13 Direct and Inverse Proportions

Learn and Remember

1. Two quantities x and y are said to be in direct proportion, if they increase or decrease together in such a manner that the ratio of their corresponding values remains constant. That is, if $\frac{x}{y} = k$, [If k is a positive number], then x and y are said to vary directly. In such a case, if y_1, y_2 are the values of y corresponding the values x_1, x_2 of x respectively, then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.

   Some examples of direct proportion has been given below.

   (*i*) If the number of articles purchased increases, the total cost also increases.

   (*ii*) More the money deposited in a bank, more is the interest earned.

2. Two quantities x and y are said to be in inverse proportion if an increase in x causes a proportional decrease in y and vice-versa, in such a manner that the product of their corresponding values remains constant. That is, if $xy = k$, then x and y are said to vary inversely. In this case, if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively, then $x_1y_1 = x_2y_2$ or $\frac{x_2}{x_1} = \frac{y_2}{y_1}$.

   (*i*) As the speed of the vehicle increases, the time taken to cover the same distance decreases.

   (*ii*) For a given job, more the number of workers, less will be the time taken to complete the work.

3. Many of some situations are not in direct proportion. For example

   (*i*) Physical changes in human beings occur with time but not necessarily in a predetermined ratio.

   (*ii*) Changes in weight and height among individuals are not in any known proportion.

   (*iii*) There is no direct relationship or ratio between the height of a tree and the number of leaves growing on its branches.

4. (*i*) When two quantities x and y are in direct proportion (or vary directly) they are also written as $x \propto y$.

   (*ii*) When two quantities x and y are in inverse proportion (or vary inversely) they are also written as $x \propto \frac{1}{y}$.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 13.1 (*Page –208*)

Q1. Following are the car parking charges near a railway station upto

4 hours	**₹ 60**
8 hours	**₹ 100**
12 hours	**₹ 140**
24 hours	**₹ 180**

Check if the parking charges are in direct proportion to the parking time.

Sol. Charges per hour, $k_1 = \frac{60}{4} = ₹\ 15$

$$k_2 = \frac{100}{8} = ₹\ 12.5$$

$$k_3 = \frac{140}{12} = ₹\ 11.67$$

$$k_4 = \frac{180}{24} = ₹\ 7.5$$

Here charges per hour are not same or $k_1 \neq k_2 \neq k_3 \neq k_4$.

So, the parking charges are not in direct proportion to the parking time.

Q2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

Parts of red pigment	1	4	7	12	20
Parts of base	8	...	...	...	...

Sol. Let the ratio of parts of red pigment and parts of base be $\frac{x}{y}$.

Here $x_1 = 1, y_1 = 8$

$$k = \frac{x_1}{y_1} = \frac{1}{8}$$

When $x_2 = 4, y_2 = ?$

$$k = \frac{x_2}{y_2} \Rightarrow y_2 = \frac{x_2}{k} = \frac{4}{\frac{1}{8}} = 4 \times 8 = 32$$

When $x_3 = 7, y_3 = ?$

$$k = \frac{x_3}{y_3} \Rightarrow y_3 = \frac{x_3}{k} = \frac{7}{\frac{1}{8}} = 7 \times 8 = 56$$

When $x_4 = 12, y_4 = ?$

$$k = \frac{x_4}{y_4} \Rightarrow y_4 = \frac{x_4}{k} = \frac{12}{\frac{1}{8}} = 12 \times 8 = 96$$

When $x_5 = 20, y_5 = ?$

$$k = \frac{x_5}{y_5} \Rightarrow y_5 = \frac{x_5}{k} = \frac{20}{\frac{1}{8}} = 20 \times 8 = 160$$

Parts of red pigment	1	4	7	12	20
Parts of base	8	52	36	96	160

Q3. In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?

Sol. Let the parts of red pigment mix with 1800 mL base be x.

Parts of red pigment	1	x
Parts of base (in mL)	75	1800

It is in direct proportion.

$$\therefore \quad \frac{1}{75} = \frac{x}{1800} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad 75 \times x = 1 \times 1800$$

$$\Rightarrow \quad x = \frac{1800}{75}$$

$$\Rightarrow \quad x = 24 \text{ parts}$$

With base 1800 mL, 24 parts red pigment should be mixed.

Q4. A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?

Sol. Let the number of bottles filled in five hours be x.

Hours	6	5
Bottles	840	x

Here, ratio of hours and bottles are in direct proportion.

