14 Factorisation

Learn and Remember

1. When we factorise an expression of polynomial, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.
2. An irreducible factor is a factor which can not be expressed further as product of factors.
3. Sometimes, all the terms in a given expression of polynomial do not have a common factor, but the terms can be grouped in such a way that all the terms in each group have a common factor. When we do this, there must emerge a common factor across all the groups leading to the required factorisation of the expression. This is called the method of **regrouping.**
4. A systematic way of factorising an expression is the common factor method. It consists of three steps (a) write each term of the expression as a product of irreducible factors (b) look for and separate the common factors. (c) combine the remaining factors in each term in accordance with the distributive law.
5. In factorisation by regrouping, we should remember that any regrouping of the terms in expression may not lead to factorisation.
6. We must observe expression and come out with the desired regrouping by trial and error method.
7. A number of expressions to be factorised are of the form or can be put into the form $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$ and $x^2 + (a + b)x + ab$. Such expressions can be easily factorised using identities.

$$a^2 + 2ab + b^2 = (a + b)^2 \quad ...(i)$$
$$a^2 - 2ab + b^2 = (a - b)^2 \quad ...(ii)$$
$$a^2 - b^2 = (a - b)(a + b) \quad ...(iii)$$
$$x^2 + (a + b)x + ab = (x + a)(x + b) \quad ...(iv)$$

8. In expressions, which have factors of the type $(x + a)(x + b)$, remember the numerical term ab. Its factors, a and b, should be so chosen that their sum, **with signs taken care of,** is the coefficient of x.

9. We know that in the case of numbers, division is the inverse of multiplication. This idea is also applicable to the division of algebraic expressions.
10. In the case of division of a polynomial, we may carry out the division either by dividing each term of the polynomial by the monomial or by the common factor method.
11. In case of division of a polynomial by a polynomial, we can not proceed by dividing each term in the dividend polynomial, by the divisor polynomial. Instead, we factorise both the polynomials and cancel their common factors.
12. Coefficient 1 of a term is usually not shown. But, while adding like terms, we include it in the sum.
13. When you multiply the expression enclosed in a bracket by a constant (or a variable) outside, each term of the expression has to be multiplied by the constant (or the variable).
14. In the case of divisions of algebraic expressions, we have

    Dividend = Divisor × Quotient

    In general, however, the relation is

    Dividend = Divisor × Quotient + Remainder
15. Students commonly make many errors while solving algebraic exercises. They should avoid making such errors.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 14.1 *(Page -220-221)*

Q1. Find the common factors of the given terms.

(i) $12x, 36$ (ii) $2y, 22xy$
(iii) $14pq, 28p^2q^2$ (iv) $2x, 3x^2, 4$
(v) $6abc, 24ab^2, 12a^2b$ (vi) $16x^3, -4x^2, 32x$
(vii) $10pq, 20qr, 30rp$ (viii) $3x^2y^3; 10x^3y^2, 6x^2y^2z$.

Sol. (i) $12x, 36$

$12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

Here, common factors are 2, 2 and 3.

The highest common factors $= 2 \times 2 \times 3 = 12$.

(ii) $2y, 22xy$

$2y = 2 \times y$

$22xy = 2 \times 11 \times x \times y$

Here, common factors are 2 and y.

The highest common factors $= 2 \times y = 2y$.

(iii) $14pq, 28p^2q^2$

$14pq = 2 \times 7 \times p \times q$

$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

Here, common factors are 2, 7, p and q.

The highest common factors $= 2 \times 7 \times p \times q = 14pq$.

(iv) $2x, 3x^2, 4$

$2x = 2 \times x \times 1$

$3x^2 = 3 \times x \times x \times 1$

$4 = 2 \times 2 \times 1$

Here, common factor is 1.

The highest common factor is 1.

(v) $6abc, 24ab^2, 12a^2b$

$6abc = 2 \times 3 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

Here, common factors are 2, 3, a, b.

The highest common factor is $6ab$..

(vi) $16x^3, -4x^2, 32x$

$16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$

$-4x^2 = (-1) \times 2 \times 2 \times x \times x$

$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$

Here, common factors are 2, 2 and x.

The highest common factor $2 \times 2 \times x = 4x$.

(vii) $10pq, 20qr, 30rp$

$10pq = 2 \times 5 \times p \times q$

$20qr = 2 \times 2 \times 5 \times q \times r$

$30rp = 2 \times 3 \times 5 \times r \times p$

Here, common factors are 2 and 5

The highest common factor $= 2 \times 5 = 10$.

(viii) $3x^2y^3, 10x^3y^2, 6x^2y^2z$

$3x^2y^3 = 3 \times x \times x \times y \times y \times y$

$10x^3y^2 = 2 \times 5 \times x \times x \times x \times y \times y$

$6x^2y^2z = 2 \times 3 \times x \times x \times y \times y \times z$

Here, common factors are x, x, y and y

The highest common factor $= x \times x \times y \times y = x^2y^2$.

Q2. Factorise the following expressions.

