6 Squares and Square Roots

Learn and Remember

1. If a natural number is m. It can be expressed as n^2, where n is also a natural number, then m is a square number *i.e.*, $25 = 5^2$, where $m = 25$ and $n^2 = 5^2$.
2. Squares of all natural or integer ends with 0, 1, 4, 5, 6 and 9 at unit's place.
3. Squares of even numbers are even numbers ending with 0, 4 and 6 at unit's place.
4. Square numbers can only have even number of zeros at the end.
5. Square root is the inverse operation of square.
6. Square of odd numbers can have odd numbers ending with 1, 5 and 9 at unit's place.
7. If sum of digits of a squared number is even, then it is square of having unit's digit 3 like 13^2, 23^2, 33^2, 43^2, ... and if sum of digits of a squared number is odd, then it is square of having unit's digit 7 like 7^2, 17^2, 27^2, 37^2, 47^2, ... leaving numbers 27^2, 57^2 and 97^2.
8. There are two integral square roots of a perfect square number. Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

   For example, $3^2 = 9$ gives $\sqrt{9} = 3$ and $(-3)^2 = 9$ gives $\sqrt{9} = -3$ square and square roots. So, there are two values of a square root number.
9. The square root of a number can be got by prime factorisation method, division method and by repeated subtraction of odd numbers starting from 1.
10. For knowing Pythagoras triplets, there are three numbers $2m$, $(m-1)$ and $(m+1)$, where m is a natural number.
11. If p is a prime factor of n, then $p \times p$ is a factor of n^2.

12. If p and q are perfect squres $(q \neq 0)$, then $\sqrt{p \times q} = \sqrt{p} \times \sqrt{q}$ and $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$.

13. If n is not a perfect square, then $\sqrt{n}$ is not a rational number.

Consider the following numbers, and their squares are given below.

Number	Square
1	$1 \times 1 = 1$
2	$2 \times 2 = 4$
3	$3 \times 3 = 9$
4	$4 \times 4 = 16$
5	$5 \times 5 = 25$
6	$6 \times 6 = 36$
7	$7 \times 7 = 49$
8	$8 \times 8 = 64$
9	$9 \times 9 = 81$
10	$10 \times 10 = 100$
11	$11 \times 11 = 121$
12	$12 \times 12 = 144$

Now we have 1, 4, 9, 16, ... are known as square numbers of natural numbers. These numbers (1, 4, 9, 16, 25, ...) are called perfect square numbers.

We have perfect squares of even natural numbers *i.e.,* 4, 16, 36, 64, 100 etc. and units place of these numbers contain 4, 6,0 and perfect squares of odd numbers are 1, 9, 25, 49, 81, ... And their unit's digit is 1, 5, 9. Therefore, there are only six digits (0, 1, 4, 5, 6, 9) which occupy their positions at unit place in perfect square numbers.

TEXTBOOK QUESTIONS SOLVED

EXERCISE 6.1 *(Page–96)*

Q1. What will be the unit digit of the squares of the following numbers?

(i) 81 (ii) 272 (iii) 799
(iv) 3853 (v) 1234 (vi) 26387

(vii) 52698 (viii) 99880 (ix) 12796
(x) 55555

Sol. (i) The number 81 contains its unit's place digit 1. So, square of 1 is 1.

Hence, unit's digit of square of 81 is 1.

(ii) Digit of unit's place of given number 272 is 2. So, square of 2 is 4. Therefore, unit's digit of the square of the given number is 4.

(iii) It contains 9 as unit's place digit. Square of 9 is 81. Unit's digit of the square of the given number 799 is 1.

(iv) This number 3853 contains its unit digit 3. So, square of 3 is 9. Therefore, unit's digit of the square of the given number 3853 is 9.

(v) You have seen here, one's place digit of the number 1234 is 4. Square of 4 is 16.

Therefore, unit's digit of the square of the given number is 6.

(vi) The given number contains unit's digit 7. Square of 7 is 49. So, unit's digit of the square of the given number 26387 is 9.

(vii) Unit's place digit of given number 52698 is 8 and square of 8 is 64. Therefore, this contains 4 as its unit digit.

(viii) As the given number 99880, has unit's place digit 0. Square of zero is zero. So, unit digit of the square of the given number is zero.

(ix) One's place digit of the given number 12796 is 6. Square of 6 is 36. Therefore, unit's digit of the square of the given number is 6.

(x) One's place digit is 5

Square of 5 is 25

Unit's place digit of the square of the given number 55555 is 5

Q2. The following numbers are obviously not perfect squares. Give reason.

(i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
(v) 64000 (vi) 89722 (vii) 222000 (viii) 505050.

Sol. (i) 1057 is not a perfect square because its unit's place digit is 7. Since, perfect square numbers contain their unit's place digit 0, 1, 4, 5, 6 and 9.

(*ii*) The given number 23453 has its unit's digit 3. So, it is not a perfect square number because perfect square numbers have unit's digits 0, 1, 4, 5, 6 and 9.

Note. If sum of their digits is even, then it is square of having unit's digit 3, *e.g.*, 13^2, 23^2, 33^2, 43^2, ... and if sum of their digits is odd then it is square of having unit digit 7 like 7^2, 17^2, 27^2, 37^2 leaving number 27^2, 57^2 and 97^2.

(*iii*) Since, perfect square numbers contain their unit digits 0, 1, 4, 5, 6 and 9. But, unit's digit of the given number is 8. So, it is not a perfect square number.

