16 Playing with Numbers

Learn and Remember

1. Numbers can be written in general form. Thus, a two digit number ab will be written as

   $$ab = 10a + b$$

   And three digit number abc can be written as $abc = 100a + 10b + c$.
2. The general forms of numbers are helpful in solving puzzles or number games.
3. The reasons for the divisibility of numbers by 10, 5, 2, 9 or 3 can be given when numbers are written in general form.
4. All these numbers 10, 2, 4, 6, 8 whose place digits or ones place digits are 0, 2, 4, 6, 8 is divisible by 2. These numbers are also called **even numbers.**
5. If the sum of digits of a two digit number and its reversing order of digits is always divisible by 11.
6. Difference of a two digit number and the number obtained by its reversing the order of digits is always divisible by 9.
7. Difference of a three digit number and the number obtained by its reversing the order of digits is always divisible by 99.
8. If the sum of the digits of a number and the number obtained by reversing the order of digits in cyclic form is always divisible by 37.
9. If the difference of the sum of digits in odd place and the sum of digits in even places from L.H.S. is obtained as 0, 11, 22, ... then the number is divisible by 11.
10. If the sum of digits of a number is divisible by 9, then that number is divisible by 9.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 16.1 *(Page -255-256)*

Find the values of the letters in each of the following and give reasons for the steps involved.

1.
$$\begin{array}{r} 3\ A \\ +\ 2\ 5 \\ \hline B\ 2 \end{array}$$

Sol. On putting A = 1, 2, 3, 4, 5, 6 that is

$1 + 5 = 6 < 12$

$2 + 5 = 7 < 12$, $3 + 5 = 8 < 12$

and so on $7 + 5 = 12$ which is equal to 12.

then, A = 7

Put 2 and carry over 1

$1 + 3 + 2 = 6 = B$

Hence, A = 7 and B = 6.

2.
$$\begin{array}{r} 4\ A \\ +\ 9\ 8 \\ \hline C\ B\ 3 \end{array}$$

Sol. On putting A = 5, we get 8 + 5 = 13.

Put 3 at ones place and carry over 1.

Now, 1 + 4 + 9 = 14, that means B = 4 and C = 1

Hence, A = 5, B = 4 and C = 1.

3.
$$\begin{array}{r} 1\ A \\ \times\ \ A \\ \hline 9\ A \\ \hline \end{array}$$

Sol. Here value of A on these three places would be same.

If A = 1, we are not getting nine at tens place.

So, on putting A = 6.

We are getting 96.

Hence, A = 6.

4.
$$\begin{array}{r} A\ B \\ +\ 3\ 7 \\ \hline 6\ A \\ \hline \end{array}$$

Sol. Here, we observe that B = 5 so that 7 + 5 = 12. Put 2 at ones place and carry over 1 or A = 2.

Now, 2 + 3 + 1 = 6.

Hence, A = 2 and B = 5.

5.
$$\begin{array}{ccc} & A & B \\ & \times & 3 \\ \hline C & A & B \\ \hline \end{array}$$

Sol. Here, on putting B = 0, we get 0 × 3 = 0.

A = 5, then 5 × 3 = 15, means A = 5, C = 1

Hence A = 5, B = 0 and C = 1

6.
$$\begin{array}{ccc} & A & B \\ & \times & 5 \\ \hline C & A & B \\ \hline \end{array}$$

Sol. On putting B = 0, we get 0 × 5 = 0 and A = 5, then 5 × 5 = 25 means A = 5, C = 2.

Hence, A = 5, B = 0 and C = 2.

7.
$$\begin{array}{ccc} & A & B \\ & \times & 6 \\ \hline B & B & B \\ \hline \end{array}$$

Sol. Here, product of B and 6 must be same as ones place digit as B.

6 × 1 = 6, 6 × 2 = 12, 6 × 3 = 18, 6 × 4 = 24.

On putting B = 4 we get the ones digit 4 and remaining two B's value must be 44.

For 6 × 7 = 42 + 2 = 44.

Hence, A = 7 and B = 4.