(i) $7x - 42$ **(ii)** $6p - 12q$

(iii) $7a^2 + 14a$ **(iv)** $-16z + 20z^3$

(v) $20l^2m + 30alm$ **(vi)** $5x^2y - 15xy^2$

(vii) $10a^2 - 15b^2 + 20c^2$ **(viii)** $-4a^2 + 4ab - 4ca$

(ix) $x^2yz + xy^2z + xyz^2$ **(x)** $ax^2y + bxy^2 + cxyz$

Sol. (i) $7x - 42$

$= 7 \times x - 7 \times 3 \times 2$

(We write each term in the prime factors' form.)

$= 7 \times (x - 3 \times 2) = 7(x - 6)$.

(ii) $6p - 12q$

$= 2 \times 3 \times p - 2 \times 2 \times 3 \times q$

(We write each term in the prime factors' form.)

$= 2 \times 3 \times (p - 2q)$

$= 6(p - 2q)$.

(iii) $7a^2 + 14a$

$= 7 \times a \times a + 7 \times 2 \times a$

(We write each term in the prime factors' form.)

$= 7 \times a \times (a + 2)$

$= 7a(a + 2)$.

(iv) $-16z + 20z^3$

$= -1 \times 2 \times 2 \times 2 \times 2 \times z + 2 \times 2 \times 5 \times z \times z \times z$

(We write each term in the prime factors' form.)

$= 2 \times 2 \times z(-1 \times 2 \times 2 + 5 \times z \times z)$

$= 4z(-4 + 5z^2)$.

(v) $20l^2m + 30alm$

$= 2 \times 2 \times 5 \times l \times l \times m + 2 \times 3 \times 5 \times a \times l \times m$

$= 2 \times 5 \times l \times m \times (2 \times l + 3 \times a)$

(We write each term in the prime factors' form.)

$= 10lm(2l + 3a)$.

(vi) $5x^2y - 15xy^2$

$= 5 \times x \times x \times y - 3 \times 5 \times x \times y \times y$

(We write each term in the prime factors' form.)

$= 5 \times x \times y(x - 3y) = 5xy(x - 3y)$.

(vii) $10a^2 - 15b^2 + 20c^2$

$= 2 \times 5 \times a \times a - 3 \times 5 \times b \times b + 2 \times 2 \times 5 \times c \times c$

(We write each term in the prime factors' form.)

$= 5(2 \times a \times a - 3 \times b \times b + 2 \times 2 \times c \times c)$

$= 5(2a^2 - 3b^2 + 4c^2)$.

(viii) $-4a^2 + 4ab - 4ca$

$= -1 \times 2 \times 2 \times a \times a + 2 \times 2 \times a \times b - 2 \times 2 \times c \times a$

(We write each term in prime factors' form.)

$= 2 \times 2 \times a(-1 \times a + b - c) = 4a(-a + b - c)$.

(ix) $x^2yz + xy^2z + xyz^2$

$= x \times x \times y \times z + x \times y \times y \times z + x \times y \times z \times z$

(We write each term in prime factors' form.)

$= x \times y \times z \times (x + y + z) = xyz(x + y + z)$.

(x) $ax^2y + bxy^2 + cxyz$

$= a \times x \times x \times y + b \times x \times y \times y + c \times x \times y \times z$

(We write each term into prime factors' form.)

$= x \times y \times (a \times x + b \times y + c \times z)$

$= xy(ax + by + cz)$.

Q3. Factorise.

(i) $x^2 + xy + 8x + 8y$ **(ii)** $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$ **(iv)** $15pq + 15 + 9p + 25p$

(v) $z - 7 + 7xy - xyz$

Sol. (i) $x^2 + xy + 8x + 8y$

$= x(x + y) + 8(x + y)$ (Taking in groups)

$= (x + y)(x + 8)$.

(ii) $15xy - 6x + 5y - 2$

$= 3x(5y - 2) + 1(5y - 2)$ (Taking in groups)

$= (5y - 2)(3x + 1)$.

(iii) $ax + bx - ay - by$

$= (ax + bx) - (ay + by)$ (Taking in groups)

$= x(a + b) - y(a + b)$

$= (a + b)(x - y).$

(iv) $15pq + 15 + 9q + 25p$

$= 15pq + 25p + 9q + 15$ (Taking in groups)

$= 5p(3q + 5) + 3(3q + 5)$

$= (3q + 5)(5p + 3).$

(v) $z - 7 + 7xy - xyz$

$= 7xy - 7 - xyz + z$ (Taking in graphs)

$= 7(xy - 1) - z(xy - 1)$

$= (xy - 1)(7 - z)$

$= (-1)(1 - xy)(-1)(z - 7) = +(1 - xy)(z - 7).$

EXERCISE 14.2 *(Page -223-224)*

Q1. Factorise the following expressions.

(i) $a^2 + 8a + 16$ (ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$ (iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$ (vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ **[Hint: Expand $(l + m)^2$ first]**

(viii) $a^4 + 2a^2b^2 + b^4$

Sol. (i) $a^2 + 8a + 16$

By using identity

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

Product of constant terms in the expression is 16.

$$ab = 16$$

Two factors of product $= 8 \times 2$

Where $a = 8$, $b = 2$

$a + b = 8 + 2 = 10$

These values are not valid.

