180°$ (By angle sum property of a triangle.)

$\Rightarrow \quad 2x + x = 180°$ [From equation (*i*)]

$\Rightarrow \quad 3x = 180°$

or $\quad x = \frac{180°}{3} = 60°$

or $\quad y = 60°$ [From equation (*i*)]

Thus, $x = 60°$ and $y = 60°$.

Exercise 6.4 *(Page No. 126-127)*

Q1. Is it possible to have a triangle with the following sides?

(*i*) 2 cm, 3 cm, 5 cm **(*ii*) 3 cm, 6 cm, 7 cm**

(*iii*) 6 cm, 3 cm, 2 cm.

Sol. Suppose such triangles are possible whose the sum of the lengths of any two sides would be greater than the length of third side.

Let us check this:

(*i*) 2 cm, 3 cm, 5 cm

Is $2 + 3 > 5$? No, it is equal.

Is $2 + 5 > 3$? Yes.

Is $3 + 5 > 2$? Yes.

But this triangle is not possible.

(*ii*) 3 cm, 6 cm, 7 cm

Is $3 + 6 > 7$? Yes.

Is $6 + 7 > 3$? Yes.

Is $3 + 7 > 6$? Yes.

Thus, this triangle is possible.

(*iii*) 6 cm, 3 cm, 2 cm

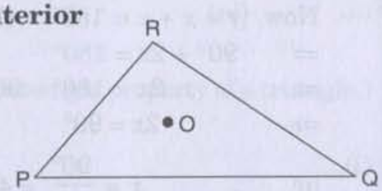
Is 6 + 3 > 2? Yes.

Is 6 + 2 > 3? Yes.

Is 2 + 3 > 6? No. It is less than 6.

So, this triangle is not possible.

Q2. Take any point O in the interior of a triangle PQR. Is



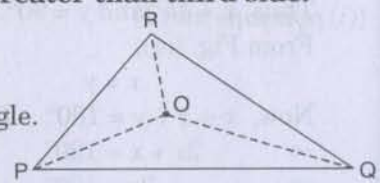
(*i*) OP + OQ > PQ?

(*ii*) OQ + OR > QR?

(*iii*) OR + OP > RP?

Since sum of two sides is greater than third side.

Sol. Join the OR, OQ and OP.



(*i*) Is OP + OQ > PQ ?

Yes, POQ form the triangle.

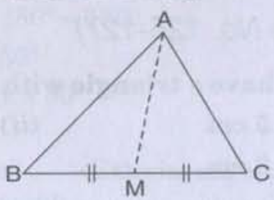
(*ii*) Is OQ + OR > QR?

Yes, ROQ form the triangle.

(*iii*) Is OR + OP > RP?

Yes, ROP form the triangle.

Q3. AM is a median of a triangle ABC. Is AB + BC + CA > 2 AM? (Consider the sides of triangles ΔABM and ΔAMC.)



Sol. We know, the sum of length of any two sides in a triangle would be greater than the length of third side.

In ΔABM, AB + BM > AM ...(*i*)

In ΔAMC, AC + MC > AM ...(*ii*)

Adding equations (*i*) and (*ii*),

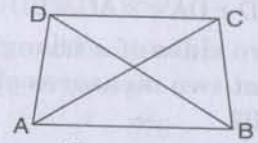
AB + BM + AC + MC > AM + AM

⇒ AB + AC + (BM + MC) > 2 AM

⇒ AB + AC + BC > 2 AM

Thus, it is true.

Q4. ABCD is a quadrilateral. Is AB + BC + CD + DA > AC + BD?



Sol. We know that sum of length of any two sides in a triangle would be greater than the length of third side.

In ΔABC, AB + BC > AC ...(*i*)

and ΔADC, AD + DC > AC ...(*ii*)

In ΔDCB, DC + CB > DB ...(*iii*)

and ΔADB, AD + AB > DB ...(*iv*)

Adding equations (*i*), (*ii*), (*iii*) and (*iv*),

AB + BC + AD + DC + DC + CB + AD + AB > (AC + AC) + (DB + DB)

⇒ (AB + AB) + (BC + BC) + (AD + AD) + (DC + DC) > 2(AC + DB)

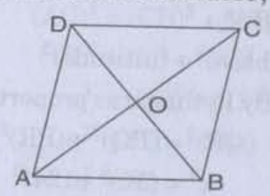
⇒ 2(AB + BC + AD + DC) > 2(AC + DB)

⇒ AB + BC + AD + DC > AC + DB

or AB + BC + CD + DA > AC + BD. **Yes.**

Q5. ABCD is quadrilateral. Is AB + BC + CD + DA < 2(AC + BD)?

Sol. We draw a quadrilateral ABCD and AC, DB intersect at point O.



We know that sum of length of any two sides in a triangle would be greater than the length of third side.