(*iv*) We observe the square of natural numbers that perfect square, numbers contain at their unit's digits 0, 1, 4, 5, 6 and 9. But, unit's digit of the given number is 2. So, it is not a perfect square number.

(*v*) Since, perfect square numbers contain zeros in pair or pairs, 64000 does not have zeros in pairs. So, it is not a perfect square number.

(*vi*) Since, all natural numbers containing their unit's digits 0, 1, 4, 5, 6 and 9, are perfect square numbers. But, unit's digit of the given number is 2. So, it is not a perfect square number.

(*vii*) Since this number is also have three zeros which are not in pairs. So, it is not a perfect square number.

(*viii*) Since this number contains one zero at unit's place which is not in pair. So, it is not a perfect square number.

Q3. The squares of which of the following would be odd number?

(*i*) 431 (*ii*) 2826 (*iii*) 7779 (*iv*) 82004

Sol. (*i*) Unit's digit of given number is 1. Since, square of 1 is 1, unit's digit of square of odd numbers contains 1, 5, 9. So, square of 431 would be an odd number.

(*ii*) Unit's digit of given number is 6. Square of 6 is 36. Therefore, unit's digit of the square of the given number is 6. So, the square of this number will be an even number or would not be an odd number.

(*iii*) Unit's place of given number is 9. Square of 9 is 81. Therefore, unit's digit of square of the given number is 1. So, square of the given number is an odd number.

(*iv*) Given number has its unit's digit 4 and square of 4 is 16.

Unit's digit of the square of the given number is 6. Therefore, it is an even number.

Hence, square of (*i*) 431 and (*iii*) 7779 would be an odd number.

Q4. Observe the following pattern and find the missing digits.

$$11^2 = 121$$
$$101^2 = 10201$$
$$1001^2 = 1002001$$
$$100001^2 = 1.....2.....1$$
$$10000001^2 = 1................$$

Sol.
$$11^2 = 121$$
$$101^2 = 10201$$
$$1001^2 = 1002001$$
$$100001^2 = 10000200001$$
$$10000001^2 = 100000020000001$$

Q5. Observe the following pattern and supply the missing numbers.

$$11^2 = 121$$
$$101^2 = 10201$$
$$10101^2 = 102030201$$
$$1010101^2 = ...$$
$$............^2 = 10203040504030201$$

Sol.
$$11^2 = 121$$
$$101^2 = 10201$$
$$10101^2 = 102030201$$
$$1010101^2 = 1020304030201$$
$$101010101^2 = 10203040504030201$$

Q6. Using the given pattern, find the missing numbers.

$$1^2 + 2^2 + 2^2 = 3^2$$
$$2^2 + 3^2 + 6^2 = 7^2$$
$$3^2 + 4^2 + 12^2 = 13^2$$
$$4^2 + 5^2 + \_^2 = 21^2$$

$$5^2 + \_^2 + 30^2 = 31^2$$
$$6^2 + 7^2 + \_^2 = \_^2$$

Sol.
$$1^2 + 2^2 + 2^2 = 3^2$$
$$2^2 + 3^2 + 6^2 = 7^2$$
$$3^2 + 4^2 + 12^2 = 13^2$$
$$4^2 + 5^2 + 20^2 = 21^2$$
$$5^2 + 6^2 + 30^2 = 31^2$$
$$6^2 + 7^2 + 42^2 = 43^2$$

Q7. Without adding, find the sum.

(i) 1 + 3 + 5 + 7 + 9

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19

(iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

Sol. (i) $1 + 3 + 5 + 7 + 9 = 5^2 = 25$

Here, there are five odd numbers. So, square of 5 is 25. Hence, sum of these numbers is 25.

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2 = 100$

Here, there are ten odd numbers. So, square of 10 is 100.

Sum of these numbers is 100.

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 = 12^2 = 144$.

Here, there are twelve odd numbers. So, square of 12 is 144.

Hence, sum of all these numbers is 144.

Q8. (i) Express 49 as the sum of 7 odd numbers.

(ii) Express 121 as the sum of 11 odd numbers.

Sol. (i) The given number is 49. So, $49 = 1+3+5+7+9+11+13$.

(ii) The given number is 121. Since, it is sum of 11 odd numbers.

So, $121 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$.

Q9. How many numbers lie between squares of the following numbers?

(i) 12 and 13 **(ii) 25 and 26** **(iii) 99 and 100**

Sol. (i) 12 and 13, we know that non perfect square numbers between n^2 and $(n+1)^2$ are $2n$.

Here, $n = 12$ and $n + 1 = 13$.

Therefore, non perfect square numbers between n^2 and $(n+1)^2$ are $2n = 2 \times 12 = 24$.

(ii) 25 and 26, we know that non perfect square numbers between n^2 and $(n+1)^2$ are $2n$.

Here, $n = 25$ and $n + 1 = 26$.

Therefore, non perfect square numbers between n^2 and $(n+1)^2$ are $2n = 2 \times 25 = 50$.

(iii) 99 and 100, we know that non perfect square numbers between n^2 and $(n+1)^2$ are $2n$.

Here, $n = 99$ and $n + 1 = 100$.

Therefore, non perfect square numbers between n^2 and $(n+1)^2$ are $2n = 2 \times 99 = 198$.

EXERCISE 6.2 (*Page –98*)

Q1. Find the square of the following numbers.

(i) 32	**(ii) 35**	**(iii) 86**
(iv) 93	**(v) 71**	**(vi) 46**

Sol. (i) $32^2 = (30 + 2)^2 = (30 + 2)(30 + 2) = 30(30 + 2) + 2(30 + 2)$
$= 30^2 + 30 \times 2 + 2 \times 30 + 2^2$
$= 900 + 60 + 60 + 4 = 1024$.

