Chapter 6

APPLICATION OF DERIVATIVES

1 Mark Questions

Q1 **How will you interpret geometrically $\frac{dy}{dx}$ of a function at a point (x_0, y_0)**

Q2 **Find the rate of change of area of a circle with respect to its radius 'r' when r = 5 cm.**

Q3 **The total cost C(x) in rupees, associated with the production of x units of an item is given by C(x)= $0.005x^2 - 0.002x + 5000$. Find the marginal cost when only two items are produced.**

Q4 **Total revenue in selling x units of a product is given by R(x) = $3x^2 + 36x + 5$. Find the marginal revenue when x = 5.**

Q5 **Show that function $f(x) = 2x - 1$ is strictly increasing on R.**

Q6 **Write the condition for a function f to be strictly increasing.** {$f'(x) > 0, \forall x \in D_f$}

Q7 **Write the condition for a function f to be strictly decreasing.** {$f'(x) < 0, \forall x \in D_f$}

Q8 **Show that function $f(x) = \cos x$ is strictly increasing on ($\pi, 2\pi$).**

Q9 **Show that function $f(x) = \cos x$ is strictly increasing on ($0, \pi$).**

Q10 **Show that function $f(x) = \cos x$ is neither increasing nor decreasing on ($0, 2\pi$).**

Q11 **Find the point of inflection for the function $f(x) = x^3$.**

Q12 **Find the intervals in which the function $f(x) = x^2 - 2x$ is strictly increasing.**

Q13 **Find the intervals in which the function $f(x) = x^2 - 2x$ is strictly decreasing.**

Q14 **Write the domain of the function $f(x) = \log(1+x) - \frac{2x}{1+x}$.**

Q15 **Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point where the curve cuts y-axis.**

4 Mark Questions

Q1 **Prove that the curves x = y^2 and x. y = k cut each other at right angle if $8.k^2 = 1$.**

Q2 **Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which tangents are**

parallel to x- axis.

Q3 **Find the points on the curve $y = x^3$ at which tangent makes angle of 45^0 tox-axis**

$$\left\{\left(\frac{1}{\sqrt{3}},\frac{1}{3\sqrt{3}}\right);\left(-\frac{1}{\sqrt{3}},-\frac{1}{3\sqrt{3}}\right)\right\}$$

Q4 **Find the points on the curve $y = x^3$ at which slope of the tangent is equal to the ordinate of the point.**

Q5 **Find the points on the curve $x^2 + 3y = 3$ where the tangent is perpendicular to the line $4y + x = -5$.**

Q6 **Find the equation of tangent to the curve $16x^2 + 9y^2 = 144$ at the point (x_1, y_1) where $x_1 = 2$ and $y_1 > 0$.**

Q7 **Find the least value of 'b' for which $f(x) = x^2 + bx + 1$ is an increasing on [1, 2]**

Q8 **Find the points of local max. and local min. for the function. Also find their max. / minimum values** $y = Sin\,x - Cos\,x,\ 0 \le x \le 2\pi$

Q9 **Find the absolute maximum and absolute minimum values of the function**

$f(x) = 2\cos 2x - \cos 4x,\ on\ [0,\pi]$

Q10 **If $f(x) = x3 + ax^2 + bx + c$ has a maximum at x = -1 and minimum at x = 3. Show that a = - 3, b = -9, c ∈ R**

Q11 **A given rectangular area is to be fenced off in a field whose length lies along a straight river. If no fencing is required along the rive show that least amount of fencing will be required when length of the field is twice its breadth.**

Q12 **Find the angle x which is increasing twice as fast as its sine.**

Q13 **Using differential, find the approximate value of $\tan^{-1}(0.999)$.**

Q14 **If $y = x^4 - 10$ and if x changes from 2 to 1.99, find the approximate change in y.**

Q15 **If the side of a cube is increased from 10 cm to 10.01 cm.. Find approximate change in the volume of the cube.**