8.
$$\begin{array}{ccc} & A & 1 \\ + & 1 & B \\ \hline & B & 0 \\ \hline \end{array}$$

Sol. On putting B = 9, 9 + 1 = 10.

Put 0 at ones place and carry over 1.

For A = 7. Now, A = 7 + 1 + 1 = 9.

Hence, A = 7 and B = 9.

9.
$$\begin{array}{cccc} & 2 & A & B \\ + & A & B & 1 \\ \hline & B & 1 & 8 \\ \hline \end{array}$$

Sol. On putting B = 7, that means 7 + 1 = 8 and

Now A = 4, then A = 4 + 7 = 11.

Put 1 at tens place and carry over 1.

Now, 2 + 4 + 1 = 7.

Hence, A = 4 and B = 7.

10.
$$\begin{array}{cccc} & 1 & 2 & A \\ + & 6 & A & B \\ \hline & A & 0 & 9 \\ \hline \end{array}$$

Sol. Here, A = 8 and B = 1.

Now, 8 + 1 = 9

Now again when we add 2 + 8 = 10.

Tens place digit is '0' and carry over 1.

Now, 1 + 6 + 1 = 8 = A

Hence, A = 8 and B = 1.

EXERCISE 16.2 *(Page –260)*

Q1. If 21*y*5 is a multiple of 9, where *y* is a digit, what is the value of *y*?

Sol. Since, 21*y*5 is a multiple of 9.

Its sum of digits 2 + 1 + *y* + 5 = 8 + *y* is a multiple of 9. So, 8 + *y* is either 0 or 9 or 18 or 27 ....

This is possible when 8 + *y* = 9 or 18 or 27 ....

If $8 + y = 9$

$\Rightarrow y = 9 - 8 = 1$

Q2. If 31*z*5 is a multiple of 9, where *z* is a digit, what is the value of *z*? You will find that there are *two* answers from the last problem. Why is this so?

Sol. Since, $31z5$ is a multiple of 9.

Sum of digits $3 + 1 + z + 5 = 9 + z$ is a multiple of 9. So, $9 + z$ is either 0 or 9 or 18 or 27 .... since z is a digit.

This is possible when $9 + z = 9$ or 18 or 27, ...

$\Rightarrow \quad z = 9 - 9$

$\Rightarrow \quad z = 0$

If $\quad 9 + z = 18$

$z = 18 - 9 = 9$

Hence, 0 and 9 are two possible answers.

Q3. If $24x$ is a multiple of 3, where x is a digit, what is the value of x?

Sol. (Since $24x$ is a multiple of 3, its sum of digits $6 + x$ is a multiple of 3; so $6 + x$ is one of these numbers: 0, 3, 6, 9, 12, 15, 18, ... But since, x is a digit. Therefore, it can only be that $6 + x = 6$ or 9 or 12 or 15, therefore $x = 0$ or 3 or 6 or 9 or 12 or 15. Therefore, $x = 0$ or 3 or 6 or 9. Thus, x can have any of four different values.

Q4. If $31z5$ is a multiple of 3, where z is a digit, what might be the values of z?

Sol. It is given that $31z5$ is a multiple of 3.

Sum of its digits is $3 + 1 + z + 5 = 9 + z$ is multiple of 3, so $9 + z$ is one of these numbers 9, 12, 15, so on.

This is possible when $9 + z = 9, 12, 15, 18,$ ......

If $\quad 9 + z = 9 \quad$ or If $\quad 9 + z = 12$

$\Rightarrow \quad z = 9 - 9 \quad \Rightarrow \quad z = 12 - 9$

$\Rightarrow \quad z = 0 \quad \Rightarrow \quad z = 3$

Possible values of z are 0 or 3.

If $\quad 9 + z = 15 \quad$ or If $\quad 9 + z = 18$

$\Rightarrow \quad z = 15 - 9 \quad \Rightarrow \quad z = 18 - 9$

$\Rightarrow \quad z = 6 \quad \Rightarrow \quad z = 9$

Hence, four possible values of z are 0, 3, 6 and 9.

□□