12 Exponents and Powers

Learn and Remember

1. We read 10^{24} as 10 raised to the power 24. And 2^{-3} is read as 2 raised to the power – 3.
2. You have that $10^3 = 1000$, $10^2 = 100$ and $10^1 = 10$, $10^0 = 1 = \frac{10}{10}$ and $10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{100} = \ldots\ldots$.

   We observe here that as the exponent decreases by 1, the value becomes one-tenth of the previous value.
3. Numbers with exponents obey the following laws.

   (*i*) $a^m \times a^n = a^{m+n}$ (*ii*) $a^m \div a^n = a^{m-n}$

   (*iii*) $(a^m)^n = a^{mn}$ (*iv*) $a^m \times b^m = (ab)^m$

   (*v*) $a^0 = 1$ (*vi*) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

   (*vii*) $a^m = \frac{1}{a^{-m}}$
4. Very small numbers can be expressed in standard form using negative exponents as $0.000000078 = 7.8 \times 10^{-8}$.
5. Very large numbers can be expressed in standard form using positive exponents, as $60200000000 = 6.02 \times 10^{10}$.
6. When we have to add numbers in standard form, we convert them into numbers with the same exponents. As we add the mass of earth which is 5.97×10^{24} kg and mass of moon is 7.35×10^{22} kg, what is the total mass.

   Total mass $= 5.97 \times 10^{24}$ kg $+ 7.35 \times 10^{22}$ kg
   $= 5.97 \times 100 \times 10^{22} + 7.35 \times 10^{22}$ kg
   $= (597 \times 10^{22} + 7.35 \times 10^{22})$ kg
   $= (597 + 7.35) \times 10^{22}$ kg
   $= 604.35 \times 10^{22}$ kg $= 6.04 \times 10^{24}$ kg

Learn some of the formulae by Heart

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $a^m \times b^m = (ab)^m$
5. $a^{-m} = \frac{1}{a^m}$
6. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
7. $a^0 = 1$
8. $\left(\frac{1}{a}\right)^{-1} = a$

TEXTBOOK QUESTIONS SOLVED

EXERCISE 12.1 (*Page 197–198*)

Q1. Evaluate.

(i) 3^{-2} **(ii)** $(-4)^{-2}$ **(iii)** $\left(\frac{1}{2}\right)^{-5}$

Sol. (i) $3^{-2} = \frac{1}{3^2}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{9}$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{16}$

(iii) $\left(\frac{1}{2}\right)^{-5} = \left[\left(\frac{1}{2}\right)^{-1}\right]^5$ $\left[\because\ \left(\frac{1}{a}\right)^{-1} = a\right]$

$= (2)^5$

$= 32.$

Q2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ **(ii)** $\left(\frac{1}{2^3}\right)^2$ **(iii)** $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ **(v)** $2^{-3} \times (-7)^{-3}$.

Sol. (i) $(-4)^5 \div (-4)^8$

$= (-4)^{5-8}$ $[\because\ x^m \div x^n = x^{m-n}]$

$= (-4)^{-3} = \frac{1}{(-4)^3}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

(ii) $\left(\frac{1}{2^3}\right)^2$

$= \frac{1^2}{(2^3)^2}$ $\left[\because\ \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$= \frac{1^2}{2^{3\times 2}} = \frac{1}{2^6}$ $[\because\ (a^m)^n = a^{mn}]$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

$= (-3)^4 \times \frac{5^4}{3^4}$

$= (-3)^4 \times 5^4 \times 3^{-4} = ((-1)^4 \times 3)^4 \times 3^{-4} \times 5^4$

$= (-1)^4 \times 3^4 \times 3^{-4} \times 5^4$ $[\because\ (-1)^4 = 1]$

$= 1 \times 3^{4+(-4)} \times 5^4$ $[\because\ a^m \times a^n = a^{m+n}]$

$= 1 \times 3^{4-4} \times 5^4$

$= 1 \times 3^0 \times 5^4 = 1 \times 1 \times 5^4$ $(\because\ a^0 = 1)$

$= 1 \times 5^4 = 5^4.$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

$= 3^{-7-(-10)} \times 3^{-5}$ $[\because\ x^m \div x^n = x^{m-n}]$

$= 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5}$

$= 3^{3+(-5)}$

$= 3^{3-5} = 3^{-2}$ $[\because\ a^m \times a^n = a^{m+n}]$

$= \frac{1}{3^2}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

(v) $2^{-3} \times (-7)^{-3}$

$= \frac{1}{2^3} \times \frac{1}{(-7)^3}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{[2\times(-7)]^3}$ $[\because\ (a^3 \times b^3) = (a \times b)^3]$

$= \frac{1}{(-14)^3}.$

Q3. Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ **(ii)** $(2^{-1} \times 4^{-1}) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$ **(iv)** $(3^{-1} + 4^{-1} + 5^{-1})^0$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Sol. (i) $(3^0 + 4^{-1}) \times 2^2$

$= \left(\frac{1}{1} + \frac{1}{4}\right) \times 2^2$ $\left[\because\ a^0 = 1 \text{ and } a^{-m} = \frac{1}{a^m}\right]$