<u>6 Mark Questions</u>

Q1 **Find the equation of tangent to the curve** $y = \cos(x + y)$**,** $-2\pi \le x \le 2\pi$ **that are parallel to the line x + 2 y = 0.**

Q2 **Find the points on the curve $y = 4x^3 - 2x^5$ at which tangent passes through the origin.**

Q3 **Find the angle of intersection of the curves** $y = x^2$ & $x^2 + y^2 = 20$

Q4 **Find the intervals in which following functions are strictly increasing or strictly decreasing f(x) = x^x, x > 0.**

Q5 **Find the intervals in which following functions are strictly increasing or strictly decreasing** **f(x) = $(x + 1)^3(x- 3)^3$**

Q6 **Find the absolute maximum and absolute minimum values of the function** $f(x) = -x + 2Sin\ x\ on\ [0, 2\pi]$

Q7 **A jet of an enemy is flying along the curve** $y = x^2 + 2$**. A soldier is placed at the point (3, 2) . Find the nearest distance between the soldier and the jet.**

Q8 **An open box with a square base in to be made out of a given quantity of sheet of area** c^2 **.Show that the maximum volume of the box is** $c^3 / 6\sqrt{3}.$

Q9 **A sheet of paper for a poster is 15000 sq. cm. in area. The margins at the top and bottom are to be 6 cm. wide and at the sides 4 cm. wide. Find the dimensions of the sheet to maximize the printed area.**

Q10 **A given quantity of metal to be cast into half cylinder with a rectangular base and semicircular ends. Show that in order that total surface area is minimum, the ratio of length of cylinder to the diameter of its semi-circular ends is** $\pi : \pi + 2$**..**

Answers: APPLICATION OF DERIVATIVES

1 Mark Questions

Q2 **{10 π }**

Q3 **{Rs.0.018}**

Q4 **{Rs. 66}**

Q11 **{ x = 0}**

Q12 **{(1, ∞)}**

Q13 **{(-∞ ,1)}**

Q14 **{ x > 1}**

Q15 **{-1}**

4 Mark Questions

Q2 **{(1,2); (1, -2)}**

Q4 **{(0,0); (3,27)}**

Q5 **{(-6, -11)}**

Q6 $\{8x + 3\sqrt{5}y = 36\}$

Q7 **{-2}**

Q8 $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}; \sqrt{2}, -\sqrt{2}\right\}$

Q9 $\left\{Ab.\max.\frac{3}{2}, Ab.\min. -3\right\}$

Q12 $\left\{\frac{\pi}{3}\right\}$

Q13 **{0.7852}**

Q14 **{-0.32}**

Q15 **{3 cm ^{3}}**

6 Mark Questions

Q1 $\left\{x + 2y = \frac{\pi}{2}; 2x + y = -\frac{3\pi}{2}\right\}$

Q2 **{(0,0);(1,2);(-1,-2)}**

Q3 $\left\{\tan^{-1}\frac{9}{2}\right\}$

Q4 $\left\{\frac{1}{e}, \infty\right\}$

Q5 **{Strictly increasing in (1,3),(3,∞) & Strictly decreasing in (-1,1),(-∞ , -1)}**

Q6 **{Ab. Min -** $\frac{3\sqrt{3} + 5\pi}{3}$ **& Ab Max.** $\frac{3\sqrt{3} - \pi}{3}$**}**

Q7 $\{\sqrt{5}\}$

Q9 **{One side 150 cm and other side 100 cm}**

Chapter 8

APPLICATIONS OF INTEGRALS

6 Mark Questions

Q1 **Using Integration find the area of the region { (x,y):0≤y≤x^2+1, 0≤y≤x+1,0≤x≤2}**

Q2 **Find the area bounded by the curve x=a(θ - sin θ), y=a(1-cos θ)**

$0 \le \theta \le 2\pi$ **.**

Q3 **Draw the rough sketch of the curve y=(**$\frac{x}{\pi} + 2\sin^2 x$**)and find the area between the x axis, the curve and the ordinates**