So, we take another values of a and b

$$b = 4 \times 4 = 16$$

Here, $a = 4$, $b = 4$ and $a + b = 4 + 4 = 8$

These values of a and b are valid.

$$a^2 + 8a + 16 = a^2 + (4 + 4)a + 4 \times 4$$

Hence, factors of $a^2 + 8a + 16 = (a + 4)(a + 4) = (a + 4)^2$.

(ii) $p^2 - 10p + 25$

By using identity

$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

Product of constant terms is 25

$$ab = 25$$

Product of two factors $= -5 \times -5$

Here $a = -5$, $b = -5$

Sum $= -5 - 5 = -10$

$$p^2 + (-5 - 5)p + (-5)(-5)$$

Hence, factors of $p^2 - 10p + 25 = [(p + (-5))(p) + (-5)]$

$= (p - 5)(p - 5) = (p - 5)^2$

(iii) $25m^2 + 30m + 9$

$= (5m)^2 + 2 \times 5m \times 3 + (3)^2$

By using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here, $a = 5m$ and $b = 3$

$= (5m + 3)^2 = (5m + 3)(5m + 3)$

Hence, factors of $25m^2 + 30m + 9 = (5m + 3)(5m + 3)$

$= (5m + 3)^2$

(iv) $49y^2 + 84yz + 36z^2$

$= (7y)^2 + 2 \times 7y \times 6z + (6z)^2$

By using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here, $a = 7y$ and $b = 6z$

$= (7y + 6z)^2 = (7y + 6z)(7y + 6z)$

Hence, factors of $49y^2 + 84yz + 36z^2 = (7y + 6z)(7y + 6z)$

$= (7y + 6z)^2.$

(v) $4x^2 - 8x + 4$

$= (2x)^2 - 2 \times 2x \times 2 + (2)^2$

By using identity $(a - b)^2 = a^2 - 2ab + b^2$

Here, $a = 2x$ and $b = 2$

$= (2x - 2)^2 = (2x - 2)(2x - 2)$

Hence, factors of $4x^2 - 8x + 4 = (2x - 2)(2x - 2)$

$= 2(x - 1)2(x - 1) = 4(x - 1)^2.$

Alternate method

$4x^2 - 8x + 4 = 4(x^2 - 2x + 1)$

By comparing expression

$x^2 + (a + b)x + ab = (x + a)(x + b)$

$= ab = 1, a = -1, b = -1$

$= 4(x^2 + (-1 - 1)x + (-1)(-1))$

$= 4[x + (-1)][x + (-1)]$

$= 4(x - 1)(x - 1).$

(vi) $121b^2 - 88bc + 16c^2$

$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

By using identity $(a - b)^2 = a^2 - 2ab + b^2$

Here, $a = 11b$ and $b = 4c$

$= (11b - 4c)^2 = (11b - 4c)(11b - 4c)$

Hence, factors of $121b^2 - 88bc + 16c^2$

$= (11b - 4c)(11b - 4c).$

(vii) $(l + m)^2 - 4lm$

By using identity $(a + b)^2 = a^2 + 2ab + b^2$

$= (l)^2 + 2 \times l \times m + (m)^2 - 4lm$

$= l^2 + 2lm + m^2 - 4lm$

$= l^2 - 2lm + m^2 = (l)^2 - 2 \times l \times m + (m)^2$

By using identity $(a - b)^2 = a^2 - 2ab + b^2$

Here, $a = l$ and $b = m$

$(l - m)^2 = (l - m)(l - m)$

Hence, factors of $(l + m)^2 - 4lm = (l - m)(l - m) = (l - m)^2$

(viii) $a^4 + 2a^2b^2 + b^4$

$= (a^2)^2 + 2a^2 \times b^2 + (b^2)^2$

By using identity $(a + b)^2 = a^2 + 2ab + b^2$

$= (a^2 + b^2)^2 = (a^2 + b^2)(a^2 + b^2)$

Hence, factors of $a^4 + 2a^2b^2 + b^4 = (a^2 + b^2)(a^2 + b^2)$

$= (a^2 + b^2)^2.$

Q2. Factorise.

(i) $4p^2 - 9q^2$ **(ii)** $63a^2 - 112b^2$

(iii) $49x^2 - 36$ **(iv)** $16x^5 - 144x^2$

(v) $(l + m)^2 - (l - m)^2$ **(vi)** $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$ **(viii)** $25a^2 - 4b^2 + 28bc - 49c^2$

Sol. (i) $4p^2 - 9q^2$

$= (2p)^2 - (3q)^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$(2p - 3q)(2p + 3q).$

(ii) $63a^2 - 112b^2$

$= 7(9a^2 - 16b^2)$

$\because$ 7 is taken common to make all terms in perfect square form

$= 7[(3a)^2 - (4b)^2]$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= 7[(3a - 4b)(3a + 4b)].$

(iii) $49x^2 - 36$

$= (7x)^2 - (6)^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= (7x - 6)(7x + 6).$

(iv) $16x^5 - 144x^3$

$= 16x^3(x^2 - 9) = 16x^3(x^2 - 3^2)$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= 16x^3 (x - 3)(x + 3).$