(ii) $35^2 = (30 + 5)^2 = (30 + 5)(30 + 5) = 30(30 + 5) + 5(30 + 5)$
$= 30^2 + 30 \times 5 + 5 \times 30 + 5^2$
$= 900 + 150 + 150 + 25 = 1225$.

(iii) $86^2 = (80 + 6)^2 = (80 + 6)(80 + 6) = 80(80 + 6) + 6(80 + 6)$
$= 80^2 + 80 \times 6 + 6 \times 80 + 6^2$
$= 6400 + 480 + 480 + 36 = 7386$.

(iv) $93^2 = (90 + 3)^2 = (90 + 3)(90 + 3) = 90(90 + 3) + 3(90 + 3)$
$= 90^2 + 90 \times 3 + 3 \times 90 + 3^2$
$= 8100 + 270 + 270 + 9 = 8649$

(v) $71^2 = (70 + 1)^2 = (70 + 1)(70 + 1) = 70(70 + 1) + 1(70 + 1)$
$= 70^2 + 70 \times 1 + 1 \times 70 + 1^2$
$= 4900 + 70 + 70 + 1 = 5041$.

(vi) $46^2 = (40 + 6)^2 = (40 + 6)(40 + 6) = 40(40 + 6) + 6(40 + 6)$
$= 40^2 + 40 \times 6 + 6 \times 40 + 6^2$
$= 1600 + 240 + 240 + 36 = 2116$.

Q2. Write a Pythagorean triplet whose one member is.

(i) 6	**(ii) 14**
(iii) 16	**(iv) 18**

Sol. (*i*) In a Pythagorean triplet, there are three numbers which are $2m$, $m^2 - 1$ and $m^2 + 1$
and first number, $2m = 6$

$$\Rightarrow \quad m = \frac{6}{2} = 3$$

Second number $m^2 - 1 = 3^2 - 1 = 9 - 1 = 8$
Third number $m^2 + 1 = 3^2 + 1 = 9 + 1 = 10$
Hence, Pythagorean triplet is (6, 8, 10).

(*ii*) Pythagorean triplet contains $2m$, $m^2 - 1$ and $m^2 + 1$
Here, first number, $2m = 14$

$$\Rightarrow \quad m = \frac{14}{2} = 7$$

Second number $= m^2 - 1 = 7^2 - 1 = 49 - 1 = 48$
Third number $= m^2 + 1 = 7^2 + 1 = 49 + 1 = 50$
Hence, Pythagorean triplet is (14, 48, 50)

(*iii*) Pythagorean triplet contains $2m$, $m^2 - 1$ and $m^2 + 1$
First number, $2m = 16$

$$\Rightarrow \quad m = \frac{16}{2} = 8$$

Second number, $m^2 - 1 = 8^2 - 1 = 64 - 1 = 63$
Third number $= m^2 + 1 = 8^2 + 1 = 64 + 1 = 65$
Hence, Pythagorean triplet is (16, 63, 65).

(*iv*) Pythagorean triplets contains $2m$, $m^2 - 1$ and $m^2 + 1$
First number $2m = 18$

$$\Rightarrow \quad m = \frac{18}{2} = 9$$

Second number $= m^2 - 1 = 9^2 - 1 = 81 - 1 = 80$
Third number $= m^2 + 1 = 9^2 + 1 = 81 + 1 = 82$
Hence, Pythagorean triplet is (18, 80, 82).

EXERCISE 6.3 (*Page –102-103*)

Q1. What could be the possible 'one's' digits of the square root of each of the following numbers?

(*i*) 9801 **(*ii*) 99856**
(*iii*) 998001 **(*iv*) 657666025**

Sol. (*i*) As we do square of any natural numbers, we get unit's digit of square of numbers 0, 1, 4, 5, 6 and 9. As we also know that square root is the inverse operation of the square of the numbers.
So, unit's digit of the following numbers are (*i*) 1 (*ii*) 6 (*iii*) 1 (*iv*) 5.

Q2. Without doing any calculation, find the numbers which are surely not perfect squares.

(*i*) 153 **(*ii*) 257** **(*iii*) 408** **(*iv*) 441**

Sol. As we know that all perfect square numbers contain their unit's place digits 0, 1, 4, 5, 6, 9.

(*i*) But given number 153 has its unit digit 3. So, it is not a perfect square number.
(*ii*) Here, 257 contains its unit's digit 7 so, it is also not a perfect square number.
(*iii*) Since, given number has its unit digit 8. It is not a perfect square.
(*iv*) Given number 441 contains 1 as unit's digit. So, it is a perfect square number.

Q3. Find the square roots of 100 and 169 by the method of repeated subtraction.

Sol. By successively subtracting odd natural numbers from 100

$100 - 1 = 99$	$75 - 11 = 64$	$99 - 3 = 96$
$64 - 13 = 51$	$96 - 5 = 91$	$51 - 15 = 36$
$91 - 7 = 84$	$36 - 17 = 19$	$84 - 9 = 75$
$19 - 19 = 0$		

We subtracted successive odd natural numbers starting from 1 to 19 and obtained 0 at 10th step.

