9 Algebraic Expressions and Identities

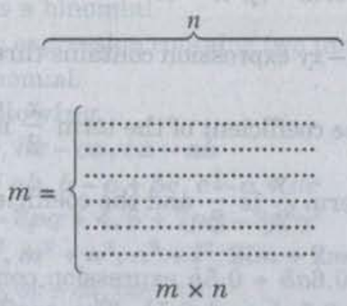
Learn and Remember

1. Expressions are formed from **variables** and **constants.**
2. Terms are added to form **expressions.**
3. These terms are formed as product of factors.
4. All those expressions that contain exactly one, two and three terms are called **monomial, binomial** and **trinomial** respectively.
5. In general, any expression containing one or more terms with non-zero coefficients (and with variables having non-negative exponents) is called a **polynomial.**
6. Terms are formed from the some variable and the powers of these variables are the same, too; having different coefficients.
7. While adding or subtracting polynomials, first look for like terms and then add or subtract these terms. Then handle the unlike terms.
8. There are number of situations in which we need to multiply algebraic expressions, for example, in finding area of rectangle, the sides of which are given as expressions.
9. Monomial multiplied by a monomial always gives a **monomial.**
10. While multiplying a polynomial by a monomial, we multiply every terms in the polynomial by the monomial.
11. In carrying out the multiplication of a polynomial by a binomial or trinomial, we multiply term by term, that means every term of the **polynomial** is multiplied by every term in the **binomial** or **trinomial.**
12. When we multiply, we may get like terms in the product which are like and have to be combined by adding or subtracting.
13. The following are the useful identities and these identities are known as **standard identities.**

$$(a + b)^2 = a^2 + 2ab + b^2 \quad ...(i)$$
$$(a - b)^2 = a^2 - 2ab + b^2 \quad ...(ii)$$

$$(a + b)(a - b) = a^2 - b^2 \quad ...(iii)$$
and $$(x + a)(x + b) = x^2 + (a + b)x + ab \quad ...(iv)$$

14. An identity is an **equality** which is true for all values of the variables in the equality.
15. An equation is true only for certain values of its variables. So, an equation is not an identity.
16. These four identities are useful in carrying out squares and products of algebraic expressions. They also allow easy alternative methods to calculate products of numbers and so on.
17. To find the number of dots we have to multiply the expression for the number of rows by the expression for the number of columns. That is as.



18. Algebraic factor of product is equal to algebraic factor of first monomial × algebraic factor of second monomial.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 9.1 (*Page – 140*)

Q1. Identify the terms, their coefficients for each of the following expressions.

(*i*) $5xyz^2 - 3zy$ (*ii*) $1 + x + x^2$

(*iii*) $4x^2y^2 - 4x^2y^2z^2 + z^2$ (*iv*) $3 - pq + qr - rp$

(*v*) $\frac{x}{2} + \frac{y}{2} - xy$ (*vi*) $0.3a - 0.6ab + 0.5b$

Sol. (*i*) $5xyz^2 - 3zy$ expression contains two terms $5xyz^2$ and $- 3zy$. The coefficient in the term $5xyz^2$ is 5 and the coefficient in the term $- 3zy$ is $- 3$.

(*ii*) $1 + x + x^2$ expression contains three terms 1, x and x^2, where 1 is a constant term, the coefficient of x is 1. And the coefficient of the term x^2 is also 1.

(*iii*) $4x^2y^2 - 4x^2y^2z^2 + z^2$ expression contains three terms $4x^2y^2$, $-4x^2y^2z^2$ and z^2. The coefficient of the term $4x^2y^2$ is 4, the coefficient of the term $-4x^2y^2z^2$ is -4 and the coefficient of the term z^2 is 1.

(*iv*) $3 - pq + qr - rp$ expression contains four terms 3, $-pq$, qr and $-rp$. 3 is a constant term the coefficient of the term $-pq$ is -1.
The coefficient in the term qr is 1 and the coefficient in the term $-rp$ is -1.

(*v*) $\frac{x}{2} + \frac{y}{2} - xy$ expression contains three terms $\frac{x}{2}$, $\frac{y}{2}$ and $-xy$. The coefficient of the term $\frac{x}{2}$ is $\frac{1}{2}$, the coefficient of the term $\frac{y}{2}$ is $\frac{1}{2}$ and the coefficient of the term $-xy$ is -1.

(*vi*) $0.3a - 0.6ab + 0.5b$ expression contains three terms $0.3a$, $-0.6ab$ and $0.5b$. The coefficient of the term $0.3a$ is 0.3, the coefficient of the term $-0.6ab$ is -0.6 and the coefficient of the term $0.5b$ is 0.5.

Q2. Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

$x + y$, **1000**, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$.

Sol. (*i*) $x + y$ expression contains two terms x and y. So, it is binomial.

(*ii*) 1000 is a single term without variable, it is a monomial.

(*iii*) $x + x^2 + x^3 + x^4$ expressions contains four terms x, x^2, x^3 and x^4. It is a polynomial, so, it does not fit in above three categories.

(*iv*) $7 + y + 5x$ expression contains three terms 7, y and $5x$. So, it is a trinomial.

(*v*) $2y - 3y^2$ expression contains two terms $2y$ and $-3y^2$. So, it is a binomials.

(*vi*) $2y - 3y^2 + 4y^3$ expression contains three terms $2y$, $-3y^2$ and $4y^3$. So, it is a trinomial.

(*vii*) $5x - 4y + 3xy$ expression contains three terms $5x$, $-4y$ and $3xy$. So, it is a trinomial.

(*viii*) $4z - 15z^2$ expression contains two terms $4z$ and $-15z^2$. So, it is a binomial.

(*ix*) $ab + bc + cd + da$ expression contains four terms ab, bc, cd and da. So, it is a polynomials and it does not fit in any three above categories.