(v) $(l + m)^2 - (l - m)^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= [(l + m) - (l - m)][(l + m) + (l - m)]$

$= (l + m - l + m)(l + m + l - m)$

$= (2m)(2l) = 4lm.$

(vi) $9x^2y^2 - 16$

$= (3xy)^2 - 4^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= (3xy - 4)(3xy + 4).$

(vii) $(x^2 - 2xy + y^2) - z^2$

By using identity, $a^2 - 2ab + b^2 = (a - b)^2$

$= (x - y)^2 - z^2$

Now, by using identity, $a^2 - b^2 = (a - b)(a + b)$

$= [(x - y) - z][(x - y) + z]$

$= (x - y - z)(x - y + z).$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$
$= 25a^2 - (4b^2 - 28bc + 49c^2)$
$= 25a^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2]$
$= (5a)^2 - (2b - 7c)^2$
Now, by using identity, $a^2 - b^2 = (a - b)(a + b)$
$(\because \ a^2 - 2ab + b^2 = (a^2 - b^2))$
$= [5a - (2b - 7c)][5a + (2b - 7c)]$
$= (5a - 2b + 7c)(5a + 2b - 7c).$

Q3. Factorise the expressions.

(i) $ax^2 + bx$ ***(ii)*** $7p^2 + 21q^2$
(iii) $2x^3 + 2xy^2 + 2xz^2$ ***(iv)*** $am^2 + bm^2 + bn^2 + an^2$
(v) $(lm + l) + m + 1$ ***(vi)*** $y(y + z) + 9(y + z)$
(vii) $5y^2 - 20y - 8z + 2yz$ ***(viii)*** $10ab + 4a + 5b + 2$
(ix) $6xy - 4y + 6 - 9x$

Sol. (i) $ax^2 + bx$
$= x(ax + b).$

(ii) $7p^2 + 21q^2$
$= 7(p^2 + 3q^2).$

(iii) $2x^3 + 2xy^2 + 2xz^2$
$= 2x(x^2 + y^2 + z^2).$

(iv) $am^2 + bm^2 + bn^2 + an^2$
$= m^2(a + b) + n^2(b + a)$
[$\because$ $(a + b)$ is common in both terms]
$= (a + b)(m^2 + n^2).$

(v) $(lm + l) + m + 1$
$= l(m + 1) + 1(m + 1)$
$= (m + 1)(l + 1).$

(vi) $y(y + z) + 9(y + z)$
$= (y + z)(y + 9).$

(vii) $5y^2 - 20y - 8z + 2yz$
$= 5y^2 - 20y + 2yz - 8z$
$= 5y(y - 4) + 2z(y - 4)$
$= (y - 4)(5y + 2z)$

(viii) $10ab + 4a + 5b + 2$
$= 2 \times 5ab + 2 \times 2a + 5b + 2$
$= 2a(5b + 2) + 1(5b + 2)$
[$\because$ 1 is multiple to both terms]
$= (5b + 2)(2a + 1)$

(ix) $6xy - 4y + 6 - 9x$
$= 6xy - 9x - 4y + 6$
$= 3x(2y - 3) - 2(2y - 3)$
$= (2y - 3)(3x - 2)$

Q4. Factorise.

(i) $a^4 - b^4$ ***(ii)*** $p^4 - 81$
(iii) $x^4 - (y + z)^4$ ***(iv)*** $x^4 - (x - z)^4$
(v) $a^4 - 2a^2b^2 + b^4$

Sol. (i) $a^4 - b^4$
$= (a^2)^2 - (b^2)^2$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]
$= (a^2 - b^2)(a^2 + b^2)$
$= (a - b)(a + b)(a^2 + b^2).$
[Again using the same identity]

(ii) $p^4 - 81$
$= (p^2)^2 - (9)^2$ [$\because$ $a^2 - b^2 = (a - b)(a + b)$]
$= (p^2 - 9)(p^2 + 9)$
[Using identity $a^2 - b^2 = (a - b)(a + b)$]
$= (p^2 - 3^2)(p^2 + 9)$
$= (p - 3)(p + 3)(p^2 + 9).$

(iii) $x^4 - (y + z)^4$
$= (x^2)^2 - [(y + z)^2]^2$
[Using identity $a^2 - b^2 = (a - b)(a + b)$]
$= [x^2 - (y + z)^2][x^2 + (y + z)^2]$
Now again using the same identity for first factor
$= [x - (y + z)][x + (y + z)][x^2 + (y + z)^2]$
$= (x - y - z)(x + y + z)[x^2 + (y + z)^2].$

(iv) $x^4 - (x - z)^4$
$= (x^2)^2 - [(x - z)^2]^2$
[Using identity $a^2 - b^2 = (a - b)(a + b)$]
$= [x^2 - (x - z)^2][x^2 + (x - z)^2]$
$= [x - (x - z)][x + (x - z)][x^2 + (x - z)^2]$
(Again using same identity)
$= (x - x + z)(x + x - z)[x^2 + (x - z)^2]$
$= z(2x - z)[x^2 + (x - z)^2] = z(2x - z)(x^2 + x^2 - 2xz + z^2)$
$= z(2x - z)(2x^2 - 2xz + z^2).$

(v) $a^4 - 2a^2b^2 + b^4$

$= (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= (a^2 - b^2)^2$ [Using identity $a^2 - b^2 = (a-b)(a+b)$]

$= [(a-b)(a+b)]^2$

$= (a-b)^2(a+b)^2$ $\because (xy)^m = x^m \cdot y^m$

Q5. Factorise the following expressions.