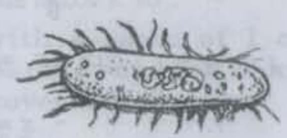
$$\therefore \quad \frac{6}{840} = \frac{5}{x} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad x \times 6 = 5 \times 840$$

$$\Rightarrow \quad x = \frac{5 \times 840}{6} = 700$$

Hence, machine will fill 700 bottles in five hours.

Q5. A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the *actual* length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?



Sol. Let enlarged length of bacteria be x.

Actual length of the bacteria $= \frac{5}{50000} = \frac{1}{10000}$

$= 1 \times 10^{-4}$ cm $= 10^{-4}$ cm

Length	5	x
Enlarged length	50,000	20,000

Here length and enlarged length of bacteria are in direct proportion.

$$\therefore \quad \frac{5}{50{,}000} = \frac{x}{20{,}000} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad x \times 50{,}000 = 5 \times 20{,}000$$

$$\Rightarrow \quad x = \frac{5 \times 20{,}000}{50{,}000} = 2$$

Hence, the enlarged length of bacteria is 2 cm.

Q6. In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?

Sol. Let the length of model ship be x.

Length of actual ship (in m)	12	28
Length of model ship (in cm)	9	x

Here, length of mast and actual length of ship are in direct proportion.

$$\therefore \quad \frac{12}{9} = \frac{28}{x} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad 12 \times x = 28 \times 9$$

$$\therefore \quad x = \frac{28 \times 9}{12} = \frac{252}{12} = 21$$

Hence, length of the model ship is 21 cm.

Q7. Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in (*i*) 5 kg of sugar? (*ii*) 1.2 kg of sugar?

Sol. (*i*) Let sugar crystals be x.

Weight of sugar (in kg)	2	5
Number of crystals	9×10^6	x

In this problem, if weight of sugar increases, the number

of crystals increases and if weight is decreased, the number of crystals is lessened. So, it is a matter of direct proportion.

$$\therefore \quad \frac{2}{9 \times 10^6} = \frac{5}{x} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad 2 \times x = 5 \times 9 \times 10^6$$

$$\Rightarrow \quad x = \frac{5 \times 9 \times 10^6}{2} = \frac{45}{2} \times 10^6$$

$$= 22.5 \times 10^6 = 2.25 \times 10^7$$

Hence, the number of sugar crystals is 2.25×10^7.

(*ii*) Let the sugar crystals be x.

Weight of sugar (in kg)	2	1.2
Number of crystals	9×10^6	x

$$\therefore \quad \frac{2}{9 \times 10^6} = \frac{1.2}{x} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad 2 \times x = 9 \times 10^6 \times 1.2$$

$$\Rightarrow \quad x = \frac{9 \times 10^6 \times 1.2}{2}$$

$$\Rightarrow \quad x = 0.6 \times 9 \times 10^6 = 5.4 \times 10^6$$

Hence, the number of sugar crystals is 5.4×10^6.

Q8. Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

Sol. Let distance covered in the map be x.

Actual distance (in km)	18	72
Distance covered in map (in cm)	1	x

Here actual distance and distance covered in the map are in direct proportion

$$\therefore \quad \frac{18}{1} = \frac{72}{x} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad 18 \times x = 1 \times 72$$

$$\Rightarrow \quad x = \frac{72}{18}$$

$$\Rightarrow \quad x = 4$$

Hence, distance covered in the map is 4 cm.

Q9. A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (*i*) the length of the shadow cast by another pole 10 m 50 cm high (*ii*) the height of a pole which casts a shadow 5 m long.

Sol. Since, height of a pole and the length of a shadow are in direct proportion.

Since 1 m = 100 cm

5 m 60 cm = 5 × 100 + 60 = 500 + 60 = 560 cm

3 m 20 cm = 3 × 100 + 20 = 300 + 20 = 320 cm

10 m 50 cm = 10 × 100 + 50 = 1000 + 50 = 1050 cm

5 m = 5 × 100 = 500 cm

(*i*) Let the length of the shadow of another pole be y.

Height of pole (in cm)	560	1050
Length of shadow (in cm)	320	y

$$\therefore \quad \frac{560}{320} = \frac{1050}{y} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad 560 \times y = 320 \times 1050$$

$$\therefore \quad y = \frac{320 \times 1050}{560} = \frac{336000}{560} = 600 \text{ cm} = 6 \text{ m}$$

Hence, length of the shadow of another pole is 6 m.

(*ii*) Let height of the pole be x.