Q4 **Find the area bounded by the curve y^2 +x = 8 + 2y, the y axis and the lines y= -1 and y=3.**

Q5 **Sketch the graph y=|x - 1|. Evaluate** $\int_{-2}^{4} |x-1| dx$ **What does the value on this integral represent on the graph.**

Q6 **Find the area bounded by the curve $(1- x^2)^{1/2}$, line x=y and the positive x-axis.**

Q7 **Find the area of the area bounded by the curve y=x $\sin x^2$, x axis and between x=0 and x=**$\sqrt{\pi/2}$ **.**

Q8 **Draw the rough sketch of y^2+1=x, x≤ 2 and find the area enclosed by the curves and ordinate x=2.**

Q9 **Sketch the rough graph of y=** $4\sqrt{x-1}$ **, 1≤x≤3 and evaluate the area between the curve, x -axis and the line x=3.**

Q10 **Determine the area enclosed between the curve y=4x-x^2.and the x-axis**

Q11 **Sketch the region bounded by the curve y=$\sqrt{5-x^2}$ and y = |x-1| and find its area.**

Answers: APPLICATIONS OF INTEGRALS

6 Mark Questions

Q1 **23/6 sq.units**

Q2 **3π a^2 Sq.units**

Q3 **$3\pi/2$ sq.units**

Q4 **92/3 sq.units**

Q5 **9 sq.units**

Q6 **π /8 sq.units**

Q7 **1/2 sq.units**

Q8 **4/3 sq.units**

Q9 **$16\sqrt{2}$ /3 sq.units**

Q10 **32/3 sq.units**

Q11 $\left(\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}+\frac{5}{2}\sin^{-1}\frac{1}{5}-\frac{1}{2}\right)$ **sq.units**

Chapter 5
CONTINUITY AND DIFFENTIABILITY

1 Mark Questions

Q1 Check the continuity of the function $f(x) = \begin{cases} 1, \text{ if } x \le 0 \\ 2, \text{ if } x > 0 \end{cases}$

Q2 **Check the continuity of the function $f(x) = |x|$ at x = 0**

Q3 **Check the continuity of the function $f(x) = \sin x + x^2$ at x = π**

Q4 **Examine the continuity of the function $f(x) = |x|$**

Q5 **Write the points of discontinuity of the function $f(x) = [x]$ where $[x]$ denotes the greatest integer function less than or equal to x.**

Q6 **Write the points of discontinuity of the function $f(x) = \dfrac{x^2 - 25}{x - 5}$.**

Q7 **Give an example of a function which is continuous but not differentiable.**

Q8 **Find the derivative of $\sin\left(\cos^2(\sqrt{x})\right)$.**

Q9 **Find the points in the open interval (0, 3) where the greatest integer function $f(x) = [x]$ is not differentiable.**

Q10 **Write the derivative of the function $f(x) = \tan^{-1}\sqrt{\sin x}$ w. r. to x.**

Q11 **Is it true that $\log\left(x^{\sin x} + \cos^{\sin x} x\right) = \sin x \log x + \sin x \log \cos x$?**

Q12 **If $x = f(t)$ and $y = g(t)$, then is $\dfrac{d^2y}{dx^2} = \dfrac{d^2y/dt^2}{d^2x/dt^2}$?**

Q13 **Check the applicability of Rolle's theorem for $f(x) = [x]$ on [1, 5].**

Q14 **Discuss the continuity of the function $f(x) = \sin|x|$**

Q15 **Discuss the continuity of the function $f(x) = |x| - |x - 1|$**

4 Mark Questions

Q1 **Differentiate log (x + $\sqrt{1 + x^2}$)**

Q2 **Differentiate $\dfrac{\sqrt{a + x} + \sqrt{a - x}}{\sqrt{a + x} - \sqrt{a - x}}$**

Q3 **Find** $\frac{dy}{dx}$ **for** $\sin(xy) + \frac{x}{y} = x^2 - y$

Q4 **If** $2^x + 2^y = 2^{x+y}$ **, show that** $\frac{dy}{dx} = -2^{y-x}$

Q5 **If** $\tan^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = a$ **, show that** $\frac{dy}{dx} = \frac{x(1 - \tan a)}{y(1 + \tan a)}$