Therefore, $\sqrt{100} = 10$.
Hence, the square root of 100 is 10.
By successively subtracting odd natural numbers from 169

$169 - 1 = 168$	$120 - 15 = 105$	$168 - 3 = 165$
$105 - 17 = 88$	$165 - 5 = 160$	$88 - 19 = 69$
$160 - 7 = 153$	$69 - 21 = 48$	$153 - 9 = 144$
$48 - 23 = 25$	$144 - 11 = 133$	$25 - 25 = 0$
$133 - 13 = 120$		

We subtracted successive odd natural numbers starting from 1 to 25 and obtained 0 (zero) at 13th step.

Therefore, $\sqrt{169} = 13$.
Hence, the square root of 169 is 13.

Q4. Find the square roots of the following numbers by the Prime Factorisation method.

(i) 729 (ii) 400 (iii) 1764 (iv) 4096
(v) 7744 (vi) 9604 (vii) 5929 (viii) 9216
(ix) 529 (x) 8100

Sol. (i) 729

3	729
3	243
3	81
3	27
3	9
3	3
	1

Prime factors of 729 = $3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\sqrt{729} = 3 \times 3 \times 3$

$\Rightarrow \quad \sqrt{729} = 27$. **Ans.**

(ii) 400

2	400
2	200
2	100
2	50
5	25
5	5
	1

Prime factors of 400 = $2 \times 2 \times 2 \times 2 \times 5 \times 5$

$\Rightarrow \quad \sqrt{400} = 2 \times 2 \times 5 = 20$. **Ans.**

(iii) 1764

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Prime factors of 1764 = $2 \times 2 \times 3 \times 3 \times 7 \times 7$

$\Rightarrow \quad \sqrt{1764} = 2 \times 3 \times 7 = 42$. **Ans.**

(iv) 4096

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of 4096 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow \quad \sqrt{4096} = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 64$. **Ans.**

(v) 7744

2	7744
2	3872
2	1936
2	968
2	484
2	242
11	121
11	11
	1

Prime factors of 7744 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11 \times 11$

$\Rightarrow \quad \sqrt{7744} = 2 \times 2 \times 2 \times 11$

$\sqrt{7744} = 88$. **Ans.**

(vi) 9604

2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

Prime factors of 9604 = $2 \times 2 \times 7 \times 7 \times 7 \times 7$

$\Rightarrow \quad \sqrt{9604} = 2 \times 7 \times 7$

$\Rightarrow \quad \sqrt{9604} = 98$. **Ans.**

(*vii*) 5929

7	5929
7	847
11	121
11	11
	1

Prime factors of 5929 = $7 \times 7 \times 11 \times 11$

$\Rightarrow \quad \sqrt{5929} = 7 \times 11 = 77$

$\Rightarrow \quad \sqrt{5929} = 77$. **Ans.**

(*viii*) 9216

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Prime factors of 9126 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow \quad \sqrt{9216} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$

$\Rightarrow \quad \sqrt{9216} = 96$. **Ans.**

(*ix*) 529

23	529
23	23
	1

Prime factors of 529 = 23×23

$\sqrt{529} = 23$

$\Rightarrow \quad \sqrt{529} = 23$. **Ans.**

(*x*) 8100

2	8100
2	4050
3	2025
3	675
3	225
3	75
5	25
5	5
	1

Prime factors of 8100 = $2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$

$\Rightarrow \quad \sqrt{8100} = 2 \times 3 \times 3 \times 5 = 90$

$\Rightarrow \quad \sqrt{8100} = 90$. **Ans.**

Q5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.

(*i*) 252 (*ii*) 180 (*iii*) 1008
(*iv*) 2028 (*v*) 1458 (*vi*) 768

Sol. (*i*) 252

2	252
2	126
3	63
3	21
7	7
	1

Prime factors of 252 = $2 \times 2 \times 3 \times 3 \times 7$

Since, prime factor 7 has no pair. So, we must multiply the number by 7 to make it a perfect square number.

$252 = 2 \times 2 \times 3 \times 3 \times 7 \times 7$

Now, each prime factor has a pair.

Therefore, $252 \times 7 = 1764$ is a perfect square number.

The smallest multiple number is 7.

Thus, $\sqrt{1764} = 2 \times 3 \times 7 = 42$.

(*ii*) 180

2	180
2	90
3	45
3	15
5	5
	1

Prime factors of $180 = 2 \times 2 \times 3 \times 3 \times 5$

Prime factor 5 does not occur in pair. So, the number is not a perfect square. If 5 gets a pair then number will be perfect square, so we multiply the number by 5 to get a perfect square number,

$$180 \times 5 = 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

Now, each prime factor has a pair. Therefore, $180 \times 5 = 900$ is a perfect square number.

Thus, required the smallest number is 5

$$\sqrt{900} = \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5}$$

$$\sqrt{900} = 2 \times 3 \times 5 = 30.$$

(*iii*) 1008

2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

Prime factors of $1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$.

Prime factor 7 does not occur in a pair, so the given number is not a perfect square number. If 7 gets a pair, then the number will be a perfect square.

The smallest number is 7 and we multiply,

$$1008 \times 7 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$$

The square root of $7056 = 2 \times 2 \times 3 \times 7$

$$\sqrt{7056} = 84.$$

(*iv*) 2028

2	2028
2	1014
3	507
13	169
13	13
	1

Prime factors of $2028 = 2 \times 2 \times 3 \times 13 \times 13$

Here, prime factor 3 does not occur in pair. So, the number is not a perfect square. If 3 has a pair, then the number will be a perfect square. So, we multiply,

$$2028 \times 3 = 2 \times 2 \times 3 \times 3 \times 13 \times 13$$

Now, each factor has a pair. Therefore, $2028 \times 3 = 6084$ is a perfect square and the required smallest number is 3.

$$\sqrt{6084} = \sqrt{2 \times 2 \times 3 \times 3 \times 13 \times 13} = 2 \times 3 \times 13 = 78.$$

(*v*) 1458

2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Prime factors of $1458 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$.