(*x*) pqr expression contains only one term. So, it is a monomial.

(*xi*) $p^2q + pq^2$ expression contains two terms p^2q and pq^2. So, it is a binomial.

(*xii*) $2p + 2q$ expression contains two terms $2p$ and $2q$. So, it a is binomial.

Q3. Add the following.

(*i*) $ab - bc$, $bc - ca$, $ca - ab$

(*ii*) $a - b + ab$, $b - c + bc$, $c - a + ac$

(*iii*) $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$

(*iv*) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$

Sol. (*i*) $ab - bc$, $bc - ca$, $ca - ab$

$$\begin{array}{rrr} ab & -bc & \\ + & +bc & -ca \\ +-ab & & +ca \\ \hline 0 & +0 & +0 \end{array}$$

Hence, the sum of expressions is zero.

(*ii*) $a - b + ab$, $b - c + bc$, $c - a + ac$

$$\begin{array}{rrrrrr} a & -b & +ab & & & \\ + & b & & -c & +bc & \\ +-a & & & +c & & +ac \\ \hline 0 & +0 & +ab & +0 & +bc & +ac \end{array}$$

Hence, the sum of expressions is $ab + bc + ac$.

(*iii*) $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$

$$\begin{array}{rrr} 2p^2q^2 & -3pq & +4 \\ +-3p^2q^2 & +7pq & +5 \\ \hline -p^2q^2 & +4pq & +9 \end{array}$$

Hence, the sum of expressions is $-p^2q^2 + 4pq + 9$.

(*iv*) $l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$

$$
\begin{array}{r}
l^2 + m^2 \qquad\qquad\qquad\qquad \\
+ \quad m^2 + n^2 \qquad\qquad\qquad \\
+ \; l^2 \qquad + n^2 \qquad\qquad\qquad \\
+ \qquad\qquad 2lm + 2mn + 2nl \\
\hline
2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl \\
\hline
\end{array}
$$

Hence, the sum of expressions is $2(l^2 + m^2 + n^2 + lm + mn + nl)$

Q4. (*a*) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$.

(*b*) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

(*c*) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$.

Sol. (*a*) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

$$
\begin{array}{r}
12a - 9ab + 5b - \;\; 3 \\
4a - 7ab + 3b + 12 \\
(-) \quad (+) \quad (-) \quad (-) \\
\hline
8a - 2ab + 2b - 15
\end{array}
$$

(*b*) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

$$
\begin{array}{r}
5xy - 2yz - 2zx + 10xyz \\
3xy + 5yz - 7zx \qquad\quad \\
(-) \quad (-) \quad (+) \qquad\quad \\
\hline
+ 2xy - 7yz + 5zx + 10xyz
\end{array}
$$

(*c*) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

$$
\begin{array}{r}
5p^2q - 2pq^2 + 5pq - 11q - 3p + 18 \\
4p^2q + 5pq^2 - 3pq + 7q - 8p - 10 \\
(-) \quad (-) \quad (+) \quad (-) \quad (+) \quad (+) \\
\hline
p^2q - 7pq^2 + 8pq - 18q + 5p + 28 \\
\hline
\end{array}
$$

EXERCISE 9.2 (*Page – 143-144*)

Q1. Find the product of the following pairs of monomials.

(*i*) 4, $7p$ (*ii*) $-4p$, $7p$ (*iii*) $-4p$, $7pq$

(*iv*) $4p^3$, $-3p$ (*v*) $4p$, 0

Sol. (*i*) $4 \times 7p$

$= 4 \times 7 \times p$

$= 28p.$

(*ii*) $-4p \times 7p$

$= (-4 \times 7) \times (p \times p)$

$= -28 \times p^2$

$= -28p^2.$

(*iii*) $-4p \times 7pq$

$= (-4 \times 7) \times (p \times pq)$

$= -28 \times p^2q$

$= -28p^2q.$

(*iv*) $4p^3 \times -3p$

$= (4 \times -3) \times (p^3 \times p)$

$= -12 \times p^4$

$= -12p^4.$

(*v*) $4p \times 0$

$= 4 \times 0 \times p$

$= 0 \times p = 0.$

Q2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

(p, q); $(10m, 5n)$; $(20x^2, 5y^2)$; $(4x, 3x^2)$; $(3mn, 4np)$

Sol. (*i*) (p, q)

Area of rectangle = length × breadth

$= p \times q = pq$ square units.

(*ii*) $10m, 5n$

Area of rectangle = length × breadth

$= 10m \times 5n = (10 \times 5) \times (m \times n)$

$= 50 \times mn$

$= 50mn$ square units.

(*iii*) $20x^2, 5y^2$

Area of rectangle = length × breadth

$= 20x^2 \times 5y^2 = (20 \times 5) \times (x^2 \times y^2)$

$= 100 \times x^2y^2 = 100x^2y^2$ square units.

(*iv*) $4x, 3x^2$

Area of rectangle = length × breadth

$= 4x \times 3x^2 = (4 \times 3) \times (x \times x^2)$

$= 12 \times x^3 = 12x^3$ square units.

(*v*) $3mn, 4np$

Area of rectangle = length × breadth

$= 3mn \times 4np = (3 \times 4) \times (mn \times np)$

$= 12 \times mn^2p = 12mn^2p$ square units.

Q3. Complete the table of products.

First monomial → Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

Sol.

First monomial → Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$+25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

Q4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$ (iii) $xy, 2x^2y, 2xy^2$

(iv) $a, 2b, 3c$

Sol. (i) $5a, 3a^2, 7a^4$

Volume of rectangular box = length × breadth × height

$= 5a \times 3a^2 \times 7a^4$

$= 5 \times 3 \times 7 \times a \times a^2 \times a^4$

$= 105a^7$ cubic units.

