CUBES AND CUBE ROOTS

Cubes and Cube Roots

Learn and Remember

1. Numbers like 1729, 4104, 13832 are known as Hardy Ramanujan Numbers. They can be expressed as sum of two cubes in two different ways *i.e.*,

1729	$= 1728 + 1 = 12^3 + 1^3$	
1729	$= 1000 + 729 = 10^3 + 9^3$	
4104	$= 4096 + 8 = 16^3 + 2^3$	
4104	$= 3375 + 729 = 15^3 + 9^3$	
13832	$= 8000 + 5832 = 20^3 + 18^3$	
13832	$= 13824 + 8 = 24^3 + 2^3$.	

- 2. Numbers obtained when a number is multiplied by itself three times are known as cube numbers. For example, 1, 8, 27, are the cube of 1, 2, 3,, etc.
- 3. If in the prime factorisation of any number, each factor appears three times, then the number is a perfect cube.
- 4. The symbol $(\sqrt[3]{})$ denotes cube root. For example $\sqrt[3]{27} = 3$.
- 5. To be a perfect cube number, similar factors must be in form of triplets.
- 6. All those prime factors which appear in groups of similar prime factors of three are perfect cubes. All those which are not in the group of three are not perfect cube numbers.
- 7. While getting prime factors of a number, you must start with factors (2, 3, 5, 7, 11, 13) in ascending orders.
- 8. All those numbers whose unit's digits are 0, 2, 4, 6 and 8 are cubes of even numbers and whose unit's digits are 1, 3, 5, 7, 9 are cubes of odd numbers.
- **9.** If n is a perfect cube and $n = m^3$, then m is the cube root of n. Cube root of n is written as $\sqrt[3]{n}$.

10. The cube root of a product of two perfect cubes is written as

the product of the cube roots of the perfect cubes *i.e.*, $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$.

11. The cube root of a quotient of two perfect cubes is the quotient of their cube roots *i.e.*, $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$, $b \neq 0$, where *a* and *b* are perfect cubes.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 7.1 (Page -114)

Q1.	Which of	the following	numbers	are not perfect cubes?
	(1) 910	(22)	100	(111) 1000

	(0)	410	(11)	140	(111) 1000	
	(iv)	100	(v)	46656		
Sol.	(<i>i</i>)	216				

2 216 Prime factors of $216 = 2 \times 2$ 2 108 ×2×3 2 54 $\times 3 \times 3$ 3 Here, all factors are in groups 27 of 3's (in triplets). 3 9 So, 216 is a perfect cube number. 3 3 (ii) 128