(i) $p^2 + 6p + 8$ (ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Sol. (i) $p^2 + 6p + 8$

$= p^2 + (4+2)p + 4 \times 2$ (as $8 = 4 \times 2$)

$= p^2 + 4p + 2p + 4 \times 2$

$= p(p+4) + 2(p+4)$

$= (p+4)(p+2)$.

(ii) $q^2 - 10q + 21$

$= q^2 - (7+3)q + 7 \times 3$ as $21 = 7 \times 3$

$= q^2 - 7q - 3q + 7 \times 3$

$= q(q-7) - 3(q-7)$

$= (q-7)(q-3)$.

(iii) $p^2 + 6p - 16$

$= p^2 + (8-2)p + 8 \times (-2)$ as $-16 = 8 \times -2$

$= p^2 + 8p - 2p + 8 \times (-2)$

$= p(p+8) - 2(p+8)$

$= (p+8)(p-2)$

EXERCISE 14.3 *(Page -227)*

Q1. Carry out the following divisions.

(i) $28x^4 \div 56x$ (ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$ (iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div (-6a^6b^4)$

Sol. (i) $28x^4 \div 56x$

$$= \frac{28x^4}{56x} = \frac{28}{2 \times 28} \times \frac{x^3 \times x}{x}$$

$$= \frac{1}{2}x^3.$$

(Cancelling the common factors 28 and x from Nr. and Dr.)

(ii) $-36y^3 \div 9y^2$

$$= \frac{-36y^3}{9y^2} = \frac{9 \times -4 \times y^2 \times y}{9 \times y^2} = -4y.$$

(Cancelling the common factors 9 and y^2 from Nr. and Dr.)

(iii) $66pq^2r^3 \div 11qr^2$

$$= \frac{66pq^2r^3}{11qr^2} = \frac{11 \times 6 \times p \times q \times q \times r^2 \times r}{11 \times q \times r^2} = 6qpr.$$

(Cancelling the common factors 11, q and r^2 from Nr. and Dr.)

(iv) $34x^3y^3z^3 \div 51xy^2z^3$

$$= \frac{34x^3y^3z^3}{51xy^2z^3} = \frac{34 \times x \times x^2 \times y \times y^2 \times z^3}{51 \times x \times y^2 \times z^3}$$

$$= \frac{2 \times 17 \times x \times x^2 \times y \times y^2 \times z^3}{3 \times 17 \times x \times y^2 \times z^3} = \frac{2}{3}x^2y.$$

(Cancelling the common factors 17, x^2, y^2 and z^2 from Nr. and Dr.)

(v) $12a^8b^8 \div (-6a^6b^4)$

$$= \frac{12a^8b^8}{-6a^6b^4} = \frac{12a^2 \times a^6 \times b^4 \times b^4}{-6 \times a^6 \times b^4}$$

$$= \frac{6 \times 2a^2 \times a^6 \times b^4 \times b^4}{-6 \times a^6b^4} = -2a^2 \times b^4 = -2a^2b^4.$$

(Cancelling the common factors 6, a^6 and b^4 from Nr. and Dr.)

Q2. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$

(ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$.

Sol. (*i*) $(5x^2 - 6x) \div 3x$

$$= \frac{5x^2 - 6x}{3x} = \frac{5x^2}{3x} - \frac{6x}{3x} \text{ (Dividing each term by } 3x)$$

$$= \frac{5}{3}x - 2 = \frac{1}{3}(5x - 6).$$

(*ii*) $(3y^8 - 4y^6 + 5y^4) \div y^4$

$$= \frac{3y^8 - 4y^6 + 5y^4}{y^4} = \frac{3y^8}{y^4} - \frac{4y^6}{y^4} + \frac{5y^4}{y^4}$$

(Dividing each term by y^4)

$$= 3y^4 - 4y^2 + 5.$$

(*iii*) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

$$= \frac{8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3)}{4x^2y^2z^2}$$

$$= \frac{8x^3y^2z^2}{4x^2y^2z^2} + \frac{8x^2y^3z^2}{4x^2y^2z^2} + \frac{8x^2y^2z^3}{4x^2y^2z^2}$$

(Dividing each term by $4x^2y^2z^2$)

$$= 2x + 2y + 2z = 2(x + y + z)$$

(*iv*) $(x^3 + 2x^2 + 3x) \div 2x$

$$= \frac{x^3 + 2x^2 + 3x}{2x} = \frac{x^3}{2x} + \frac{2x^2}{2x} + \frac{3x}{2x}$$

(Dividing each term by $2x$)

$$= \frac{x^2}{2} + \frac{2x}{2} + \frac{3}{2} = \frac{1}{2}(x^2 + 2x + 3)$$

(Taking out as common factor $\frac{1}{2}$)

(*v*) $(p^3q^6 - p^6q^3) \div p^3q^3$

$$= \frac{p^3q^6 - p^6q^3}{p^3q^3} = \frac{p^3q^6}{p^3q^3} - \frac{p^6q^3}{p^3q^3}$$

$$= q^3 - p^3.$$

Q3. Work out the following divisions.