Q6 **Differentiate** $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

Q7 **Differentiate** $x^x + x^a + a^x + a^a$

Q8 **Differentiate** $\left(x + \frac{1}{x}\right)^x + x^{\left(x + \frac{1}{x}\right)}$

Q9 **Find** $\frac{dy}{dx}$ **for x =** $\frac{\sin^3\theta}{\sqrt{\cos 2\theta}}$ **, y =** $\frac{\cos^3\theta}{\sqrt{\cos 2\theta}}$

Q10 **Find** $\frac{dy}{dx}$ **for x =** $\sqrt{a^{\sin^{-1}t}}$ **, y =** $\sqrt{a^{\cos^{-1}t}}$

Q11 **If y =** $\left(\tan^{-1}x\right)^2$ **show that** $(1 + x^2)^2\frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} - 2 = 0$

Q12 **If sin y = x cos (a + y), show that** $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\cos a}$

Q13 **If** $y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1$**, show that** $\frac{dy}{dx} = -\sqrt{\frac{1 - y^2}{1 - x^2}}$

Q14 **If x sin (a + y) + sin a cos (a + y) = 0 , show that** $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

Q15 **Check the continuity of the function** $f(x) = \begin{cases} \frac{x - |x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ **at x = 0**

<u>6 Mark Questions</u>

Q1 **If y =** $\left(\sin^{-1}x\right)^2$ **show that** $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

Q2 **Find** $\frac{d^2y}{dx^2}$ **for the function x =** $a(\theta + \sin\theta)$**, y =** $a(1+\cos\theta)$ **at** $\theta = \frac{\pi}{2}$

Q3 **Find** $\frac{dy}{dx}$ **for** $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$

Q4 **Find** $\frac{dy}{dx}$ **for tan (x + y) + tan (x – y) = 1**

Q5 **Show that the function defined by** $g(x) = x - [x]$ **is discontinuous at all integral points.**

Answers: Continuity and Differentiability
1 Mark Questions

Q1 **{Not Continuous}**

Q2 **{Continuous}**

Q3 **{Continuous}**

Q4 **{Continuous}**

Q5 **{All integers}**

Q6 **{x = 5}**

Q7 $\{f(x) = |x|\}$

Q8 $\frac{-\cos\left(\cos^2\sqrt{x}\right)\sin\sqrt{x}\ \cos\sqrt{x}}{\sqrt{x}}$

Q9 **{1, 2}**

Q10 $\{\frac{\cos x}{2\sqrt{\sin x}\ (1+\sin x)}\}$

Q11 **{No}**

Q12 **{No}**

Q13 **{ Not applicable}**

Q15 **{Continuous}**

Q14 **{Continuous}**

4 Marks Questions

Q1 $\left\{\frac{1}{\sqrt{1+x^2}}\right\}$

Q2 $\left\{\frac{a}{\sqrt{a^2-x^2}\left(a+\sqrt{a^2-x^2}\right)}\right\}$

Q3 $\left\{\frac{2xy^2-y-y^3\cos(xy)}{y^2x\cos(xy)-x+y^2}\right\}$

Q4 $\left\{-2^{y-x}\right\}$

Q6 **{-1}**

Q7 **{x^x (1 + log x) + a x^{a-1} + a^x loga}**

Q8 $\left\{\left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]+x^{1+\frac{1}{x}}\left(\frac{x+1-\log x}{x^2}\right)\right\}$

Q9 **{- cot 3θ }**

Q10 $\left\{-\frac{y}{x}\right\}$

Q15 **{Not Continuous}**

6 Mark Questions

Q2 $\left\{-\frac{1}{a}\right\}$

Q3 $\left\{\frac{1}{\sqrt{1+x^2}(2x^2+1+2x\sqrt{1+x^2}}\right\}$

Q4 $\left\{-\frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x+y)-\sec^2(x-y)}\right\}$