$= \left(\frac{4+1}{4}\right) \times 2^2 = \frac{5}{4} \times 2^2 = \frac{5 \times 2^2}{2^2}$

$= 5 \times 2^2 \times 2^{-2} = 5 \times 2^{2+(-2)}$ $\left[\because\ \frac{1}{a^m} = a^{-m}\right]$

$= 5 \times 2^{2-2} = 5 \times 2^0 = 5 \times 1 = 5.$ $[\because\ a^m \times a^n = a^{m+n}]$

(ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$

$= \left(\frac{1}{2^1} \times \frac{1}{4^1}\right) \div 2^{-2}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \left(\frac{1}{2} \times \frac{1}{2 \times 2}\right) \div 2^{-2} = \left(\frac{1}{2^3}\right) \div 2^{-2}$

$= 2^{-3} \div 2^{-2} = 2^{-3-(-2)} = 2^{-3+2}$ $[\because\ x^m \div x^n = x^{m-n}]$

$= 2^{-1} = \frac{1}{2^1}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \frac{1}{2}.$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$= (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= 2^{-1 \times (-2)} + 3^{-1 \times (-2)} + 4^{-1 \times (-2)}$ $[\because\ (a^m)^n = a^{mn}]$

$= 2^2 + 3^2 + 4^2$

$= 4 + 9 + 16 = 29.$

(iv) $[3^{-1} + 4^{-1} + 5^{-1}]^0$

$= \left[\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right]^0$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$= \left[\frac{20+15+12}{60}\right]^0$

$= \left[\frac{47}{60}\right]^0 = 1.$ $[\because\ a^0 = 1]$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

$= \left(\frac{-2}{3}\right)^{-2 \times 2}$ $[\because\ (x^m)^n = x^{m \times n} = x^{mn}]$

$= \left(\frac{-2}{3}\right)^{-4}$

$= \left[\left(\frac{-2}{3}\right)^{-1}\right]^4 = \left(\frac{-3}{2}\right)^4$ $\left[\because\ \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right]$

$= \frac{-3 \times -3 \times -3 \times -3}{2 \times 2 \times 2 \times 2} = \frac{81}{16}.$

Q4. Evaluate

(i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ **(ii)** $(5^{-1} \times 2^{-1}) \times 6^{-1}$

Sol. (i) $\frac{8^{-1} \times 5^3}{2^{-4}} = (2^3)^{-1} \times 5^3 \times 2^4$ $\left[\because\ \frac{1}{a^{-m}} = a^m\right]$

$= 2^{-3} \times 5^3 \times 2^4$ $[\because\ (a^m)^n = a^{m \times n}]$

$= 2^{4-3} \times 5^3$ $[\because\ a^m \times a^n = a^{m+n}]$

$= 2^1 \times 5^3 = 2 \times 125 = 250$

(ii) $(5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5^1} \times \frac{1}{2^1}\right) \times \frac{1}{6^1}$ $\left[\because\ a^{-m} = \frac{1}{a^m}\right]$

$$= \left(\frac{1}{5}\times\frac{1}{2}\right)\times\frac{1}{6}$$

$$= \frac{1}{10}\times\frac{1}{6} = \frac{1}{60}.$$

Q5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Sol. $5^m \div 5^{-3} = 5^5$

$\Rightarrow \quad 5^{m-(-3)} = 5^5 \qquad [\because\ a^m \div a^n = a^{m-n}]$

$\Rightarrow \quad 5^{m+3} = 5^5$

Comparing exponents both sides, we get

$\Rightarrow \quad m + 3 = 5$

$\Rightarrow \quad m = 5 - 3$ (Transposing 3 to R.H.S.)

$\Rightarrow \quad m = 2.$

Q6. Evaluate.

(i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ **(ii)** $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

Sol. (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1} = \{(3^{-1})^{-1} - (4^{-1})^{-1}\}^{-1}$

$$= \{3^{-1\times(-1)} - 4^{-1\times(-1)}\}^{-1} \qquad \left[\because\ \frac{1}{a^m} = a^{-m}\right]$$

$$= \{3^1 - 4^1\}^{-1} \qquad [\because\ (a^m)^n = a^{mn}]$$

$$= [-1]^{-1} = -1.$$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \qquad \left[\because\ \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$

$$= 5^{-7} \times 8^7 \times 8^{-4} \times 5^4 \qquad \left[\because\ \frac{1}{a^{-m}} = a^m\right]$$

$$= 5^{-7} \times 5^4 \times 8^{-4+7}$$

$$= 5^{-7+4} \times 8^{7-4} \qquad [\because\ a^m \times a^n = a^{m+n}]$$

$$= 5^{-3} \times 8^3 = \frac{8^3}{5^3} \qquad \left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$$= \frac{8\times8\times8}{5\times5\times5} = \frac{512}{125}.$$

Q7. Simplify.