(*i*) $(10x - 25) \div 5$ (*ii*) $(10x - 25) \div (2x - 5)$

(*iii*) $10y(6y + 21) \div 5(2y + 7)$ (*iv*) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(*v*) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

Sol. (*i*) $(10x - 25) \div 5$

$$= \frac{10x - 25}{5} = \frac{5(2x - 5)}{5} = (2x - 5).$$

(Cancelling the similar factors 5)

(*ii*) $(10x - 25) \div (2x - 5)$

$$= \frac{(10x - 25)}{(2x - 5)} = \frac{5(2x - 5)}{(2x - 5)}$$

(Cancelling the similar factors $(2x - 5)$)

$$= 5.$$

(*iii*) $10y(6y + 21) \div 5(2y + 7)$

$$= \frac{10y(6y + 21)}{5(2y + 7)} = \frac{2 \times 5 \times y \times 3(2y + 7)}{5(2y + 7)}$$

(Cancelling the factors 5 and $(2y + 7)$)

$$= 2 \times y \times 3 = 6y.$$

(*iv*) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

$$= \frac{9x^2y^2(3z - 24)}{27xy(z - 8)} = \frac{9}{9 \times 3} \quad \frac{xy \times xy \times 3(z - 8)}{xy \times (z - 8)}$$

(Cancelling the factors 3, 9, xy and $(z - 8)$)

$$= xy \times 1 = xy.$$

(*v*) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

$$= \frac{96abc(3a - 12)(5b - 30)}{144(a - 4)(b - 6)}$$

$$= \frac{12 \times 4 \times 2 \times abc \times 3(a - 4) \times 5(b - 6)}{12 \times 4 \times 3\,(a - 4)(b - 6)}$$

(Cancelling the common factors 12, 4, $3(a - 4)$ and $(b - 6)$)

$$= 10abc.$$

Q4. Divide as directed.

(*i*) $5(2x + 1)(3x + 5) \div (2x + 1)$

(*ii*) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

(*iii*) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

(*iv*) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$

(*v*) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

Sol. (*i*) $5(2x + 1)(3x + 5) \div (2x + 1)$

$= \frac{5(2x+1)(3x+5)}{(2x+1)} = 5(3x+5)$

(Cancelling the common factors $(2x + 1)$ from Nr. and Dr.)

(ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

$= \frac{26xy(x+5)(y-4)}{13x(y-4)} = \frac{13 \times 2 \times xy(x+5)(y-4)}{13x(y-4)}$

$= 2y(x + 5)$

(Cancelling the common factors 13, x and $(y - 4)$ from Nr. and Dr.)

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

$= \frac{52pqr(p+q)(q+r)(r+p)}{104pq(q+r)(r+p)}$

$= \frac{52pqr(p+q)(q+r)(r+p)}{52 \times 2 \times pq(q+r)(r+p)}$

(Cancelling the common factors 52, p, $q(q + r)$ and $(r + p)$ from Nr. and Dr.)

$= \frac{1}{2} r(p+q)$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$

$= \frac{20(y+4)(y^2+5y+3)}{5(y+4)} = \frac{5 \times 4(y+4)(y^2+5y+3)}{5(y+4)}$

(Cancelling the common factors 5 and $(y + 4)$ from Nr. and Dr.)

$= 4(y^2 + 5y + 3)$

(v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

$= \frac{x(x+1)(x+2)(x+3)}{x(x+1)} = (x + 2)(x + 3).$

(Cancelling the common factors x and $(x + 1)$ from Nr. and Dr.)

Q5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$

(ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Sol. (i) $(y^2 + 7y + 10) \div (y + 5)$

$= \frac{y^2+7y+10}{(y+5)} = \frac{y^2+2y+5y+2\times 5}{(y+5)}$

$= \frac{y^2+(2+5)y+2\times 5}{(y+5)} = \frac{(y+2)(y+5)}{(y+5)}$

$[\because\ x^2 + (a + b)x + ab = (x + a)(x + b)]$

$= (y + 2)$ (Cancelling the similar factors $(y + 5)$)

(ii) $(m^2 - 14m - 32) \div (m + 2)$

$= \frac{(m^2-14m-32)}{(m+2)} = \frac{m^2-16m+2m+(-16)\times 2}{m+2}$

$= \frac{m^2+(-16+2)m+(-16)(2)}{(m+2)} = \frac{(m-16)(m+2)}{(m+2)}$

$[\because\ x^2 + (a + b)x + a \times b = (x + a)(x + b)]$

$= (m - 16).$ (Cancelling the similar factors $(m + 2)$)

(iii) $(5p^2 - 25p + 20) \div (p - 1)$

$= \frac{(5p^2-25p+20)}{(p-1)} = \frac{5p^2-20p-5p+20}{p-1}$

$= \frac{5p(p-4)-5(p-4)}{(p-1)} = \frac{(5p-5)(p-4)}{(p-1)}$

$= \frac{5(p-1)(p-4)}{(p-1)}$

$= 5(p - 4)$ (Cancelling the similar factors $(p - 1)$)

(iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

$= \frac{4yz(z^2+6z-16)}{2y(z+8)} = \frac{4yz(z^2+8z-2z+8\times(-2))}{2y(z+8)}$

$= \frac{4yz[z^2+(8-2)z+8\times -2]}{2y(z+8)}$

$= \frac{2\times 2\times y\times z(z-2)(z+8)}{2y(z+8)}$

$[\because (x^2 + (a + b)x + a \times b = (x + a)(x + b)]$

$= 2z(z - 2)$ (Cancelling the factors 2, y and $(z + 8)$)

(*v*) $5pq(p^2 - q^2) \div 2p(p + q)$

$$= \frac{5pq(p^2 - q^2)}{2p(p + q)} = \frac{5pq(p - q)(p + q)}{2p(p + q)}$$

$[\because a^2 - b^2 = (a - b)(a + b)]$

$= \frac{5}{2}q(p - q)$ (Cancelling the factors p and $(p + q)$)

(*vi*) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$

$$= \frac{12xy(9x^2 - 16y^2)}{4xy(3x + 4y)} = \frac{12xy[(3x)^2 - (4y)^2]}{4xy(3x + 4y)}$$

$$= \frac{4 \times 3 \times xy(3x - 4y)(3x + 4y)}{4xy(3x + 4y)}$$

$[\because a^2 - b^2 = (a - b)(a + b)]$

$= 3(3x - 4y)$

(Cancelling the factors 4, x, y and $(3x + 4y)$)

(*vii*) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

$$= \frac{39y^3(50y^2 - 98)}{26y^2(5y + 7)} = \frac{39y^3 \times 2(25y^2 - 49)}{26y^2(5y + 7)}$$

$$= \frac{39y^3 \times 2[(5y)^2 - 7^2]}{26y^2(5y + 7)} \quad [\because a^2 - b^2 = (a - b)(a + b)]$$

$$= \frac{3 \times 13 \times y^2 \times y \times 2(5y - 7)(5y + 7)}{2 \times 13y^2 \times (5y + 7)} = 3y(5y - 7).$$

(Cancelling the factors 13, y^2, 2 and $(5y + 7)$)

EXERCISE 14.4 (*Page -228-229*)

Find and correct the errors in the following mathematical statements.

1. $4(x - 5) = 4x - 5$ **2.** $x(3x + 2) = 3x^2 + 2$

3. $2x + 3y = 5xy$ **4.** $x + 2x + 3x = 5x$

5. $5y + 2y + y - 7y = 0$ **6.** $3x + 2x = 5x^2$

7. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

8. $(2x)^2 + 5x = 4x + 5x = 9x$

9. $(3x + 2)^2 = 3x^2 + 6x + 4$

Sol. 1. $4(x - 5) = 4x - 5$

L.H.S. $= 4(x - 5)$

$= 4x - 20 \neq$ R.H.S.

Hence, the correct statement is $4(x - 5) = 4x - 20$.

2. $x(3x + 2) = 3x^2 + 2$

L.H.S. $= x(3x + 2)$

$= 3x^2 + 2x \neq$ R.H.S.

Hence, the correct statement is $x(3x + 2) = 3x^2 + 2x$.

3. $2x + 3y = 5xy$

L.H.S. $= 2x + 3y \neq$ R.H.S.

Hence, the correct statement is $2x + 3y = 2x + 3y$.

4. $x + 2x + 3x = 5x$

L.H.S. $= x + 2x + 3x = 6x \neq$ R.H.S.

Hence, is $x + 2x + 3x = 6x$.

5. $5y + 2y + y - 7y = 0$

L.H.S. $= 5y + 2y + y - 7y$

$= 8y - 7y = y \neq$ R.H.S.

Hence, the correct statement is $5y + 2y + y - 7y = y$.

6. $3x + 2x = 5x^2$

L.H.S. $= 3x + 2x = 5x \neq$ R.H.S.

Hence, the correct statement is $3x + 2x = 5x$.

7. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

L.H.S. $= (2x)^2 + 4(2x) + 7$

$= 4x^2 + 8x + 7 \neq$ R.H.S.

Hence, the correct statement is $(2x)^2 + 4(2x) + 7$

$= 4x^2 + 8x + 7$.

8. $(2x)^2 + 5x = 4x + 5x = 9x$

L.H.S. $= (2x)^2 + 5x = 4x^2 + 5x \neq$ R.H.S.

Hence, the correct statement is $(2x)^2 + 5x = 4x^2 + 5x$.

9. $(3x + 2)^2 = 3x^2 + 6x + 4$

L.H.S. $= (3x + 2)^2 = 9x^2 + 12x + 4 \neq$ R.H.S.

Hence, the correct statement is

$(3x)^2 + 2 \times 3x \times 2 + (2)^2$

$= (3x + 2)^2 = 9x^2 + 12x + 4.$

Q10. Substituting $x = -3$ in

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

Sol. (a) L.H.S. $= x^2 + 5x + 4$

Putting $x = -3$

$= (-3)^2 + 5(-3) + 4$

$= 9 - 15 + 4 = -2 \neq$ R.H.S.