Chapter 4
DETERMINANTS

1 Mark Questions

Q1 **If A is a square matrix of order 3 and | A | = 5, find the value of** $|-3A|$

Q2 **If ω is cube root of unity find the value of**

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & \omega \\ \omega^2 & 1 & 1 \end{vmatrix}$$

Q3 **Find the value of determinant**

$$\Delta = \begin{vmatrix} 1 & 2 & 4 \\ 8 & 16 & 32 \\ 64 & 128 & 256 \end{vmatrix}$$

Q4 **Find the value of determinant**

$$\Delta = \begin{vmatrix} 2 & 2 & 2 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$$

Q5 **If a, b, care in A.P. find the value of determinant**

$$\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

Q6 For what value of k, the matrix $\begin{bmatrix} 2 & k \\ 3 & 5 \end{bmatrix}$ has no inverse

Q7 **A B C are three non zero matrices of same order, then find the condition on A such that AB = AC ⇒ B = C**

Q8 **Let Abe a non singular matrix of order 3 x 3, such that** $|AdjA|$ **=100, find** $|A|$ **.**

Q9 If A is a non singular matrix of order n, then write the value of Adj(AdjA) and hence write the value of Adj(AdjA) if order of A is 3 and | A | = 5

Q10 **Let A be a diagonal A = (d_1, d_2, ..., d_n) write the value of | A |.**

Q11 Using determinants find the value of the line passing through the points (-1,3) and (0,2).

Q12 **Using determinants find the value of k for which the following system of equations has unique solution,**

$$2x - 5y = 26$$
$$3x + ky = 5$$

4 Mark Questions

Q1 **If $A = [a_{ij}]$ is a 3 x 3 matrix and A_{ij}'s denote cofactors of the corresponding elements a_{ij}'s then write the value of ,**

(i) $a_{11} A_{11} + a_{11}A_{11} + a_{11}A_{11}$

(ii) $a_{12} A_{12} + a_{22}A_{22} + a_{32}A_{32}$

(iii) $a_{21} A_{11} + a_{22}A_{12} + a_{23}A_{13}$

(iv) $a_{11} A_{13} + a_{21}A_{23} + a_{31}A_{33}$

Q2 **If $x \in R$, $\pi/2 \geq x \geq 0$ and** $\begin{vmatrix} 2\sin x & -1 \\ 1 & \sin x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ -4 & \sin x \end{vmatrix}$

find the values of x.

Q3 **If a ,b, c are all distinct and** $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$**, find the values of a, band c.**

Q4 **If x is a real number then show that if**

$$\Delta = \begin{vmatrix} 1 & \sin x & 1 \\ -\sin x & 1 & \sin x \\ -1 & -\sin x & 1 \end{vmatrix}$$ **then ,** $2 \leq \Delta \leq 4$

Q5 **If x , y, z are real numbers such that $x + y + z = \pi$ then find the value of ,**

$$\begin{vmatrix} \sin(x+y+z) & \sin(x+z) & \cos z \\ -\sin y & 0 & \tan x \\ \cos(x+y) & \tan(y+z) & 0 \end{vmatrix}$$

Q6 **Without expanding find the value of the following determinant,**

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}$$

Q7 **Find value of k, if area of the triangle with vertices P (k ,0) , Q (4,0) and R(0,2) is 4 square units**

Q8 **If** $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ **show that** $A' A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

Q9 **Prove that,** $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$

Q10 **Using properties prove that,** $\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$

where α, β, γ **are in A.P.**

Q11 **Prove that,** $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

6 Mark Questions

Q1 **If** $a \neq p$ $b \neq q$ $c \neq r$ **and** $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ **find the value of**

$$\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$$

Q2 **For the matrix** $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ **find the numbers a and b such that**

$A^2 + aA + bI = O$. **Hence find** A^{-1}.