Here, factor 2 does not occur in pair, so, 1458 is not a perfect square number.

If 2 gets its pair, then it will be a perfect square number. We multiply the number by 2

$$1458 \times 2 = 2916$$

So, 2916 is a perfect square number and the smallest multiple number is 2.

Prime factors of $2916 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$$\sqrt{2916} = 2 \times 3 \times 3 \times 3 = 54.$$

(*vi*) 768

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Prime factors of $768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Here, prime factor 3 does not occur in pair, so it is not a perfect square number.

To be a perfect square number, 3 must be multiplied to the number.

$$768 \times 3 = 2304 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$\sqrt{2304} = 2 \times 2 \times 2 \times 2 \times 3 = 48.$$

The required smallest number is 3 and square root of 2304 is 48.

Q6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.

(*i*) 252 **(*ii*) 2925** **(*iii*) 396**

(*iv*) 2645 **(*v*) 2800** **(*vi*) 1620**

Sol. (*i*) 252

2	252
2	126
3	63
3	21
7	7
	1

Prime factors of $252 = 2 \times 2 \times 3 \times 3 \times 7$

Here, factor 7 does not occur in pair. So, it is not a perfect square number.

To make it a perfect square, it must be divided by 7.

We get $252 \div 7 = 36 = 2 \times 2 \times 3 \times 3$

$$\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3} = 2 \times 3 = 6.$$

(*ii*) 2925

3	2925
3	975
5	325
5	65
13	13
	1

Prime factors of $2925 = 3 \times 3 \times 5 \times 5 \times 13$

Here, factor 13 does not occur in pair. So, it is not a perfect square number. It must be divided by 13 to make the perfect square number.

So, $2925 \div 13 = 225$

$$225 = 3 \times 3 \times 5 \times 5$$

Hence, $\sqrt{225} = \sqrt{3 \times 3 \times 5 \times 5} = 5 \times 3 = 15.$

The smallest number is 13 and the square root is 15.

(*iii*) 396

2	396
2	198
3	99
3	33
11	11
	1

Prime factors of $396 = 2 \times 2 \times 3 \times 3 \times 11$

Here, factor 11 does not occur in pair. So, it is not a perfect square number. It must be divided by 11 to get a perfect square.

Therefore, $396 \div 11 = 36$

$$36 = 2 \times 2 \times 3 \times 3$$

$$\sqrt{36} = \sqrt{2\times2\times3\times3} = 2 \times 3 = 6$$

The smallest number is 11 and the square root of the number is 6.

(*iv*) 2645

5	2645
23	529
23	23
	1

Prime factors of $2645 = 5 \times 23 \times 23$

Here, 5 is not in pair. So, it is not a perfect square number.

2645 must be divided by 5 to get a perfect square number.

$$2645 \div 5 = 529 = 23 \times 23$$

$$\sqrt{529} = \sqrt{23\times23} = 23$$

$\sqrt{529} = 23$ and the smallest number is 5.

(*v*) 2800

2	2800
2	1400
2	700
2	350
5	175
5	35
7	7
	1

Prime factors of $2800 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7$

Here, prime factor 7 does not occur in a pair. So, it is not a perfect square number.

So, 2800 must be divided by 7 to get a perfect square number

$$2800 \div 7 = 400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$$

$$\sqrt{400} = \sqrt{2\times2\times2\times2\times5\times5} = 2 \times 2 \times 5 = 20$$

Hence, the smallest number is 7 and the square root of the number is 20.

(*vi*) 1620

2	1620
2	810
3	405
3	135
3	45
3	15
5	5
	1

Prime factors of $1620 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5$

Here, factor 5 does not occur in pair. So, it is not a perfect square number.

So, 1620 must be divided by 5 to get a perfect square number.

$$1620 \div 5 = 324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

$$\sqrt{324} = \sqrt{2\times2\times3\times3\times3\times3} = 2 \times 3 \times 3 = 18.$$

Q7. The students of Class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Sol. Given, donated money = ₹ 2401

Let the number of students be x

Also, donated same amounts to be equal to the number of students = ₹ x

Total donated money = $x \times x$

According to the condition,

Donated money = 2401

$x \times x = 2401$

$x \times x = 7 \times 7 \times 7 \times 7$

7	2401
7	343
7	49
7	7
	1

Taking square roots both sides,

$$\sqrt{x \times x} = \sqrt{7 \times 7 \times 7 \times 7}$$

$$x = 7 \times 7 = 49$$

Hence, the number of students is 49.

Q8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Sol. Number of plants = 2025

Let the number of rows of planted plants be x

and each row contains number of plants = x

Total number of plants = $x \times x$

According to the given condition,

$x \times x = 2025$

$x \times x = 3 \times 3 \times 3 \times 3 \times 5 \times 5$

3	2025
3	675
3	225
3	75
5	25
5	5
	1

Taking square roots on both sides,

$$\sqrt{x \times x} = \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$$

$$x = 3 \times 3 \times 5 = 45$$

Therefore, number of rows = 45

Each row contains number of plants is 45.

Q9. Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.

Sol. L.C.M. of 4, 9 and 10 is 180

Now prime factorisation of 180

2	180
2	90
3	45
3	15
5	5
	1

Prime factors of 180 = $2 \times 2 \times 3 \times 3 \times 5$

Here, we see that factors 2 and 3 are in pairs and 5 is lone factor.