(ii) $2p, 4q, 8r$

Volume of rectangular box = length × breadth × height

$= 2p \times 4q \times 8r$

$= 2 \times 4 \times 8 \times p \times q \times r$

$= 64pqr$ cubic units

(iii) $xy, 2x^2y, 2xy^2$

Volume of rectangular box = length × breadth × height

$= xy \times 2x^2y \times 2xy^2$

$= 2 \times 2 \times xy \times x^2 \times y \times xy^2$

$= 4x^4y^4$ cubic units.

(iv) $a, 2b, 3c$

Volume of rectangular box = length × breadth × height

$= a \times 2b \times 3c$

$= 2 \times 3 \times a \times b \times c$

$= 6abc$ cubic units.

Q5. Obtain the product of

(i) xy, yz, zx (ii) $a, -a^2, a^3$ (iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$ (v) $m, -mn, mnp$

Sol. (i) The product of xy, yz and $zx = xy \times yz \times zx$

$= x \times y \times y \times z \times z \times x$

$= x \times x \times y \times y \times z \times z$

$= x^2 \times y^2 \times z^2$

$= x^2y^2z^2$.

(ii) The product of $a, -a^2$ and $a^3 = a \times -a^2 \times a^3$

$= a \times (-1) \times a^2 \times a^3$

$= (-1) \times a \times a^2 \times a^3$

$= -1 \times a^6 = -a^6.$

(iii) The product of 2, 4y, $8y^2$ and

$16y^3 = 2 \times 4y \times 8y^2 \times 16y^3$

$= 2 \times 4 \times 8 \times 16 \times y \times y^2 \times y^3$

$= 1024 \times y^6 = 1024y^6.$

(iv) The product of a, 2b, 3c and

$6abc = a \times 2b \times 3c \times 6abc$

$= 2 \times 3 \times 6 \times a \times b \times c \times a \times b \times c$

$= 36 \times a^2 \times b^2 \times c^2$

$= 36a^2b^2c^2.$

(v) The product of m, – mn and

$mnp = m \times -mn \times mnp$

$= (-1) \times m \times m \times n \times m \times n \times p$

$= (-1)\, m^3 \times n^2 \times p$

$= -m^3n^2p.$

EXERCISE 9.3 (*Page – 146*)

Q1. Carry out the multiplication of the expressions in each of the following pairs.

(i) 4p, q + r (ii) ab, a – b (iii) a + b, $7a^2b^2$

(iv) $a^2 - 9$, 4a (v) pq + qr + rp, 0

Sol. (i) $4p, q + r$

$4p \times (q + r)$

$= (4p \times q + 4p \times r)$

$= 4pq + 4pr$

(ii) $ab, a - b$

$ab \times (a - b)$

$= (ab \times a - ab \times b)$

$= a^2b - ab^2$

(iii) $a + b, 7a^2b^2$

$(a + b) \times 7a^2b^2$

$= (7a^2b^2 \times a) + (7a^2b^2 \times b)$

$= 7a^3b^2 + 7a^2b^3$

(iv) $a^2 - 9, 4a$

$(a^2 - 9) \times 4a$

$= (4a \times a^2) - (4a \times 9)$

$= 4a^3 - 36a$

(v) $pq + qr + rp, 0$

$(pq + qr + rp) \times 0$

$= 0 \times (pq + qr + rp)$

$= (0 \times pq) + (0 \times qr) + (0 \times rp) = 0 + 0 + 0 = 0.$

Q2. Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

Sol.

	First expression	Second expression	Product
(i)	a	$b + c + d$	$a(b + c + d)$ $= a \times b + a \times c + a \times d$ $= ab + ac + ad.$
(ii)	$x + y - 5$	$5xy$	$5xy(x + y - 5)$ $= 5xy \times x + 5xy \times y - 5xy \times 5$ $= 5x^2y + 5xy^2 - 25xy.$
(iii)	p	$6p^2 - 7p + 5$	$p(6p^2 - 7p + 5)$ $= p \times 6p^2 - p \times 7p + p \times 5$ $= 6p^3 - 7p^2 + 5p.$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^2q^2(p^2 - q^2)$ $= 4p^2q^2 \times p^2 - 4p^2q^2 \times q^2$ $= 4p^4q^2 - 4p^2q^4.$
(v)	$a + b + c$	abc	$abc(a + b + c)$ $= abc \times a + abc \times b + abc \times c$ $= a^2bc + ab^2c + abc^2.$

Q3. Find the product.

(i) $(a^2) \times (2a^{22}) \times (4a^{26})$ (ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$ ***(iv)*** $x \times x^2 \times x^3 \times x^4$

Sol. (*i*) $(a^2) \times (2a^{22}) \times (4a^{26})$

$= 2 \times 4 \times a^2 \times a^{22} \times a^{26}$

$= 8a^{2+22+26}$

$= 8 \times a^{50} = 8a^{50}$. (In multiplication, powers are added)

(*ii*) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

$= \frac{2}{3} \times \frac{-9}{10} \times x \times y \times x^2 \times y^2$

$= \frac{-3}{5} \times x^3 \times y^3$

$= \frac{-3}{5}x^3y^3$.

(*iii*) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$

$= \frac{-10}{3} \times \frac{6}{5} \times p \times q^3 \times p^3 \times q$

$= -4 \times p^4 \times q^4 = -4p^4q^4$.

(*iv*) $x^1 \times x^2 \times x^3 \times x^4$

$= (x^3 \times x^3) \times x^4$ (Taking two terms at a line)

$= x^6 \times x^4 = x^{10}$.

Q4. (*a*) Simplify: $3x(4x - 5) + 3$ and find its values for

(*i*) $x = 3$ (*ii*) $x = \frac{1}{2}$.

(*b*) Simplify: $a(a^2 + a + 1) + 5$ and find its value for

(*i*) $a = 0$ (*ii*) $a = 1$ (*iii*) $a = -1$.