200 AM 100		
	2	128
ign mai	2	64
(etalojo	2	32
Contra	2	16
	2	8
	2	4
-	2	2
-		1

132			MATHEMATICS-VIII	De Desere	contraction of the second second second
(iii) 1000			ercora an O and	CUBES AND CUBE ROOTS	
cubes and the	2	1000	the module of the cube tosts :	Q2. Find the small following num	lest number by which each of t abers must be multiplied to obtain
and the second	2	500	Prime factors of $1000 = 2 \times 2$	perfect cube.	(iii) 256 (iiii) 72
MISHON SOLALISO	2	250	X 2 X 5 X 5 X 5	(<i>i</i>) 243 (<i>iv</i>) 675	(v) 100
and has a rail	5	125	group of 3's (in triplets)	Fol. (i) 243	121 a volume sign from the hills
the second second second	5	25	So, 1000 is a perfect cube number	3	243 Prime factors of $243 - 3 \times 3 \times 3$
tearn and Herry	5	5	,	3	81 × 3
	18	1	PATEXTBOOK QUES	100 C x S = 001 3	27 Here, 3 do not appear in a group
(<i>iv</i>) 100			orly can be expensed in such of the	toa oh 8 bargan	9 of 3's (in triplets).
	2	100	Prime factors of $100 - 2 \times 2 \times 5 \times 5$	publication all all to gat	So, 243 must be multiplied by 3 t
	2	50	Here factors 2 and 5 do not appear	and beilg dlum	make it a perfect cube number.
	5	25	in group of 3's (in triplets).	porten a di estado la	of Or and the matter bindescript, cabe min
	5	5	So. 100 is not a perfect cube	(11) 256	956
	-	1	number.	paired to the following	
() 10050		1 1	= 25 Million W	2	$\frac{128}{2}$ Prime factors of $256 = 2 \times 2 \times 2$
(v) 46656	9	AGGEG	a summing in an applied by the if your	2	64 × 2 × 2
Linna araches	4	40000	appropriate Party and pole 1, 8, 27, 1, 1	2	32 × 4 × 4 × 2 × 2
and the second	Z	23328	the second se	2	16 a group of 3's (in triplets).
moleum while the	2	11664	and a support of the second second	2	8 So, 256 must be multiplied by 2
An Brinning Party	2	5832	Prime factors of $46656 = 2 \times 2 \times 2$	2	4 make it a perfect cube number.
A. The symbol	2	2916	×2×2×2	2	2
- 6. To be a parte	2	1458	×3×3×3	N = 1 + 18 volume alle	to 11 mer han a manufact of a perfect cube
of National	3	729	× 3 × 3 × 3	(iii) 72	
	3	243	Here, all the factors appear in a $\int 2^{2} e^{i t} dt = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$	2×2×2×2×2=8212	72 Prime factors of $72 - 2 \times 2 \times 2 \times 2$
The set of the set	3	81	So 46656 is a perfect cube number	2	36 × 3
at an adda los sa	3	27	too, toooo is a perfect cube number.	2	18 Factors of 3 are not in group of
rabidiz	3	9	in growthing gederes.	3	9 3's (in triplets).
8. All those out	3	3	and the design of the State of States	And the set is a set	So, 72 must be multiplied by 3 to
cubes afovo-	0	1	- A WINDOW WITH THE ALL ROOM IN THE PARTY OF	parted puls	make it a perfect cube number.
		I	and the state of the	The the mathematic	supering the main in the 1 x 3 - 20 in
			- demonstration of the second		

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(<i>iv</i>) 675		AND DUE HOUSE	(iii) 135	Prime factors of $135 = 3 \times 3 \times 3 \times 5$
with to where doi 3	675	Prime factors of $675 = 3 \times 3 \times 3 \times 5$	3 13	5 The prime factor of 5 does not
a minido ob boiling	225	×5" moderan antwolton	3 4	appear in group of three.
3	75	Here, factor 5 does not appear in	3 1	5 5 to make it a perfect cube number.
5	25	group of 3's (in triplets).	5	$\overline{5}$ Hence, the smallest number is 5.
5	5	make it a perfect cube number.	(4) 175616	And perfect cube number = $135 \div 5$ 1 _ 97
5×5×5×5×5×5×5	1	survey subband generation in survey and	(<i>iv</i>) 192	= 21.
(v) 100		8 4 18 1	2 19	Prime factors of $192 = 2 \times 2 \times 2 \times 2$
2	100	Prime factors of $100 = 2 \times 2 \times 5 \times 5$	2 9	⁻ × 2 × 2 × 3 ⁰⁶ The prime factor 3 does not appear
2	50	appear in group of 3's (in triplets)	2 4	¹⁸ in a group of three.
5	25	So, 100 must be multiplied by	2 2	24 So, this number must be divided by
5	5	$2 \times 5 = 10$ to make it a perfect	2	12 3 to make it a perfect cube number.
	1	cube number.	2	6 Hence, the smallest number is 3.
Q3. Find the smalle	st num	ber by which each of the following	3	3 And perfect cube number = $192 \div 3$
numbers must	be div	ided to obtain a perfect cube.		$\overline{1} = 64.$
(i) 81 (iv) 192	(1	(11) 128 (11) 135 (11) 175 (
Sol. (i) 81	2.91000	Prime factors of $81 = 3 \times 3 \times 3 \times 3$	(v) 704	$\mathbf{P}_{\text{interfeature}} = 5704 = 9 \times 9 \times 9 \times 9$
3	81	One 3's factors is not grouped in	2 7	$\frac{104}{2} \times 2 \times 2 \times 11$
S of S wild by 2 to 5	27	triplets.	2 3	52 The prime factor 11 does not appear
midmon refer 3	9	So, this number must be divided by 3 to make it a perfect cube.	2 1	76 in a group of three.
3	3	Hence, the smallest number is 3	2	88 So, this number must be divided
	1	and perfect cube number $81 \div 3 = 27$.	2	number.
(<i>ii</i>) 128		Prime factors of $128 = 9 \times 9 \times 9 \times 9$	2	$\frac{22}{11}$ Hence, the smallest number is 11
2	128	×2×2×2	11	and perfect cube number = $704 \div 11$
_2	64	The prime factor 2 does not appear	04 D 11 11 mehre	1 = 04.
2	32	in group of three except two	2 cm, 5 cm. How	many such cuboids will he need to
2	16	So, this number must be divided	form a cube?	2 26ng Prin
2	8	by 2 to make it a perfect cube	Sol. Given number = 5	(2×5)
2	4	number.	So the number mus	at be multiplied by $2 \times 2 \times 5 = 20$ to mak
2	2	And perfect cube number = 128 - 2	it a perfect cube nu	mber.
	1	= 64.	Hence, he needs 20	cuboids.