In ΔAOB, AB < OA + OB ...(*i*)

In ΔBOC, BC < OB + OC ...(*ii*)

In ΔCOD, CD < OC + OD ...(*iii*)

In ΔAOD, DA < OD + OA ...(*iv*)

Adding equations (*i*), (*ii*), (*iii*) and (*iv*),

AB + BC + CD + DA < OA + OB + OB + OC + OC + OD + OD + OA

AB + BC + CD + DA < 2OA + 2OB + 2OC + 2OD

$AB + BC + CD + DA < 2[(AO + OC) + (DO + OB)]$

Thus, $AB + BC + CD + DA < 2(AC + BD)$. **Proved.**

Q6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Sol. We know that the sum of two sides of a triangle is always greater than the third side.

Given, two sides of triangle are 12 cm and 15 cm.

Therefore, the third side is less than $12 + 15 = 27$ cm.

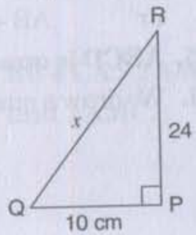
And the third side cannot be less than the difference of the two sides.

Thus, the third side has to be more than $15 - 12 = 3$ cm.

The length of the third side could be any length greater than 3 cm but less than 27 cm.

Exercise 6.5 *(Page No. 130)*

Q1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.



Sol. Given, PQ = 10 cm, PR = 24 cm

Let QR be x cm.

In right-angled ΔQPR,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$ (By Pythagoras property)

$\Rightarrow \quad (QR)^2 = (PQ)^2 + (PR)^2$

$\Rightarrow \quad x^2 = (10)^2 + (24)^2$ (Given)

$\Rightarrow \quad x^2 = 100 + 576 = 676$

or $\quad x = \sqrt{676} = 26$

Thus, the length of QR is 26 cm.

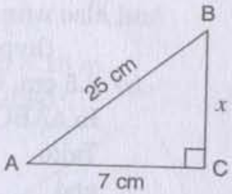
Q2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Sol. Given, AB = 25 cm and AC = 7 cm

Let BC be x cm.

In right-angled ΔACB,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$ (By Pythagoras property)



Now, $\quad (AB)^2 = (AC)^2 + (BC)^2$

$\Rightarrow \quad (25)^2 = (7)^2 + x^2$

$\Rightarrow \quad 625 = 49 + x^2$

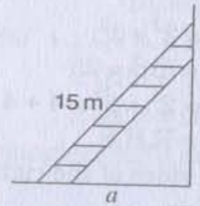
$\Rightarrow \quad 625 - 49 = x^2$

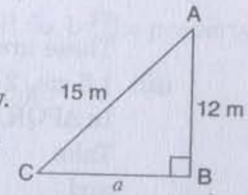
$\Rightarrow \quad x^2 = 576$

or $\quad x = \sqrt{576} = 24$ cm

Thus, the length of BC is 24 cm.

Q3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance *a*. Find the distance of the foot of the ladder from the wall.





Sol. Let AC be the ladder and A be the window.

Given, $\quad AC = 15$ m, $AB = 12$ m, $CB = a$ m

In right-angled ΔABC,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$ (By Pythagoras property)

Now, $\quad (AC)^2 = (CB)^2 + (AB)^2$

$\Rightarrow \quad (15)^2 = (a)^2 + (12)^2$

$\Rightarrow \quad a^2 = (15)^2 - (12)^2$

$\Rightarrow \quad a^2 = 225 - 144$

$\Rightarrow \quad a^2 = 81$

or $\quad a = \sqrt{81} = 9$

Thus, the distance of the foot of the ladder from the wall is 9 m.

Q4. Which of the following can be the sides of a right triangle?

(*i*) 2.5 cm, 6.5 cm, 6 cm. (*ii*) 2 cm, 2 cm, 5 cm.

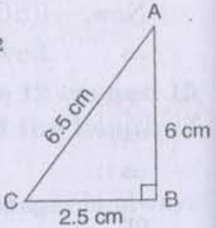
(*iii*) 1.5 cm, 2 cm, 2.5 cm.

In the case of right-angled triangles, identify the right angles.

Sol. As we consider, the larger side in the given sides of triangles be the hypotenuse.

And also using Pythagoras property,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$



(*i*) 2.5 cm, 6.5 cm, 6 cm

In ΔABC, $(AC)^2 = (AB)^2 + (CB)^2$

Take L.H.S. $= (6.5)^2 = 42.25$

and R.H.S. $= (6)^2 + (2.5)^2$

$= 36 + 6.25 = 42.25$

$\Rightarrow$ L.H.S. = R.H.S.

Then given sides are the sides of the right-angled triangle. Right angle lies on the opposite side of 6.5 cm in ΔABC right-angled at B.