Q3 **Solve the equation if** $a \neq 0$ **and**

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix}$$

Q4 **Show that the value of the determinant** $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **is negative , It is**

given that a , b , c , are positive and unequal.

Q5 **Using matrix method, determine whether the following system of equations is consistent or inconsistent.**

$$3x - y - 2z = 2$$

$$2y - z = -1$$

$$3x - 5y = 3$$

Q6 **. If** $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ **and** $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ **find** $(AB)^{-1}$.

Q7 **Let** $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ **then, verify that,** $(AdjA)^{-1} = Adj(A^{-1})$

Answers: DETERMINANTS
1 Mark Questions

Q1 **-135**

Q2 **0 (using properties)**

Q3 **0 (using properties)**

Q4 **0 (using properties)**

Q5 **0 (using properties)**

Q6 $k = \frac{10}{3}$

Q7 $|A| \neq 0$

Q8 $|A| = \pm 10$

Q9 **For a non-singular matrix of order n > 1,** $Adj(AdjA) = (|A|)^{n-2} . A$

∴ if order of matrix A is 3 x 3 and $|A| = 5$**, then**

$Adj(AdjA) = 5A$

Q10 $|A| = d_1 . d_2 . d_3 \ldots\ldots\ldots d_n$

Q11 $x + y = 2$

Q12 $k \neq -\frac{15}{2}$ **(all real numbers except** $-\frac{15}{2}$**)**

4 Mark Questions

Q1 **(i) Value of |A|**

(ii) Value of |A|

(iii) 0 (iv) 0

Q2 $x = \frac{\pi}{6}, \frac{\pi}{2}$

Q3 abc = -1

Q5 **0**

Q6 **0**

Q7 **k = 8 , 0**

6 Mark Questions

Q1 **0**

Q2 **a = -4, b = 1**

$$A^{-1} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

Q3 $x = -\frac{a}{3}$ **or** $x = -\frac{a}{2}$

Q5 **Inconsistent**

Q6

$$(AB)^{-1} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Chapter 9

Differential Equations

1 Mark Questions:

1	Find the order of the differential equation satisfying, $\sqrt{1-x^4} + \sqrt{1-y^4} = a(x^2 - y^2)$
2	What is the order and degree of the differential equation whose solution is $y = cx + c^2 - 3c^{3/2} + 2$, where c is a parameter.
3	Find the Equation of the curve passing through (1, 1) and satisfying the differential equation $\frac{dy}{dx} = \frac{2y}{x} : (x > 0, y > 0)$
4	What is the order and degree of the differential equation, $\left(\frac{d^2y}{dx^2}\right) = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$
5	Five the general solution of $x(1+y^2)^{1/2}dx + y(1+x^2)^{1/2}dy = 0$
6	**Find the equation of the curve which passes through (3,9) and satisfies differential equation** $\frac{dy}{dx} = x + \frac{1}{x^2}$
7	Form the differential equation representing the family of curves given by $y^2 = 4ax$
8	Show that $y = -\cos x + x$ is a solution of the differential equation $y'' = \cos x$.
9	Verify that the given function is a solution of the corresponding differential equation: $y = e^x + 1 \quad : \quad y'' - y' = 0$
10	Find the solution of $\frac{dy}{dx}$ = y tan 2x, where y (0) = 2
11	If $\frac{dy}{dx} + \left(\frac{1-y^2}{1-x^2}\right)^{1/2} = 0$ Find the equation corresponding to the Diff. equation.
12	Find the solution of the equation (y – x) dy – (y2– x2) dx = 0, where y (0) = –1
13	. Form the differential equation $c(y+c)^2 = x^3$
14	Find whether $y = \frac{a}{x} + b, is \cdots solution of \quad \frac{d^2y}{d^2x} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$