Sol. (*a*) $3x(4x - 5) + 3$

$= 3x \times 4x - 3x \times 5 + 3$

$= 12x^2 - 15x + 3$.

(*i*) For $x = 3$, $12x^2 - 15x + 3 = 12 \times (3)^2 - 15 \times 3 + 3$

$= 12 \times 9 - 45 + 3$

$= 108 - 45 + 3$

$= 66$.

(*ii*) For $x = \frac{1}{2}$, $12x^2 - 15x + 3 = 12 \times \left(\frac{1}{2}\right)^2 - 15 \times \frac{1}{2} + 3$

$= 12 \times \frac{1}{4} - \frac{15}{2} + 3 = 3 - \frac{15}{2} + 3$

$= \frac{6}{1} - \frac{15}{2} = \frac{12-15}{2} = \frac{-3}{2}$.

(*b*) $a(a^2 + a + 1) + 5$

$= a \times a^2 + a \times a + a \times 1 + 5$

$= a^3 + a^2 + a + 5$.

(*i*) For $a = 0$, $a^3 + a^2 + a + 5 = (0)^3 + (0)^2 + (0) + 5$

$= 0 + 0 + 0 + 5$

$= 5$.

(*ii*) For $a = 1$, $a^3 + a^2 + a + 5 = (1)^3 + (1)^2 + 1 + 5$

$= 1 + 1 + 1 + 5$

$= 8$.

(*iii*) For $a = -1$, $a^3 + a^2 + a + 5 = (-1)^3 + (-1)^2 + (-1) + 5$

$= -1 + 1 - 1 + 5$

$= -2 + 6 = 4$.

Q5. (*a*) Add: $p(p - q)$, $q(q - r)$ and $r(r - p)$

(*b*) Add: $2x(z - x - y)$ and $2y(z - y - x)$

(*c*) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

(*d*) Subtract: $3a(a + b + c) - 2b(a - b + c)$ from $4c(-a + b + c)$

Sol. (*a*) First expression $p(p - q) = p^2 - pq$

Second expression $q(q - r) = q^2 - qr$

Third expression $r(r - p) = r^2 - rp$

On adding the three expressions,

$= p^2 - pq + q^2 - qr + r^2 - rp$

$= p^2 + q^2 + r^2 - pq - qr - pr$.

(*b*) First expression $2x(z - x - y) = 2x \times z - 2x \times x - 2x \times y$

$= 2xz - 2x^2 - 2xy$

Second expression $2y(z - y - x) = 2y \times z - 2y \times y - 2y \times x$

$= 2yz - 2y^2 - 2xy$

On adding both expressions,

$= 2xz - 2x^2 - 2xy + 2yz - 2y^2 - 2xy$

$= 2xz - 4xy + 2yz - 2x^2 - 2y^2$

$= -2x^2 - 2y^2 - 4xy + 2yz + 2zx.$

(*c*) First expression $3l(l - 4m + 5n)$

$= 3l \times l - 3l \times 4m + 3l \times 5n$

$= 3l^2 - 12lm + 15ln$

Second expression $4l(10n - 3m + 2l)$

$= 4l \times 10n - 4l \times 3m + 4l \times 2l$

$= 40ln - 12lm + 8l^2$

$= 8l^2 - 12lm + 40ln$

On subtracting first expressions from second expression

$$\begin{array}{rrr} 8l^2 & - 12lm & + 40ln \\ 3l^2 & - 12lm & + 15ln \\ (-) & (+) & (-) \\ \hline +5l^2 & + 0 & + 25ln \\ \hline \end{array}$$

(*d*) First expression $3a(a + b + c) - 2b(a - b + c)$

$= 3a \times a + 3a \times b + 3a \times c - 2b \times a + 2b \times b - 2b \times c$

$= 3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc$

$= 3a^2 + ab + 3ac - 2bc + 2b^2$

Second expression $4c(-a + b + c)$

$= 4c \times -a + 4c \times b + 4c \times c$

$= -4ac + 4bc + 4c^2$

On subtracting first expression from second expression

$$\begin{array}{rrrrrr} -4ac & + 4bc & + 4c^2 & & & \\ + 3ac & - 2bc & & + 3a^2 & + ab & + 2b^2 \\ (-) & (+) & & (-) & (-) & (-) \\ \hline -7ac & + 6bc & + 4c^2 & - 3a^2 & - ab & - 2b^2 \\ \hline \end{array}$$

Hence, required answer is $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac.$

EXERCISE 9.4 (*Page -148*)

Q1. Multiply the binomials.

(*i*) $(2x + 5)$ and $(4x - 3)$

(*ii*) $(y - 8)$ and $(3y - 4)$

(*iii*) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$

(*iv*) $(a + 3b)$ and $(x + 5)$

(*v*) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

(*vi*) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$

Sol. (*i*) $(2x + 5) \times (4x - 3)$

$= 2x(4x - 3) + 5(4x - 3)$

$= (2x \times 4x) - (2x \times 3) + (5 \times 4x) - (5 \times 3)$

$= 8x^2 - 6x + 20x - 15$

$= 8x^2 + 14x - 15$ ($\because$ $6x$ and $20x$ are like terms)

(*ii*) $(y - 8) \times (3y - 4)$

$= y(3y - 4) - 8(3y - 4)$

$= (y \times 3y) - (y \times 4) - (8 \times 3y) + (8 \times 4)$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$ ($\because$ $-4y$ and $-24y$ are like terms)

(*iii*) $(2.5l - 0.5m) \times (2.5l + 0.5m)$

$= 2.5l(2.5l + 0.5m) - 0.5m(2.5l + 0.5m)$

$= (2.5l \times 2.5l) + (2.5l \times 0.5m) - (0.5m \times 2.5l) - (0.5m \times 0.5m)$

$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$

$= 6.25l^2 - 0.25n^2$ ($\because$ $1.25lm$ and $-1.25lm$ are like terms)