(i) $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}$ $(t \neq 0)$ **(ii)** $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$

Sol. (i) $\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}$ $(t \neq 0)$

$$= \frac{5^2\times t^{-4}}{5^{-3}\times5\times2\times t^{-8}} = \frac{5^2\times5^3\times5^{-1}\times t^{-4}\times t^8}{2}$$

$$\left[\because\ \frac{1}{a^m} = a^{-m} \text{ and } \frac{1}{a^{-m}} = a^m\right]$$

$$= \frac{5^{2+3+(-1)}\times t^{-4+8}}{2} = \frac{5^{2+3-1}}{2}\, t^{-4+8}$$

$$= \frac{5^4}{2}\, t^4 = \frac{625}{2}\, t^4$$

(ii) $\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$

$$= \frac{3^{-5}\times(2\times5)^{-5}\times5^3}{5^{-7}\times(2\times3)^{-5}}$$

$$= \frac{3^{-5}\times2^{-5}\times5^{-5}\times5^3}{5^{-7}\times2^{-5}\times3^{-5}} \qquad [\because\ (ab)^m = a^m \times b^m]$$

$$= 3^{-5}\times2^{-5}\times5^{-5}\times5^3\times5^7\times2^5\times3^5 \qquad \left[\because\ \frac{1}{a^{-m}} = a^m\right]$$

$$= 3^{5+(-5)}\times2^{5+(-5)}\times5^{7+3+(-5)} \qquad [\because\ a^m\times a^n = a^{mn}]$$

$$= 3^{5-5}\times2^{5-5}\times5^{7+3-5} = 3^0\times2^0\times5^5 \qquad [\because\ a^0 = 1]$$

$$= 1\times1\times5^5$$

$$= 5^5.$$

EXERCISE 12.2 (*Page 200*)

Q1. Express the following numbers in standard form.

(i) 0.0000000000085 **(ii) 0.00000000000942**

(iii) 6020000000000000 **(iv) 0.00000000837**

(v) 31860000000

Sol. (*i*) Given number = 0.0000000000085

$= 8.5 \times 10^{-12}$

Since, decimal is moved 12 places to the right.

(*ii*) Given number = 0.00000000000942

$= 9.42 \times 10^{-12}$

Since, decimal is moved 12 places to the right.

(*iii*) Given number = 6020000000000000

$= 6.02 \times 10^{15}$

Since, decimal is moved 15 places to the left.

(*iv*) Given number is 0.00000000837

$= 8.37 \times 10^{-9}$

Since, decimal is moved 9 places to the right.

(*v*) Given number = 31860000000

$= 3.186 \times 10^{10}$

Since, decimal is moved 10 places to the left.

Q2. Express the following numbers in usual form.

(*i*) 3.02×10^{-6} **(*ii*) 4.5×10^{4}**

(*iii*) 3×10^{-8} **(*iv*) 1.0001×10^{9}**

(*v*) 5.8×10^{12} **(*vi*) 3.61492×10^{6}**

Sol. (*i*) $3.02 \times 10^{-6} = \dfrac{3.02}{10^6} = \dfrac{3.02}{1000000} = 0.00000302$

(*ii*) $4.5 \times 10^{4} = 4.5 \times 10000 = 45000$

(*iii*) $3 \times 10^{-8} = \dfrac{3}{10^8} = \dfrac{3}{100000000} = 0.00000003$

(*iv*) $1.0001 \times 10^{9} = 1000100000$

(*v*) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$

(*vi*) $3.61492 \times 10^{6} = 3.61492 \times 1000000 = 3614920.$

Q3. Express the number appearing in the following statements in standard form.

(*i*) 1 micron is equal to $\dfrac{1}{1000000}$ m.

(*ii*) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.

(*iii*) Size of a bacteria is 0.0000005 m.

(*iv*) Size of a plant cell is 0.00001275 m.

(*v*) Thickness of a thick paper is 0.07 mm.

Sol. (*i*) 1 micron $= \dfrac{1}{1000000} = \dfrac{1}{10^6} = 1 \times 10^{-6}$ m

(*ii*) Charge of an electron is 0.00000000000000000016 coulombs

$= 1.6 \times 10^{-19}$ coulombs

$= 1.6 \times 10^{-19}$ coulombs

(*iii*) Size of bacteria = 0.0000005 m

In the standard form $= \dfrac{5}{10000000}$ m $= \dfrac{5}{10^7}$ m

$= 5 \times 10^{-7}$ m.

(*iv*) Size of a plant cell is 0.00001275 m

$= \dfrac{1.275}{100000}$ m $= \dfrac{1.275}{10^5} = 1.275 \times 10^{-5}$ m.

(*v*) Thickness of a thick paper = 0.07 mm.

In the standard form

$= \dfrac{7}{100}$ mm $= \dfrac{7}{10^2}$ mm $= 7.0 \times 10^{-2}$ mm.

Q4. In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack.

Sol. Thickness of one book = 20 mm

Thickness of 5 books = 20 mm × 5 = 100 mm

Thickness of one paper = 0.016 mm

Thickness of 5 papers = 0.016 mm × 5 = 0.08 mm

Total thickness of a stack = 100 mm + 0.08 mm = 100.08 mm

In standard form $= \dfrac{100.08}{100} \times 100 = 1.0008 \times 10^2$ mm.

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