So, it is not a perfect square number.

It must be multiplied by 5 to make a perfect square number.

So, $180 \times 5 = 900$.

Hence, the smallest square number which is divisible by 4, 9 and 10 is 900.

Q10. Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

Sol. L.C.M. of 8, 15 and 20 = 120

Now, prime factorisation of 120

2	120
2	60
2	30
3	15
5	5
	1

So, prime factors of 120 = $2 \times 2 \times 2 \times 3 \times 5$

Here, we find that factors 2, 3 and 5 are not in pairs. It is not a perfect square number.

Therefore, the number must be multiplied by $2 \times 3 \times 5$, we get

$120 \times (2 \times 3 \times 5) = 3600$

Hence, the smallest square number is $120 \times 2 \times 3 \times 5 = 3600$, which is divisible by 8, 15 and 20.

EXERCISE 6.4 *(Page –107-108)*

Q1. Find the square root of each of the following numbers by Division method.

(*i*) 2304	**(*ii*) 4489**	**(*iii*) 3481**
(*iv*) 529	**(*v*) 3249**	**(*vi*) 1369**
(*vii*) 5776	**(*viii*) 7921**	**(*ix*) 576**
(*x*) 1024	**(*xi*) 3136**	**(*xii*) 900**

Sol. (*i*) The given number is 2304. Its square root

$$\begin{array}{r|l} & 48 \\ \hline 4 & \overline{23}\ \overline{04} \\ & -16 \\ \hline 88 & 704 \\ & -704 \\ \hline & 000 \end{array}$$

Hence, the square root of 2304 is **48.**

(*ii*) The given number is 4489. Its square root

$$\begin{array}{r|l} & 67 \\ \hline 6 & \overline{44}\ \overline{89} \\ & -36 \\ \hline 127 & 889 \\ & -889 \\ \hline & 0 \end{array}$$

Hence, the square root of 4489 is **67.**

(*iii*) The given number is 3481. Its square root

$$\begin{array}{r|l} & 59 \\ \hline 5 & \overline{34}\ \overline{81} \\ & -25 \\ \hline 109 & 981 \\ & -981 \\ \hline & 0 \end{array}$$

Hence, the square root of 3481 is **59.**

(*iv*) The given number is 529. Its square root

$$\begin{array}{r|l} & 23 \\ \hline 2 & \overline{5}\ \overline{29} \\ & -4 \\ \hline 43 & 129 \\ & -129 \\ \hline & 0 \end{array}$$

Hence, the square root of 529 is **23.**

(*v*) The given number is 3249. Its square root

$$\begin{array}{r|l} & 57 \\ \hline 5 & \overline{32}\ \overline{49} \\ & -25 \\ \hline 107 & 749 \\ & -749 \\ \hline & 0 \end{array}$$

Hence, square root of 3249 is **57.**

(*vi*) The given number is 1369. Its square root

$$\begin{array}{r|l} & 37 \\ \hline 3 & \overline{13}\ \overline{69} \\ & -9 \\ \hline 67 & 469 \\ & -469 \\ \hline & 0 \end{array}$$

Hence, the square root of 1369 is **37.**

(*vii*) The given number is 5776. Its square root

$$\begin{array}{r|l} & 76 \\ \hline 7 & \overline{57}\ \overline{76} \\ & -49 \\ \hline 146 & 876 \\ & -876 \\ \hline & \times \end{array}$$

Hence, the square root of 5776 is **76.**

(*viii*) The given number is 7921. Its square root

$$\begin{array}{r|l} & 89 \\ \hline 8 & \overline{79}\ \overline{21} \\ & -64 \\ \hline 169 & 1521 \\ & -1521 \\ \hline & 0 \end{array}$$

Hence, the square root of 7921 is **89.**

(*ix*) The given number is 576. Its square root

$$\begin{array}{r|l} & 24 \\ \hline 2 & \overline{5}\ \overline{76} \\ & -4 \\ \hline 44 & 176 \\ & -176 \\ \hline & 0 \end{array}$$

Hence, the square root of 576 is **24.**

(*x*) The given number is 1024. Its square root

$$\begin{array}{r|l} & 32 \\ \hline 3 & \overline{10}\ \overline{24} \\ & -9 \\ \hline 62 & 124 \\ & -124 \\ \hline & 0 \end{array}$$

Hence, the square root of 1024 is **32.**

(*xi*) The given number is 3136. Its square root

$$\begin{array}{r|l} & 56 \\ \hline 5 & \overline{31}\ \overline{36} \\ & -25 \\ \hline 106 & 636 \\ & -636 \\ \hline & 0 \end{array}$$

Hence, the square root of 3136 is **56.**

(*xii*) The given number is 900. Its square root

$$\begin{array}{r|l} & 30 \\ \hline 3 & \overline{9}\ \overline{00} \\ & -9 \\ \hline 60 & 000 \\ & 000 \\ \hline & 0 \end{array}$$

Hence, the square root of 900 is 30.

Q2. Find the number of digits in the square root of each of the following numbers (without any calculation).

(*i*) 64 (*ii*) 144 (*iii*) 4489

(*iv*) 27225 (*v*) 390625

Sol. (*i*) 64. Here, 64 contains two digits which is even. So,

number of digits in square root = $\frac{n}{2} = \frac{2}{2} = 1$.

Hence, the number of digits in the square root is 1.

(*ii*) 144. Here, there are three digits which is odd. Therefore,

number of digits in the square root = $\frac{n+1}{2} = \frac{3+1}{2} = \frac{4}{2}$ = 2.