(*iv*) $(a + 3b) \times (x + 5)$

$= a(x + 5) + 3b(x + 5)$

$= (a \times x) + (5 \times a) + (3b \times x) + (3b \times 5)$

$= ax + 5a + 3bx + 15b$

(*v*) $(2pq + 3q^2) \times (3pq - 2q^2)$

$= 2pq(3pq - 2q^2) + 3q^2(3pq - 2q^2)$

$= (2pq \times 3pq) - (2pq \times 2q^2) + (3q^2 \times 3pq) - (3q^2 \times 2q^2)$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 + 5pq^3 - 6q^4$

($\because$ $-4pq^3$ and $9pq^3$ are like terms with opposite signs)

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right) \times 4\left(a^2 - \frac{2}{3}b^2\right)$

$= \left(\frac{3}{4}a^2 + 3b^2\right) \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= \frac{3}{4}a^2\left(4a^2 - \frac{8}{3}b^2\right) + 3b^2\left(4a^2 - \frac{8}{3}b^2\right)$

$= \left(\frac{3}{4}a^2 \times 4a^2\right) - \left(\frac{3}{4}a^2 \times \frac{8}{3}b^2\right) + (3b^2 \times 4a^2)$

$- \left(3b^2 \times \frac{8}{3}b^2\right)$

$= 3a^4 - 2a^2b^2 + 12a^2b^2 - 8b^4$

$= 3a^4 + 10a^2b^2 - 8b^4.$

Q2. Find the product.

(i) $(5 - 2x)(3 + x)$ (ii) $(x + 7y)(7x - y)$

(iii) $(a^2 + b)(a + b^2)$ (iv) $(p^2 - q^2)(2p + q)$

Sol. (i) $(5 - 2x)(3 + x) = 5(3 + x) - 2x(3 + x)$

$= 5 \times 3 + 5 \times x - 2x \times 3 - 2x \times x$

$= 15 + 5x - 6x - 2x^2$

$= 15 - x - 2x^2$

(ii) $(x + 7y)(7x - y) = x(7x - y) + 7y(7x - y)$

$= x \times 7x - x \times y + 7y \times 7x - 7y \times y$

$= 7x^2 - xy + 49xy - 7y^2$

$= 7x^2 + 48xy - 7y^2$

(iii) $(a^2 + b)(a + b^2) = a^2(a + b^2) + b(a + b^2)$

$= a^2 \times a + a^2 \times b^2 + b \times a + b \times b^2$

$= a^3 + a^2b^2 + ab + b^3$

(iv) $(p^2 - q^2)(2p + q) = p^2(2p + q) - q^2(2p + q)$

$= p^2 \times 2p + p^2 \times q - q^2 \times 2p - q^2 \times q$

$= 2p^3 + p^2q - 2pq^2 - q^3.$

Q3. Simplify.

(i) $(x^2 - 5)(x + 5) + 25$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

(iii) $(t + s^2)(t^2 - s)$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$

(vi) $(x + y)(x^2 - xy + y^2)$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

(viii) $(a + b + c)(a + b - c)$

Sol. (i) $(x^2 - 5)(x + 5) + 25$

$= x^2(x + 5) - 5(x + 5) + 25$

$= x^2 \times x + x^2 \times 5 - 5 \times x - 5 \times 5 + 25$

$= x^3 + 5x^2 - 5x - 25 + 25$

$= x^3 + 5x^2 - 5x$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

$= a^2(b^3 + 3) + 5(b^3 + 3) + 5$

$= a^2 \times b^3 + a^2 \times 3 + 5 \times b^3 + 5 \times 3 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $(t + s^2)(t^2 - s)$

$= t(t^2 - s) + s^2(t^2 - s)$

$= t^3 - st + s^2t^2 - s^3$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

$= a(c - d) + b(c - d) + a(c + d) - b(c + d) + 2(ac + bd)$

$= a \times c - a \times d + b \times c - b \times d + a \times c + a \times d - b \times c - b \times d + 2ac + 2bd$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= 4ac$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$

$= x(2x + y) + y(2x + y) + x(x - y) + 2y(x - y)$

$= x \times 2x + x \times y + y \times 2x + y \times y + x \times x - x \times y + 2y \times x - 2y \times y$

$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= 3x^2 + 4xy - y^2$

(*vi*) $(x + y)(x^2 - xy + y^2)$
$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$
$= x \times x^2 - x \times xy + x \times y^2 + y \times x^2 - y \times xy + y \times y^2$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$
$= x^3 + y^3$

(*vii*) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$
$= 1.5x(1.5x + 4y + 3) - 4y(1.5x + 4y + 3) - 4.5x + 12y$
$= 1.5x \times 1.5x + 1.5x \times 4y + 1.5x \times 3 - 4y \times 1.5x - 4y \times 4y - 4y \times 3 - 4.5x + 12y$
$= 2.25x^2 + 6.0xy + 4.5x - 6.0xy - 16y^2 - 12y - 4.5x + 12y$
$= 2.25x^2 - 16y^2$

(*viii*) $(a + b + c)(a + b - c)$
$= a(a + b - c) + b(a + b - c) + c(a + b - c)$
$= a \times a + a \times b - a \times c + b \times a + b \times b - b \times c + c \times a + c \times b - c \times c$
$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$
$= a^2 + b^2 + 2ab - c^2$
$= a^2 + b^2 - c^2 + 2ab.$

EXERCISE 9.5 (*Page – 151-152*)

Q1. Use a suitable identity to get each of the following products.