Height of pole (in cm)	560	x
Length of shadow (in cm)	320	500

$$\therefore \quad \frac{560}{320} = \frac{x}{500} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad x \times 320 = 560 \times 500$$

$$\therefore \quad x = \frac{560 \times 500}{320} = \frac{280000}{320} = 875 = 8 \text{ m } 75 \text{ cm}$$

Hence, height of the pole is 8 m 75 cm.

Q10. A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Sol. Let distance covered in 5 hours be x km.

1 hour = 60 minutes

5 hours = 5 × 60 = 300 minutes

Distance (km)	14	x
Time (in minutes)	25	300

$$\therefore \quad \frac{14}{25} = \frac{x}{300} \qquad \left(\because \frac{x_1}{y_1} = \frac{x_2}{y_2}\right)$$

$$\Rightarrow \quad 25 \times x = 300 \times 14$$

$$\Rightarrow \quad x = \frac{300 \times 14}{25} = 168$$

Hence, the distance covered in 5 hours is 168 km.

EXERCISE 13.2 *(Page – 213-214)*

Q1. Which of the following are in inverse proportion?

(*i*) **The number of workers on a job and the time to complete the job.**

(*ii*) **The time taken for a journey and the distance travelled in a uniform speed.**

(*iii*) **Area of cultivated land and the crop harvested.**

(*iv*) **The time taken for a fixed journey and the speed of the vehicle.**

(*v*) **The population of a country and the area of land per person.**

Sol. (*i*) The number of workers and the time to complete the job is inverse proportion. Since, less workers will take more time and more workers will take less time, so it is a case of inverse proportion.

(*ii*) Time and distance covered is a direct proportion.

(*iii*) Direct proportion, since more area of cultivated land will yield more crops.

(*iv*) Time and speed are inverse proportion. Since, time is less, speed will be more.

(*v*) It is a inverse proportion. If the population of a country increases, the area of land per person decreases.

Q2. In a Television game show, the prize money of ₹ 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in ₹)	1,00,000	50,000	...	...	...	...	...

Sol. Here, number of winners and prize money are in inverse proportion.

Since, winners are increasing, prize money is decreasing, so it is a case of inverse proportion.

When the number of winners are 4, each winner will get

$$= \frac{100000}{4} = ₹\ 25,000$$

When the number of winners are five, each winner will get

$$= \frac{100000}{5} = ₹\ 20,000$$

When the number of winners are 8, each winner will get

$$= \frac{100000}{8} = ₹\ 12,500$$

When the number of winners are 10, each winner will get

$$= \frac{100000}{10} = ₹\ 10,000$$

When the number of winners are 20, each winner will get

$$= \frac{100000}{20} = ₹\ 5000.$$

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in ₹)	1,00,000	50,000	25,000	20,000	12,500	10,000	5,000

Q3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	...	...	...

(*i*) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(*ii*) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

(*iii*) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40°?

Sol. Here, in this problem, the number of spokes are increasing and the angle between a pair of consecutive spokes is decreasing. So, it is a inverse proportion and angle at the centre of a circle is 360°.

When, the number of spokes is 8, then angle between a pair of consecutive spokes $= \frac{360°}{8} = 45°$.

When there are 10 spokes, then angle between a pair of consecutive spokes = $\frac{360°}{10} = 36°$.

When there are 12 spokes, then angle between a pair of consecutive spokes = $\frac{360°}{12} = 30°$.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	45°	36°	30°

(*i*) Yes, the number of spokes and the angles formed between a pair of consecutive spokes is inverse proportion.

(*ii*) When there are 15 spokes, then the angle between a pair of consecutive spokes on a wheel = $\frac{360°}{15} = 24°$.

(*iii*) The number of spokes would be needed = $\frac{360°}{40°} = 9$.

Q4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Sol. Each child gets = 5 sweets.

24 children will get = $24 \times 5 = 120$ sweets.

Total number of sweets = 120.

If the number of children is reduced by 4, then left children = $24 - 4 = 20$.

Now, each child will get sweets = $\frac{120}{20} = 6$ sweets.

Q5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

Sol. Let the number of days be x.

Total number of animals = $20 + 10 = 30$.

Animals	20	30
Days	6	x

If the number of animals increases then the number of days decreases. Therefore, it is a case of inverse proportion.

$\therefore \quad 20 \times 6 = 30 \times x$

or $\quad 30 \times x = 20 \times 6 \qquad (\because \; x_1y_1 = x_2y_2)$

$\Rightarrow \quad x = \frac{20 \times 6}{30} = 4$

Hence, the food will last for four days.