Hence, the correct statement is $x^2 + 5x + 4$ gives

$(-3)^2 + 5(-3) + 4$

$= 9 - 15 + 4 = -2$

(b) L.H.S. $= x^2 - 5x + 4$

Putting $x = -3$

$= (-3)^2 - 5(-3) + 4$

$= 9 + 15 + 4 = 28 \neq$ R.H.S.

Hence, correct statement is $x^2 - 5x + 4$ gives

$(-3)^2 - 5(-3) + 4$

$= 9 + 15 + 4 = 28$

(c) L.H.S. $= x^2 + 5x$

Putting $x = -3$

$= (-3)^2 + 5(-3) = 9 - 15 = -6 \neq$ R.H.S.

Hence, correct statement is $x^2 + 5x = (-3)^2 + 5(-3)$

$= 9 - 15 = -6.$

Q11. $(y - 3)^2 = y^2 - 9$ **Q12.** $(z + 5)^2 = z^2 + 25$

Q13. $(2a + 3b)(a - b) = 2a^2 - 3b^2$ **Q14.** $(a + b)(a + 2) = a^2 + 8$

Q15. $(a - 4)(a - 2) = a^2 - 8$ **Q16.** $\frac{3x^2}{3x^2} = 0$

Q17. $\frac{3x^2 + 1}{3x^2} = 1 + 1 = 2$ **Q18.** $\frac{3x}{3x + 2} = \frac{1}{2}$

Q19. $\frac{3}{4x + 3} = \frac{1}{4x}$ **Q20.** $\frac{4x + 5}{4x} = 5$

Q21. $\frac{7x + 5}{5} = 7x.$

Sol. 11. $(y - 3)^2 = y^2 - 9$

L.H.S. $= (y - 3)^2$

$= (y)^2 - 2 \times y \times 3 + (3)^2$ $[\because (a - b)^2 = a^2 - 2ab + b^2]$

$= y^2 - 6y + 9 \neq$ R.H.S.

Hence, the correct statement is $(y - 3)^2 = y^2 - 6y + 9.$

12. $(z + 5)^2 = z^2 + 25$

L.H.S. $= (z + 5)^2 = (z)^2 + 2 \times z \times 5 + (5)^2$

$[\because (a + b)^2 = a^2 + 2ab + b^2]$

$= z^2 + 10z + 25 \neq$ R.H.S.

Hence, the correct statement is $(z + 5)^2 = z^2 + 10z + 25.$

13. $(2a + 3b)(a - b) = 2a^2 - 3b^2$

L.H.S. $= (2a + 3b)(a - b)$

$= 2a^2 - 2ab + 3ab - 3b^2$

$= 2a^2 + ab - 3b^2 \neq$ R.H.S.

Hence, the correct statement is $(2a + 3b)(a - b)$

$= 2a^2 + ab - 3b^2.$

14. $(a + b)(a + 2) = a^2 + 8$

L.H.S. $= (a + 4)(a + 2)$

$= a^2 + 2a + 4a + 8$

$= a^2 + 6a + 8 \neq$ R.H.S.

Hence, the correct statement is $(a + 4)(a + 2) = a^2 + 6a + 8.$

15. $(a - 4)(a - 2) = a^2 - 8$

L.H.S. $= (a - 4)(a - 2)$

$= a^2 - 2a - 4a + 8$

$= a^2 - 6a + 8 \neq$ R.H.S.

Hence, the correct statement is $(a - 4)(a - 2) = a^2 - 6a + 8.$

16. $\frac{3x^2}{3x^2} = 0$

L.H.S. $= \frac{3x^2}{3x^2} = \frac{1}{1} = 1 \neq$ R.H.S.

(Cancelling the factors 3 and x^2)

Hence, the correct statement is $\frac{3x^2}{3x^2} = 1$.

17. $\frac{3x^2+1}{3x^2} = 1 + 1 = 2$

L.H.S. $= \frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2}$

$= 1 + \frac{1}{3x^2} \neq$ R.H.S. (Cancelling the factors 3 and x^2)

Hence, the correct statement is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$.

18. $\frac{3x}{3x+2} = \frac{1}{2}$

L.H.S. $= \frac{3x}{3x+2} \neq$ R.H.S.

Hence, the correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$.

19. $\frac{3}{4x+3} = \frac{1}{4x}$

L.H.S. $= \frac{3}{4x+3} \neq$ R.H.S.

Hence, the correct statement is $\frac{3}{4x+3} = \frac{3}{4x+3}$.

20. $\frac{4x+5}{4x} = 5$

L.H.S. $= \frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x}$

$= 1 + \frac{5}{4x} \neq$ R.H.S. (Cancelling the factors 4 and x.)

Hence, the correct statement is $\frac{4x+5}{4x} = 1 + \frac{5}{4x}$.

21. $\frac{7x+5}{5} = 7x$

L.H.S. $= \frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1 \neq$ R.H.S.

(Cancelling the factors 5.)

Hence, the correct statement is $\frac{7x+5}{5} = \frac{7x}{5} + 1$.

□□