4 Mark Questions:

1	**Find the equation of the curve that passes through the point (1,2) and satisfies the differential equation,** $\frac{dy}{dx}=\frac{-2xy}{(x^2+1)}$**.**
2	**Solve** $(1+y^2)dx=(\tan^{-1}y-x)dy$
3	**Solve** $\frac{dy}{dx}-x\sin^2 x=\frac{1}{x\log x}$
4	**Form the differential equation of the curve** $(x-a)^2+(y-b)^2=r^2$ **where** $a,b,$ **are arbitrary constants.**
5	**Solve** $\frac{dy}{dx}=\sin^3 x\cos^2 x+xe^x$
6	**Form the differential equation representing the family of ellipses** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$
7	**If** $y=x\log\left(\frac{x}{a+bx}\right)$**, prove that** $x^3.\frac{d^2y}{dx^2}=\left(x.\frac{dy}{dx}-y\right)^2$
8	**Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of *x*-axis.**
9	**Solve the differential equation** $\frac{y}{x}Cos\left(\frac{y}{x}\right)dx-\left\{\frac{x}{y}Sin\left(\frac{y}{x}\right)+Cos\frac{y}{x}\right\}dy=0$
10	**Solve the equation** $\left(1+x^3\right)\frac{dy}{dx}+6x^2y=\left(1+x^2\right)$

6 Mark Questions:

1	**Solve the differential equation** $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$
2	**Solve:** $\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$
3	**Find the particular solution of the differential equation** $\log\left(\frac{dy}{dx}\right)=3x+4y$ **at** $y(0)=0$**.**
4	**Solve** $x.dy-y.dx=\sqrt{x^2+y^2}dx$
5	Solve $x^2dy+y(x+y)dx=0$ *for* $y=1,x=1$

Answers Differential Equations

Answer-1 Mark questions	
Q1	**1**
Q2	**1 and 4**
Q3	**x2=y**
Q4	**2,2**
Q5	**$(1+x^2)^{1/2}+(1+y^2)^{1/2}=C$**
Q6	**$6xy=3x^3+29x-6$**
Q7	**2xy'-y=0**
Q10	$Y = \frac{2}{\sqrt{COS2X}}$
Q11	$\sqrt{I - X^2} + \sqrt{I - Y^2} = 0$
Q12	**Y= -(I+X)**
Answer-4 Mark questions	
Q1	**$y(x^2+1)=4$**
Q2	**$x=(\tan^{-1}y)-1+ce^{\tan^{-1}y}$**
Q3	$\frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{2} + \log\lvert\log x\rvert + c$
Q4	$\left[1 + (y')^2\right]^3 = r^2(y'')^2$
Q5	$\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + xe^x - e^x + c$
Q6	$xy'y'' + x(y')^2 = yy'$
Q8	$2xy' - y = 0$
Answer-6 Mark questions	
Q1	$\sin\frac{y}{x} = \log\lvert x\rvert + c$
Q2	$\sqrt{x^2+1} + \sqrt{1+y^2} - \log\left\lvert\frac{1+\sqrt{1+x^2}}{x} + c\right\rvert$ **+c=0**
Q3	**$4e^{3x}+3e^{-4y}-7=0$**