Q6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

Sol. Let time taken to complete the job be x.

Persons	3	4
Days	4	x

If number of persons increases, the number of days decreases. Therefore, it is a matter of inverse proportion.

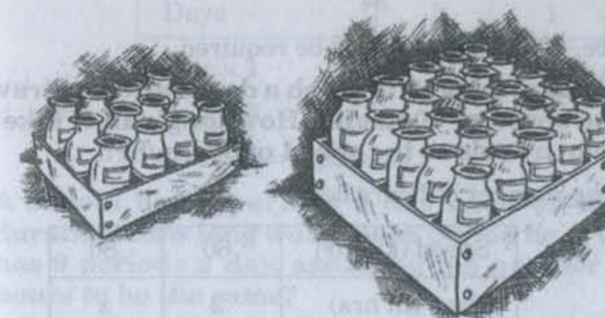
$\therefore \quad 3 \times 4 = 4 \times x$

or $\quad 4 \times x = 3 \times 4 \qquad (\because \; x_1y_1 = x_2y_2)$

$\Rightarrow \quad x = \frac{3 \times 4}{4} = 3$

Hence, they will complete the job in 3 days.

Q7. A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?



Sol. Let the number of boxes be x.

Number of bottles in each box	12	20
Boxes	25	x

Here, number of bottles increases and the number of boxes decreases.

So, it is a case of inverse proportion.

$\therefore \quad 12 \times 25 = 20 \times x \qquad (\because \; x_1y_1 = x_2y_2)$

$\Rightarrow \quad 20 \times x = 12 \times 25$

$\Rightarrow \quad x = \dfrac{12 \times 25}{20} = 15$

Hence, 15 boxes would be filled.

Q8. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

Sol. Let the number of machines required be x.

Days	63	54
Machines	42	x

Here, days are decreasing, the number of machines are increasing, therefore, it is a case of inverse proportion.

$\therefore \quad 63 \times 42 = 54 \times x \qquad (\because \; x_1y_1 = x_2y_2)$

$\Rightarrow \quad 54 \times x = 63 \times 42$

$\Rightarrow \quad x = \dfrac{63 \times 42}{54} = 49$

Hence, 49 machines would be required.

Q9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/hr? How long will it take when the car travels at the speed of 80 km/hr?

Sol. Let the number of hours be x.

Speed (km/hr)	60	80
Time (in hrs)	2	x

In this problem, if speed of the car decreases, time increases and if speed increases, time decreases. So, it is a case of inverse proportion.

$\therefore \quad 60 \times 2 = 80 \times x \qquad (\because \; x_1y_1 = x_2y_2)$

$\Rightarrow \quad 80 \times x = 60 \times 2$

$\therefore \quad x = \dfrac{60 \times 2}{80} = \dfrac{3}{2} \text{ hrs} = 1\dfrac{1}{2} \text{ hrs}$

Hence, the car will take $1\dfrac{1}{2}$ hrs to reach its destination.

Q10. Two persons could fit new windows in a house in 3 days.

(*i*) One of the persons fell ill before the work started. How long would the job take now?

(*ii*) How many persons would be needed to fit the windows in one day?

Sol. (*i*) Let number of days be x.

Persons	2	1
Days	3	x

$\therefore \quad 2 \times 3 = 1 \times x \qquad (\because \; x_1y_1 = x_2y_2)$

$\Rightarrow \quad 1 \times x = 2 \times 3$

$\Rightarrow \quad x = 2 \times 3 = 6$ days

(*ii*) Let number of persons be x.

Persons	2	x
Days	3	1

$\therefore \quad 2 \times 3 = x \times 1 \qquad (\because \; x_1y_1 = x_2y_2)$

$\Rightarrow \quad x \times 1 = 2 \times 3$

$\Rightarrow \quad x = 2 \times 3 = 6$ persons.

Q11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

Sol. Let the duration of each period be x.

Period	8	9
Duration of period (min.)	45	x

If the period increases then the duration of period decreases. So, it is a case of inverse proportion.

$$\therefore \quad 8 \times 45 = 9 \times x \qquad (\because \ x_1y_1 = x_2y_2)$$

$$\Rightarrow \quad 9 \times x = 8 \times 45$$

$$\Rightarrow \quad x = \frac{45 \times 8}{9} = 40$$

Hence, duration of each period would be 40 minutes. □□