(*i*) $(x + 3)(x + 3)$ (*ii*) $(2y + 5)(2y + 5)$

(*iii*) $(2a - 7)(2a - 7)$ (*iv*) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right)$

(*v*) $(1.1m - 0.4)(1.1m + 0.4)$ (*vi*) $(a^2 + b^2)(-a^2 + b^2)$

(*vii*) $(6x - 7)(6x + 7)$ (*viii*) $(-a + c)(-a + c)$

(*ix*) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$ (*x*) $(7a - 9b)(7a - 9b)$

Sol. (*i*) $(x + 3)(x + 3) = (x + 3)^2$
$= (x)^2 + 2 \times x \times 3 + (3)^2$
Using identity $(a + b)^2 = a^2 + 2ab + b^2$
$= x^2 + 6x + 9$

(*ii*) $(2y + 5)(2y + 5) = (2y + 5)^2$
$= (2y)^2 + 2 \times 2y \times 5 + (5)^2$
Using identity $(a + b)^2 = a^2 + 2ab + b^2$
$= 4y^2 + 20y + 25$

(*iii*) $(2a - 7)(2a - 7) = (2a - 7)^2$
$= (2a)^2 - 2 \times 2a \times 7 + (7)^2$
Using identity $(a - b)^2 = a^2 - 2ab + b^2$
$= 4a^2 - 28a + 49$
$= 4a^2 - 28a + 49$

(*iv*) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right) = \left(3a - \frac{1}{2}\right)^2$
$= (3a)^2 - 2 \times 3a \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$
Using identity $(a - b)^2 = a^2 - 2ab + b^2$
$= 9a^2 - 3a + \frac{1}{4}$

(*v*) $(1.1m - 0.4)(1.1m + 0.4)$
$= (1.1m - 0.4)(1.1m + 0.4) = (1.1m)^2 - (0.4)^2$
Using identity $(a - b)(a + b) = a^2 - b^2$
$= 1.21m^2 - 0.16$

(*vi*) $(a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$
$= (b^2)^2 - (a^2)^2$ Using identity $(a + b)(a - b) = a^2 - b^2$
$= b^4 - a^4$

(*vii*) $(6x - 7)(6x + 7)$
$= (6x)^2 - (7)^2$ Using identity $(a - b)(a + b) = a^2 - b^2$
$= 36x^2 - 49$

(*viii*) $(-a + c)(-a + c) = (-a + c)^2$
$= (-a)^2 + 2(-a) \times c + (c)^2$
Using identity $(a + b)^2 = a^2 + 2ab + b^2$
$= a^2 - 2ac + c^2$

Aliter. $(-a + c)(-a + c) = (-a + c)^2$
$= (c - a)^2$
$= (c)^2 - 2 \times c \times a + a^2$ Using identity $(a - b)^2 = a^2 - 2ab + b^2$
$= c^2 - 2ac + a^2$

(*ix*) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)=\left(\frac{x}{2}+\frac{3y}{4}\right)^2$

$=\left(\frac{x}{2}\right)^2+2\times\frac{x}{2}\times\frac{3y}{4}+\left(\frac{3y}{4}\right)^2$

Using identity $(a+b)^2=a^2+2ab+b^2$

$=\frac{x^2}{4}+\frac{3}{4}xy+\frac{9}{16}y^2$

(*x*) $(7a-9b)(7a-9b)=(7a-9b)^2$

$=(7a)^2-2\times 7a\times 9b+(9b)^2$

Using identity $(a-b)^2=a^2-2ab+b^2$

$=49a^2-126ab+81b^2.$

Q2. Use the identity $(x+a)(x+b)=x^2+(a+b)x+ab$ to find the following products.

(*i*) $(x+3)(x+7)$ **(*ii*) $(4x+5)(4x+1)$**

(*iii*) $(4x-5)(4x-1)$ **(*iv*) $(4x+5)(4x-1)$**

(*v*) $(2x+5y)(2x+3y)$ **(*vi*) $(2a^2+9)(2a^2+5)$**

(*vii*) $(xyz-4)(xyz-2)$

Sol. (*i*) $(x+3)(x+7)=x^2+(3+7)\times x+3\times 7$

By using the above identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$=x^2+10x+21$

(*ii*) $(4x+5)(4x+1)$

$(4x+5)(4x+1)=(4x)^2+(5+1)\times 4x+5\times 1$

By using above identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$=16x^2+6\times 4x+5$

$=16x^2+24x+5$

(*iii*) $(4x-5)(4x-1)$

$(4x-5)(4x-1)=(4x)^2+(-5-1)\times 4x+(-5)(-1)$

By using the above identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$=16x^2+(-5-1)\times 4x+5$

$=16x^2+(-6)\times 4x+5$

$=16x^2-24x+5$

(*iv*) $(4x+5)(4x-1)$

$(4x+5)(4x-1)=(4x)^2+[5+(-1)]\times 4x+(5)\times(-1)$

By using the above identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$=16x^2+(5-1)\times 4x-5$

$=16x^2+4\times 4x-5$

$=16x^2+16x-5$

(*v*) $(2x+5y)(2x+3y)$

$(2x+5y)(2x+3y)=(2x)^2+(5y+3y)\times 2x+5y\times 3y$

By using the above identity $(x+a)(x+b)=x^2+(a+b)x+a\times b$

$=4x^2+8y\times 2x+15y^2$

$=4x^2+16xy+15y^2$

(*vi*) $(2a^2+9)(2a^2+5)$

$(2a^2+9)(2a^2+5)=(2a^2)^2+(9+5)\times 2a^2+9\times 5$

By using the above identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$=4a^4+14\times 2a^2+45$

$=4a^4+28a^2+45$

(*vii*) $(xyz-4)(xyz-2)$

$(xyz-4)(xyz-2)=(xyz)^2+(-4-2)\times xyz+(-4)\times(-2)$

By using the above identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$=x^2y^2z^2+(-4-2)\times xyz+8$

$=x^2y^2z^2+(-6)xyz+8$

$=x^2y^2z^2-6xyz+8.$

Q3. Find the following squares by using the identities.