SHOULD CHOI EXERCISE 7.2 (Page 114) (iv) 27000 Q1. Find the cube root of each of the following numbers by prime factorisation method. Prime factors of $27000 = 2 \times$ (*i*) 64 (*ii*) 512 (iii) 10648 $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$ (*iv*) 27000 (*v*) 15625 (vi) 13824 (vii) 110592 (viii) 46656 (ix) 175616 Therefore, $\sqrt[3]{27000} = 2 \times 3$ (x) 91125 $\times 5 = 30$. Ans. Sol. (i) 64 Prime Langered 109 Prime factors of $64 = 2 \times 2$ $\times 2 \times 2 \times 2 \times 2$ TIC Therefore, $\sqrt[3]{64} = 2 \times 2$ (v) 15625 = 4. Ans. (ii) 512 Prime factors of $15625 = 5 \times 5$ $\times 5 \times 5 \times 5 \times 5$ Therefore, $\sqrt[3]{15625} = 5 \times 5$ = 25. Ans. Prime factors of $512 = 2 \times 2$ $\times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ (vi) 13824 Therefore, $\sqrt[3]{512} = 2 \times 2 \times 2$ = 8. adam langung all Prime factors of $13824 = 2 \times$ $2 \times 2 \times 2$ $\times 3 \times 3 \times 3$ (iii) 10648 Therefore, $\sqrt[3]{13824} = 2 \times 2$ of broad of line ships 2 Sente Gran D $\times 2 \times 3 = 24$. Ans. Prime factors of $10648 = 2 \times 2$ ×2×11×11×11 Therefore, $\sqrt[3]{10648} = 2 \times 11$ = 22. Ans.