Hence, number of digits in the square root is 2.

(*iii*) 4489. Here, number of digits is 4, which is even. So,

number of digits in the square root = $\frac{n}{2} = \frac{4}{2} = 2$.

Hence, the number of digits in the square is 2.

(*iv*) 27225. Here, we see that there are 5 digits which is odd.

So, number of digits in the square root = $\frac{n+1}{2} = \frac{5+1}{2}$

$= \frac{6}{2} = 3$.

Hence, the number of digits in the square root is 3.

(*v*) 390625. Here, number of digits are six which is even.

So, number of digits in square root = $\frac{n}{2} = \frac{6}{2} = 3$.

Hence, the number of digits in the square root is 3.

Q3. Find the square root of the following decimal numbers.

(*i*) 2.56 (*ii*) 7.29 (*iii*) 51.84 (*iv*) 42.25 (*v*) 31.36

Sol. (*i*) The given number is 2.56. Its square root

$$\begin{array}{r|l} & 1.6 \\ \hline 1 & \overline{2}.\overline{56} \\ & -1 \\ \hline 26 & 156 \\ & -156 \\ \hline & 0 \end{array}$$

Therefore, $\sqrt{2.56}$ = **1.6.**

(*ii*) The given number is 7.29. Its square root

	2.7
2	$\overline{7}.\overline{29}$
	−4
47	329
	−329
	0

Therefore, $\sqrt{7.29}$ = **2.7**

(*iii*) The given number is 51.84. Its square root

	7.2
7	$\overline{51}.\overline{84}$
	−49
142	284
	−284
	0

Therefore, $\sqrt{51.84}$ = **7.2**

(*iv*) The given number is 42.25. Its square root

	6.5
6	$\overline{42}.\overline{25}$
	−36
125	625
	−625
	0

Therefore, $\sqrt{42.25}$ = **6.5**

(*v*) The given number is 31.36. Its square root

	5.6
5	$\overline{31}.\overline{36}$
	−25
106	636
	−636
	0

Therefore, $\sqrt{31.36}$ = **5.6.**

Q4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(*i*) 402 (*ii*) 1989 (*iii*) 3250

(*iv*) 825 (*v*) 4000

Sol. (*i*) The given number is 402

	20
2	$\overline{4}\ \overline{02}$
	−4
40	02
	−00
	2

Here, we get the remainder 2. So, if we subtract the remainder from the number, we get a perfect square number.

Therefore, we get a perfect square number = 402 − 2 = 400, then 2 must be subtracted from the number.

The square root of 400

	20
2	$\overline{4}\ \overline{00}$
	−4
40	000
	−000
	0

Then, perfect square number is 400. And $\sqrt{400}$ = 20.

(*ii*) The given number is 1989

	44
4	$\overline{19}\ \overline{89}$
	−16
84	389
	−336
	53

Here, we get the remainder 53. So, if we subtract the remainder from the given number.

Therefore, we get a perfect square number = 1989 – 53 = 1936

$$\begin{array}{r|l} & 44 \\ \hline 4 & \overline{19}\,\overline{36} \\ & 16 \\ \hline 84 & 336 \\ & 336 \\ \hline & \times \end{array}$$

Hence, perfect square number is 1936. And $\sqrt{1936} = 44$.

(*iii*) The given number is 3250

$$\begin{array}{r|l} & 57 \\ \hline 5 & \overline{32}\,\overline{50} \\ & -25 \\ \hline 107 & 750 \\ & -749 \\ \hline & 1 \end{array}$$

Therefore, perfect square number = 3250 – 1 = 3249, then 1 must be subtracted from the number.

$$\begin{array}{r|l} & 57 \\ \hline 5 & \overline{32}\,\overline{49} \\ & -25 \\ \hline 107 & 749 \\ & -749 \\ \hline & \times \end{array}$$

Hence, perfect square number is 3249. And $\sqrt{3249} = 57$.

(*iv*) The given number is 825

$$\begin{array}{r|l} & 28 \\ \hline 2 & \overline{8}\,\overline{25} \\ & -4 \\ \hline 48 & 425 \\ & -384 \\ \hline & 41 \end{array}$$

Here, we get the remainder 41. So, if we subtract the remainder from number.

Therefore, we get a perfect square number = 825 – 41 = 784, then, 41 must be subtracted from the number.

$$\begin{array}{r|l} & 28 \\ \hline 2 & \overline{7}\,\overline{84} \\ & -4 \\ \hline 48 & 384 \\ & -384 \\ \hline & 0 \end{array}$$

Hence, the perfect square number is 784. And $\sqrt{784} = 28$.

(*v*) The given number is 4000

$$\begin{array}{r|l} & 63 \\ \hline 6 & \overline{40}\,\overline{00} \\ & -36 \\ \hline 123 & 400 \\ & -369 \\ \hline & 31 \end{array}$$

Here, we get the remainder 31. So, if we subtract the remainder from the number.

We get perfect square number = 4000 – 31 = 3969.

$$\begin{array}{r|l} & 63 \\ \hline 6 & \overline{39}\,\overline{69} \\ & -36 \\ \hline 123 & 369 \\ & -369 \\ \hline & 0 \end{array}$$

Hence, the perfect square number is 3969.

And $\sqrt{3969} = 63$.

Q5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also, find the square root of the perfect square so obtained.