(*i*) $(b-7)^2$ **(*ii*) $(xy+3z)^2$** **(*iii*) $(6x^2-5y)^2$**

(*iv*) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$ **(*v*) $(0.4p-0.5q)^2$** **(*vi*) $(2xy+5y)^2$**

Sol. (*i*) $(b-7)^2$

Using identity $(a-b)^2=a^2-2ab+b^2$

$(b-7)^2 = (b)^2 - 2 \times b \times 7 + (7)^2$
$= b^2 - 14b + 49.$

(*ii*) $(xy + 3z)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$(xy + 3z)^2 = (xy)^2 + 2 \times xy \times 3z + (3z)^2$
$= x^2y^2 + 6xyz + 9z^2.$

(*iii*) $(6x^2 - 5y)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$(6x^2 - 5y)^2 = (6x^2)^2 - 2 \times 6x^2 \times 5y + (5y)^2$
$= 36x^4 - 60x^2y + 25y^2.$

(*iv*) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$\left(\frac{2}{3}m + \frac{3}{2}n\right)^2 = \left(\frac{2}{3}m\right)^2 + 2 \times \frac{2}{3}m \times \frac{3}{2}n + \left(\frac{3}{2}n\right)^2$$

$$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2.$$

(*v*) $(0.4p - 0.5q)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$(0.4p - 0.5q)^2 = (0.4p)^2 - 2 \times 0.4p \times 0.5q + (0.5q)^2$
$= 0.16p^2 - 0.40pq + 0.25q^2.$

(*vi*) $(2xy + 5y)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$(2xy + 5y)^2 = (2xy)^2 + 2 \times 2xy \times 5y + (5y)^2$
$= 4x^2y^2 + 20xy^2 + 25y^2.$

Q4. Simplify.

(*i*) $(a^2 - b^2)^2$

(*ii*) $(2x + 5)^2 - (2x - 5)^2$

(*iii*) $(7m - 8n)^2 + (7m + 8n)^2$

(*iv*) $(4m + 5n)^2 + (5m + 4n)^2$

(*v*) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(*vi*) $(ab + bc)^2 - 2ab^2c$

(*vii*) $(m^2 - n^2m)^2 + 2m^3n^2.$

Sol. (*i*) $(a^2 - b^2)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$(a^2 - b^2)^2 = (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$
$= a^4 - 2a^2b^2 + b^4$

(*ii*) $(2x + 5)^2 - (2x - 5)^2$

Using identities $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$(2x + 5)^2 - (2x - 5)^2 = (2x)^2 + 2 \times 2x \times 5 + (5)^2 - [(2x)^2 - 2 \times 2x \times 5 + (5)^2]$
$= 4x^2 + 20x + 25 - (4x^2 - 20x + 25)$
$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25$
$= 40x$ ($\because$ Like terms with equal coefficient having opposite sign will be cancelled.)

(*iii*) $(7m - 8n)^2 + (7m + 8n)^2$

Using identities $(a - b)^2 = a^2 - 2ab + b^2$ and $(a + b)^2 = a^2 + 2ab + b^2$

$(7m - 8n)^2 + (7m + 8n)^2 = (7m)^2 - 2 \times 7m \times 8n + (8n)^2 + (7m)^2 + 2 \times 7m \times 8n + (8n)^2$
$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$
$= 98m^2 + 128n^2$ ($\because$ Having equal coefficients of like terms with opposite sign will be cancelled.)

(*iv*) $(4m + 5n)^2 + (5m + 4n)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$(4m + 5n)^2 + (5m + 4n)^2 = [(4m)^2 + 2 \times 4m \times 5n + (5n)^2] + [(5m)^2 + 2 \times 5m \times 4n + (4n)^2]$
$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 41m^2 + 80mn + 41n^2$

(*v*) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2 = (2.5p)^2 - 2 \times 2.5p \times 1.5q + (1.5q)^2 - [(1.5p)^2 - 2 \times 1.5p \times 2.5q + (2.5q)^2]$
$= 6.25p^2 - 7.50pq + 2.25q^2 - (2.25p^2 - 7.50pq + 6.25q^2)$
$= 6.25p^2 - 7.50pq + 2.25q^2 - 2.25p^2 + 7.50pq - 6.25q^2$

($\because$ Equal like variables with opposite signs will be cancelled.)

$= 4p^2 - 4q^2.$

(vi) $(ab + bc)^2 - 2ab^2c$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$(ab + bc)^2 - 2ab^2c = (ab)^2 + 2 \times ab \times bc + (bc)^2 - 2ab^2c$

$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$

$= a^2b^2 + b^2c^2.$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$(m^2 - n^2m)^2 + 2m^3n^2 = (m^2)^2 - 2 \times m^2 \times n^2m + (n^2m)^2 + 2m^3n^2$

$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$

$= m^4 + n^4m^2$

Q5. Show that.

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$

(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

(v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Sol. (i) $(3x + 7)^2 - 84x = (3x - 7)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

L.H.S. $= (3x + 7)^2 - 84x$

$= (3x)^2 + 2 \times 3x \times 7 + (7)^2 - 84x$

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 - 42x + 49$

$= (3x)^2 - 2 \times 3x \times 7 + (7)^2$ [$\because$ $a^2 - 2ab + b^2 = (a - b)^2$]

$= (3x - 7)^2$

L.H.S. = R.H.S.

(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

L.H.S. $= (9p - 5q)^2 + 180pq$

$= (9p)^2 - 2 \times 9p \times 5q + (5q)^2 + 180pq$

$= 81p^2 - 90pq + 25q^2 + 180pq$

$= 81p^2 + 90pq + 25q^2$

$= (9p)^2 + 2 \times 9p \times 5q + (5q)^2$

$= (9p + 5q)^2$ [$\because$ $a^2 + 2ab + b^2 = (a + b)^2$]

$\therefore$ L.H.S. = R.H.S.