Henro, he meda 20 cubrida

MATHEMATICS-VIII

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(vii) 110592		all value and and a	(<i>ix</i>) 175616		
2	110592	(40) 27000 1	and a st yam and 2	175616	
2	55296	Prime factors of 110509 - 9	2	87808	nation in any in the second second
2	27648	$2 \times 2 \times$	2	43904	or more digita.
A + 8 + 1 + 2 + 2 +	13824	×2×2×2×3×3×3	and all all shink the 2	21952	
2	6912	Therefore, $\sqrt[3]{110592} = 2 \times 2$	2	10976	Prime factors of $175616 = 2$
	3456	$\times 2 \times 2 \times 3 = 48$. Ans.	2	5488	$\times 2 \times 2$
2	1728		2	2744	$\times 2 \times 7 \times 7 \times 7$
2	864		2	1372	Therefore, $\sqrt[3]{175616} = 2 \times 2$
2	432		2	686	$\times 2 \times 7 = 56$. Ans.
2	216	The second state of the second state	7	343	
2	108	(c) 15625 and 2 -	7	49	
2	54	6 16	7	7	
3	27	8 6		1	
3	9		(x) 91125		
3	3	8	(Suffer abdE) 3	91125	
0 × 0 = 02010 0	1		3	30375	
(viii) 46656		Print Sectors of 512 = 2 M2	(Cour digit namba) 3	10125	
2	46656	(m) 13824c	8 (Four digit mambed)	3375	Prime factors of $91125 = 3 \times$
2	23328	82, 19 2 - 2012 - 2 - 2 - 2 - 2	3	1125	3 × 3 × 3 × 3 × 3 × 5 × 5 × 5
2	11664	Prime factors of 46656 - 9	3	375	Therefore, $\sqrt[3]{91125} = 3 \times 3$
2	5832	$x 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$	5 Shade digits abadie 5	125	× 5 = 45. Ans.
	2916	×3×3×3×3	(Single digit number)	25	$2^0 = 8$
	1458	Therefore, $\sqrt[3]{46656} = 2 \times 2$	- Maeson woy and adar 5	5	QC. You are told that 1,831
3	729	× 3 × 3 = 36. Ans.		1	
	243 01	3 3	Q2. State true or fals	e.	
- 2	97	Trituna dischare of Hop-Dy = 2 = 0	(i) Cube of any	odd num	ber is even.
3	9		(ii) A perfect cu	be does n	ot end with two zeros.
3	3	strend for ollower in 2 x 11	(iii) If square of ends with 2	a numbe	r enus with 5, then its cube
	1		(iv) There is no	perfect cu	ube which ends with 8.

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(v) The cube of a two digit number.	number may be a three					
(vi	The cube of a two digit number may have seven or more digits.						
(vii) The cube of a single digit digit number.	The cube of a single digit number may be a single digit number.					
Sol. (i) False						
	Since, $1^3 = 1$, $3^3 = 27$, $5^3 = 12$	25,, are all odd.					
(ii) True						
	Since, a perfect cube ends wi	ith three zeros.					
	For example, $1000 = 10^3$, 800	$00 = 20^3, 27000 = 30^3, \dots,$					
	so on.						
(iii)) False						
	Since, $5^2 = 25$, $5^3 = 125$, $15^2 = 125$	$= 225, 15^3 = 3375.$					
	(Did not end with 25)						
<i>(iv)</i>) False						
	Since, $12^3 = 1728$	(Ends with 8)					
	$22^3 = 10648$	(Ends with 8)					
(v)	False						
	Since, $10^3 = 1000$	(Four digit number)					
	$11^3 = 1331$	(Four digit number)					
(vi)	False						
	Since, $99^3 = 970299$	(Six digit number)					
(vii)	True						
	$1^3 = 1$ and $3^3 = 1 \times 10^3$	(Single digit number)					
	$2^3 = 8$	(Single digit number)					
Q3. You with gue	are told that 1,331 is a perfe hout factorisation what is i ss the cube roots of 4913, 12	ect cube. Can you guess ts cube root? Similarly, 2167, 32768.					
Sol. We	know that $10^3 = 1000$.	Q2. State true of false					
Poss	sible cube of $11^3 = 1331$.						
Sinc	e, cube of unit's digit $1^3 = 1$.						
So, o	cube root of 1331 is 11.						
4913	3						
We l	know that $7^3 = 343$						

Next number comes $17^3 = 4913$ Hence, cube root of 4913 is 17. 12167 We know that $3^3 = 27$ One's digit is 7. Now, next number comes $13^3 = 2197$ Now, next number comes $23^3 = 12167$ Hence, cube root of 12167 is 23. 32768 We know that $2^3 = 8$ which is unit digit. Now, comes $12^3 = 1728$. Now, next comes $22^3 = 10648$. Now, next number comes $32^3 = 32768$. Hence, cube root of 32768 is 32.

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