(i) 525 **(ii) 1750** **(iii) 252**
(iv) 1825 **(v) 6412**

Sol. (*i*) The given number is 525

$$\begin{array}{r|l} & 22 \\ \hline 2 & \overline{5}\ \overline{25} \\ & -4 \\ \hline 42 & 125 \\ & -84 \\ \hline & 41 \end{array}$$

Here, we get the remainder 41, this shows that $22^2 < 525$

Next perfect square number is $23^2 = 529$

Hence, the number to be added is $23^2 - 525 = 529 - 525 = 4$

Therefore, 4 must be added to make a perfect square number.

So, perfect square number = 525 + 4 = 529.

So, $\sqrt{529} = 23$.

(*ii*) The given number is 1750

$$\begin{array}{r|l} & 41 \\ \hline 4 & \overline{17}\ \overline{50} \\ & -16 \\ \hline 81 & 150 \\ & -81 \\ \hline & 69 \end{array}$$

Here, we get the remainder 69, this shows that $41^2 < 1750$.

Next perfect square number is $42^2 = 1764$.

Hence, the number to be added $= 42^2 - 1750$

$= 1764 - 1750 = 14$

Hence, 14 must be added to the number to make it a perfect square number. Therefore, perfect square number = 1750 + 14 = 1764. and $\sqrt{1764} = 42$.

(*iii*) The given number is 252

$$\begin{array}{r|l} & 15 \\ \hline 1 & \overline{2}\ \overline{52} \\ & -1 \\ \hline 25 & 152 \\ & -125 \\ \hline & 27 \end{array}$$

Here, we get the remainder 27, this shows that $15^2 < 252$.

Next perfect square number is $16^2 = 256$

Hence, the number to be added $= 16^2 - 252 = 256 - 252 = 4$

Therefore, perfect square number = 252 + 4 = 256.

(*iv*) The given number is 1825

$$\begin{array}{r|l} & 42 \\ \hline 4 & \overline{18}\ \overline{25} \\ & -16 \\ \hline 82 & 225 \\ & -164 \\ \hline & 61 \end{array}$$

Here, we get the remainder 61, this shows that $42^2 < 1825$.

Next perfect square number $= 43^2 = 1849$.

Hence, the number is to be added $= 43^2 - 1825$

$= 1849 - 1825 = 24$

Therefore, the perfect square number = 1825 + 24 = 1849.

(*v*) 6412

$$\begin{array}{r|l} & 80 \\ \hline 8 & \overline{64}\ \overline{12} \\ & -64 \\ \hline 160 & 0012 \\ & -0000 \\ \hline & 12 \end{array}$$

Here, we get the remainder 12, this shows that $80^2 < 6412$

Next perfect square number = $81^2 = 6561$.

Hence, the number is to be added $= 81^2 - 6412$

$= 6561 - 6412 = 149$

Therefore, 149 should be added to the number to make it a perfect square and $\sqrt{6561} = 81$.

Q6. Find the length of the side of a square whose area is 441 m².

Sol. Let the length of side of a square be x metre

Area of square $= (\text{side})^2 = x \times x$

Given area of square $= 441\ \text{m}^2$

Then, $x \times x = 441$

$\Rightarrow \quad x^2 = 441$

$\Rightarrow \quad x = \sqrt{441}$ (Taking square root both sides)

$\Rightarrow \quad x = 21$

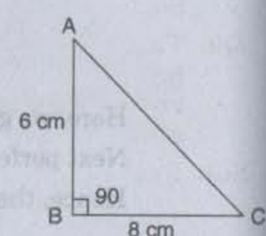
Hence, the length of side of a square is 21 m.

Q7. In a right triangle ABC, ∠B = 90°.

(a) If AB = 6 cm, BC = 8 cm, find AC

(b) If AC = 13 cm, BC = 5 cm, find AB.

Sol. (a) By Pythagoras theorem



$AC^2 = AB^2 + BC^2$

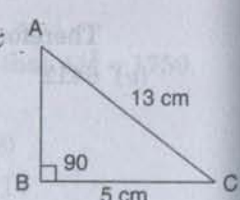
$AC^2 = (6)^2 + (8)^2$

$AC^2 = 36 + 64$

$AC^2 = 100$

$\Rightarrow AC = \sqrt{100} = 10$ cm.

(b) By Pythagoras theorem and AB = x



$\Rightarrow \quad x^2 = 441$

$AC^2 = AB^2 + BC^2$

$(13)^2 = AB^2 + (5)^2$

$169 = AB^2 + 25$

or $\quad x^2 + 25 = 169$ $\quad$ [AB = x (let)]

$x^2 = 169 - 25 = 144$

$x^2 = 144$

$x = \sqrt{144} = 12$ cm

AB = 12 cm.

Q8. A gardners has 1000 plants. He wants to plant these in such a way that the number of rows and number of columns remain same. Find the minimum number of plants he needs more for this.

Sol.

$$\begin{array}{r|l} & \;\; 31 \\ \hline 3 & \overline{10}\;\overline{00} \\ & -9 \\ \hline 61 & 100 \\ & -61 \\ \hline & \;\; 39 \end{array}$$

Here, $\quad 31^2 < 1000 < 32^2$

Hence, required minimum number of plants $= 32^2 - 1000$

$= 1024 - 1000$

$= 24.$

So, the gardener requires 24 more plants.

Q9. There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement.

Sol. By getting the square root of this number, we know that

$$\begin{array}{r|l} & \;\; 22 \\ \hline 2 & \bar{5}\;\overline{00} \\ & -4 \\ \hline 42 & 100 \\ & -84 \\ \hline & \;\; 16 \end{array}$$

In each row, the number of children is 22.

And the left out children are 16.

□□