(*ii*) 2 cm, 2 cm, 5 cm

$\Rightarrow$ $(5)^2 = 2^2 + 2^2$

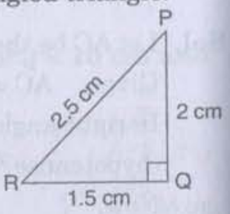
Take L.H.S. $= (5)^2 = 25$

and R.H.S. $= 2^2 + 2^2 = 4 + 4 = 8$

$\Rightarrow$ L.H.S. $\neq$ R.H.S.

These are not the sides of the right-angled triangle.

(*iii*) 1.5 cm, 2 cm, 2.5 cm



In ΔPQR, $(PR)^2 = (PQ)^2 + (RQ)^2$

Take L.H.S. $= (2.5)^2 = 6.25$

and R.H.S. $= (1.5)^2 + (2)^2$

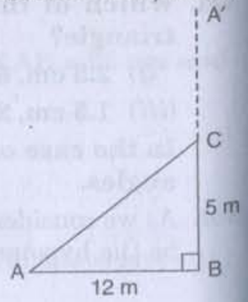
$= 2.25 + 4 = 6.25$

$\Rightarrow$ L.H.S. = R.H.S.

Then given sides are the sides of a right-angled triangle. Right angle lies on the opposite side of 2.5 cm in ΔPQR right-angled at Q.

Q5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Sol. Let A′CB represent the tree before it broken at the point C and let the top A′ touch the ground at A after it broke. Then, ΔABC is a right-angled triangle at B such that



AB = 12 and BC = 5

Using Pythagoras property in ΔABC, we have

$(AC)^2 = (AB)^2 + (BC)^2$

$= (12)^2 + (5)^2$

$= 144 + 25$

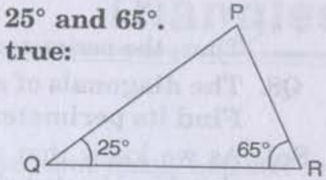
$AC^2 = 169$

$AC = \sqrt{169} = 13$

Hence, the total height of the tree = AC + CB

$= 13 + 5 = 18$ m.

Q6. Angles Q and R of a ΔPQR are 25° and 65°. Write which of the following is true:



(*i*) $PQ^2 + QR^2 = RP^2$

(*ii*) $PQ^2 + RP^2 = QR^2$

(*iii*) $RP^2 + QR^2 = PQ^2$.

Sol. First we find the third angle of the given triangle PQR.

Now, in ΔPQR,

$\Rightarrow$ $25° + 65° + \angle QPR = 180°$

(By angle sum property of triangle.)

$\Rightarrow$ $90° + \angle QPR = 180°$

$\Rightarrow$ $\angle QPR = 180° - 90°$

or $\angle QPR = 90°$

Now the given triangle is right angled at P. So, by the property of Pythagoras,

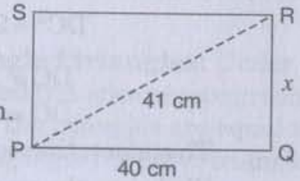
$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{altitude})^2$

$(QR)^2 = (PR)^2 + (QP)^2$.

Thus, (*ii*) option is true.

Q7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Sol. Draw a rough figure of the rectangle PQRS



Given, diagonal = 41 cm, side = 40 cm.

Let another side be x,

Now, in right-angled ΔPQR,

$(PR)^2 = (RQ)^2 + (PQ)^2$ (By Pythagoras property)

$(41)^2 = x^2 + (40)^2$

$\Rightarrow$ $x^2 = (41)^2 - (40)^2$

$\Rightarrow$ $x^2 = 1681 - 1600$

$\Rightarrow$ $x^2 = 81$

or $x = \sqrt{81} = 9$

Therefore, the another side of the rectangle is 9 cm.

Now, we find the perimeter of the rectangle through given sides.

We know that,

Perimeter of the rectangle = 2(length + breadth)

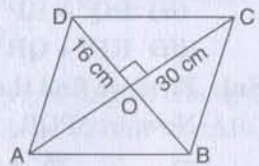
= 2(9 + 40)

= 2(49) = 98

Thus, the perimeter of the rectangle is 98 cm.

Q8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Sol. As we know that the diagonals of rhombus bisect at right angle to each other.



From the rough diagram of rhombus.

Given, AC and DB is 30 cm and 16 cm respectively.

For the calculation we have,

$$OD = \frac{DB}{2} = \frac{16}{2} = 8 \text{ cm}$$

$$OC = \frac{AC}{2} = \frac{30}{2} = 15 \text{ cm}$$

In right-angled ΔDOC,

$(DC)^2 = (OD)^2 + (OC)^2$ (By Pythagoras property)

$(DC)^2 = (8)^2 + (15)^2$

$DC^2 = 64 + 225$

$DC^2 = 289$

$DC = \sqrt{289}$

$DC = 17$

Therefore, the side of the rhombus is 17 cm.

We know, the perimeter of rhombus = 4 × side

= 4 × 17 = 68.

Thus, the perimeter of the rhombus is 68 cm.

□□