

CLASS – XII

SUBJECT – MATHEMATICS

ASSIGNMENT NO. 4

1. A particle moves along the curve $y = x^2 + 2x$ at what points on the curve are the x & y co-ordinates of the particle changing at the same rate?
2. A balloon in form of a circular cone surmounted by a hemisphere having a diam. Equal to the height of the cone is being inflated. How fast is its vol. Is changing with respect to the total length h , when $h = 9$ cm.
3. The radius of cylinder is increasing at the rate 2cm/sec & its altitude decreasing at the rate of 3cm/sec. Find the rate of change volume where radius is 3 cm & alt. 5 cm.
4. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the points on the curve at which the y -co-ordinate is changing twice as fast as x -co-ordinate.
5. Use differentiation to find the approx. values of (i) $(0.009)^{1/3}$ (ii) $(0.007)^{1/3}$ (iii) $(255)^{1/4}$
6. Find approx. value of $f(5.00)$ if $f(x) = x^3 - 7x^2 + 15$
7. Find the percentage error in finding the surface area of a cubical box if an error 1% is made in measuring the length of edges of the cube.
8. Verify Rolle's Theorem for the functions $f(x) = (x - 9)^m (x - b)^n$ on the interval (a, b) where m, n are +ve integers.
9. Verify Rolle's theorem for (i) $f(x) = e^x / \sin x - \cos x$ on $[\frac{n}{4}, \frac{5n}{4}]$
10. Verify mean value theorem for (i) $f(x) = 2\sin x + \sin 2x$ on $(0, \pi)$ (ii) $f(x) = x^3 - 2x^2 - x + 3$ on $(0, 1)$ (iii) $f(x) = 10e^x$ on $[1, 2]$
11. Prove : tangents to the curve $y = x^2 - 5x + 6$ at points $(2, 0)$ and $(3, 0)$ are at right angles
12. Find the eqn. of normal to the curve $x = a \cos^3 \theta, y = a \sin^3 \theta$ at $\theta = \frac{\pi}{4}$
13. Find the points on curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with axes.
14. Find points on the curve $xy + 4 = 0$ at which tangents to curve are inclined at an angle of 45° with x -axis
15. Find the equations of tangent & normal to the curves (i) $y = a(1 + \cos \theta)$ at $\theta = \frac{-\pi}{2}$ (ii) $y = x^3 - x$ at $x = 2$ (iii) $x^2 + 3y + y^2 = 5$ at $(1, 1)$
16. Prove that one $\left(\frac{x}{9}\right)^n + \left(\frac{y}{9}\right)^n = 2$ touches the st. line $\frac{x}{a} + \frac{y}{b} = 2$ for all $n \in \mathbb{N}$ at (a, b)
17. Show that curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.
18. Find interval in which $f(x)$ is (i) increase (ii) decrease (a) $f(x) = 2x^3 + px^2 + 12x + 20$ (ii) $f(x) = x^4 = x^3/3$ (iii) $f(x) = x^3 + \frac{1}{x^3}$
19. Separate $(0, \frac{\pi}{2})$ in sub-intervals in which $f(x) = \sin 3x$ is increasing or decreasing is also $f(x) = \sin^4 x + \cos^4 x$.
20. Find the intervals on which $f(x) = 2x^3 - 3x^2 - 3(x+7)$ is (i) strictly increasing (ii) strictly decreasing.
21. Find local max & local min. If (i) $f(x) = (\sin x - \cos x)$ where $0 < x < \pi/2$ (ii) $f(x) = \sin 4x + \cos 4x$ in $(0, \pi/2)$
22. Show that \square of max area inscribed in a given circle is an equivalent D.
23. Show that semi-vertical angle of cone of max vol. of given slant height is $\tan^{-1} \sqrt{2}$