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

L.H.S. $= \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$= \left(\frac{4}{3}m\right)^2 - 2 \times \frac{4}{3}m \times \frac{3}{4}n + \left(\frac{3}{4}n\right)^2 + 2mn$

$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$

$= \frac{16}{9}m^2 + \frac{9}{16}n^2$

$\therefore$ L.H.S. = R.H.S.

(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$

Using identities $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

L.H.S. $= (4pq + 3q)^2 - (4pq - 3q)^2$

$= (4pq)^2 + 2 \times 4pq \times 3q + (3q)^2 - [(4pq)^2 - 2 \times 4pq \times 3q + (3q)^2]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 24pq^2 + 9q^2]$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$

$= 48pq^2$

$\therefore$ L.H.S. = R.H.S.

(*v*) $(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)=0$

Using identity $(a-b)(a+b)=(a^2-b^2)$

L.H.S. $=(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)$

$=(a^2-b^2)+(b^2-c^2)+(c^2-a^2)$

$=a^2-b^2+b^2-c^2+c^2-a^2$

$=0$

$\therefore$ L.H.S. = R.H.S.

Q6. Using identities, evaluate.

(*i*) 71^2 (*ii*) 99^2 (*iii*) 102^2

(*iv*) 998^2 (*v*) 5.2^2 (*vi*) 297×303

(*vii*) 78×82 (*viii*) 8.9^2 (*ix*) 1.05×9.5

Sol. (*i*) $71^2=(70+1)^2$

Using identity $(a+b)^2=a^2+2ab+b^2$

$=(70)^2+2\times 70\times 1+(1)^2$

$=4900+140+1=5041$

(*ii*) $99^2=(100-1)^2$

Using identity $(a-b)^2=a^2-2ab+b^2$

$=(100)^2-2\times 100\times 1+(1)^2$

$=10000-200+1=9801$

(*iii*) $102^2=(100+2)^2$

Using identity $(a+b)^2=a^2+2ab+b^2$

$=(100)^2+2\times 100\times 2+(2)^2$

$=10000+400+4=10404$

(*iv*) $998^2=(1000-2)^2$

Using identity $(a-b)^2=a^2-2ab+b^2$

$=(1000)^2-2\times 1000\times 2+(2)^2$

$=1000000-4000+4=996004$

(*v*) $5.2^2=(5+0.2)^2$

Using identity $(a+b)^2=a^2+2ab+b^2$

$=(5)^2+2\times 5\times 0.2+(0.2)^2$

$=25+2.0+0.04=27.04$

(*vi*) $297\times 303=(300-3)\times(300+3)$

Using identity $(a+b)(a-b)=a^2-b^2$

$=(300)^2-(3)^2$

$=90000-9=89991$

(*vii*) $78\times 82=(80-2)(80+2)$

Using identity $(a+b)(a-b)=a^2-b^2$

$=(80)^2-(2)^2$

$=6400-4=6396$

(*viii*) $8.9^2=(8+0.9)^2$

Using identity $(a+b)^2=a^2+2ab+b^2$

$=(8)^2+2\times 8\times 0.9+(0.9)^2$

$=64+14.4+0.81=79.21$

(*ix*) $1.05\times 9.5=(10+0.5)(10-0.5)$

Using identity $(a+b)(a-b)=a^2-b^2$

$=(10)^2-(0.5)^2=100-0.25$

$=99.75$

Q7. Using $a^2-b^2=(a+b)(a-b)$, find

(*i*) 51^2-49^2 (*ii*) $(1.02)^2-(0.98)^2$

(*iii*) 153^2-147^2 (*iv*) $12.1^2-7.9^2$

Sol. (*i*) 51^2-49^2

By using above identity

$=(51+49)(51-49)$

$=100\times 2=200$

(*ii*) $(1.02)^2-(0.98)^2$

By using above identity

$=(1.02+(0.98)(1.02-0.98)$

$=2.00\times 0.04=0.08$

(*iii*) 153^2-147^2

By using above identity

$=(153+147)(153-147)$

$=300\times 6=1800$

(*iv*) $12.1^2-7.9^2$

By using above identity

$=(12.1+7.9)(12.1-7.9)$

$=20.0\times 4.2=84.0=84.$

Q8. Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

(*i*) **103 × 104** (*ii*) **5.1 × 5.2** (*iii*) **103 × 98**

(*iv*) **9.7 × 9.8**

Sol. (*i*) $103 \times 104 = (100 + 3) \times (100 + 4)$

By using above identity

$= (100)^2 + (3 + 4) \times 100 + 3 \times 4$

$= 10000 + 7 \times 100 + 12$

$= 10000 + 700 + 12 = 10712$

(*ii*) $5.1 \times 5.2 = (5 + 0.1) \times (5 + 0.2)$

By using above identity

$= (5)^2 + (0.1 + 0.2) \times 5 + 0.1 \times 0.2$

$= 25 + 0.3 \times 5 + 0.02$

$= 25 + 1.5 + 0.02 = 26.52$

(*iii*) $103 \times 98 = (100 + 3) \times (100 - 2)$

By using above identity

$= (100)^2 + [3 + (-2)] \times 100 + 3 \times (-2)$

$= 10000 + (3 - 2) \times 100 - 6$

$= 10000 + 1 \times 100 - 6$

$= 10000 + 100 - 6 = 10094$

(*iv*) $9.7 \times 9.8 = (10 - 0.3) \times (10 - 0.2)$

By using above identity

$= (10)^2 + \{(-0.3) + (-0.2)\} \times 10 + (-0.3) \times (-0.2)$

$= 100 + \{-0.3 - 0.2\} \times 10 + 0.06$

$= 100 - (0.5) \times 10 + 0.06$

$= 100 - 5 + 0.06$

$= 95.06.$

□□