Chapter 7

INTEGRATION

1 Mark Questions

Find the value of the following integrals

Q1 $\int \text{Cosec } ax \, dx$

Q2 $\int \sin x / \sin[x-a] \, dx$

Q3 $\int (\log x) / x^2 \, dx$

Q4 $\int (x + \cos 6x)/(3x^2+\sin 6x) \, dx$

Q5 $\int \sqrt{1-4x-x^2} \, dx$

Q6 $\int \cos 2x/(\sin x + \cos x)^2 \, dx$

Q7 $\int dx/(e^x + e^{-x}) \, dx$

Q8 $\int (1+\tan^2 x)/(1+\tan x) \, dx$

Q9 $\int (\sin 2x \cos 2x) / \sqrt{(9-\cos^4 2x)} \, dx$

Q10 $\int_1^{\sqrt{3}} dx/(1+x^2) \, dx$

Q11 $\int_{1/3}^{1} (x-x^3)^{1/3}/x^4 \, dx$

Q12 $\int_0^{\pi/2} (\cos^2 x/2 - \sin^2 x/2) \, dx$

Q13 $\int_0^{\pi/2} \log [(4+3\sin x) / (4+3\cos x)] \, dx$

Q14 $\int_0^{\pi} \sin^6 x \cos^5 x \, dx$

Q15 $\int_0^{\pi/2} \log(\cot x) \, dx$

Q16 $\int_{-\pi/4}^{\pi/4} dx/\sec^2 x(1+\sin x)$

Q17 $\int_{0}^{1} x(1-x)^5\,dx$

Q18 $\int_{\pi/8}^{3\pi/8} \cos x/(\cos x + \sin x)\,dx$

Q19 **If** $f(x)= \int_{0}^{x} t\sin t\,dt$, **then find f '(x**

4 Mark Questions

Evaluate

Q1

$$\int_{-1}^{3/2} |\, x \sin(\pi x) \,|\, dx$$

Q2

$$\int_{0}^{\pi} \theta \sin^3 \theta\, d\theta$$

$\int$

Q3 $\int_{0}^{\pi} x \log(\sin x)\,dx$

Q4 $\int_{0}^{2\pi} dx/(e^{\sin x}+1)$

Q6 $\int e^x (1-\sin x)/(1-\cos x)\, dx$

Q7

$$\int_0^{\pi/2} dx/(1+\cot^3 x)$$

Q8

$$\int_0^1 \log(1+x)/(1+x^2)\, dx$$

Q9

$$\int_{-\pi/2}^{\pi/2} [\sin|x| + \cos|x|]\, dx$$

Q10

$$\int_0^{\pi} x/(a^2\cos^2 x + b^2\sin^2 x)\, dx$$

Q11

$$\int_0^{\pi/2} \sin 2x \log(\tan x)\, dx$$

Q12

$$\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$$

Q13

$$\int_0^{\pi/4} \log(1+\tan x)\, dx$$

Q14

$$\int_{\pi/3}^{\pi/2} \sqrt{(1+\cos x)}/(1-\cos x)^{5/2}\, dx$$

Q15

$$\int_{\pi/4}^{\pi/2} \cos\Theta / (\cos\Theta/2 + \sin\Theta/2)^3\, d\Theta$$

Q16 $\int_0^a$ **sin $^{-1}$ (x/a+x) dx**

Q17 $\int_0^{\pi/4}$ **√ tanx dx**

Q18 $\int_{-a}^{a}$ **√ [(a-x)/(a+x)]dx**

Q19 $\int_0^{\pi/2}$ **dx / (4sin^{2}x+5cos^{2}x)**

Q20 $\int_{\pi/6}^{\pi/3}$ **dx / [1+√tanx]**

Q21 $\int_1^3$ **logx / (1+x)^2 dx**

Q22 $\int_0^{e^2}$ **[** $\frac{1}{\log x}$ **-** $\left(\frac{1}{\log x}\right)^2$ **] dx**

Q23 $\int_1^4$ **(x^2+x) / √(2x+1) dx**

Q24 $\int_0^a$ **dx / √(ax-x^2)dx**

Answers: INTEGRATION

1 Mark Questions

Q1 **1/a log | cosec ax-cot ax | +c**

Q2 **(Hint:Put x-a=t)**

(x-a) Cos a + Sin a log|Sin (x-a)| +c

Q3 **Hint: Write in the form ∫ log.x^{-2} and use integration by parts.**

-1/x log x- 1/x +c or -1/x [log x+1] +c

Q4 **1/6 log |3x^2 + Sin 6x |+c**

Q5 $5/2\ [\sin^{-1}(x+2)/\sqrt{5}] + (x+2)(\sqrt{1-4x-x^2})/2 + c$

Q6 **. log | Sin x + Cos x | +c**

Q7 **$\tan^{-1}(e^x)$ +c**

Q8 **log |1+t an x| +c**

Q9 **$-1/4\ \sin^{-1}[1/3\ \cos^2 2x]$ +c**

Q10 **π /12**

Q11 **4**

Q12 **1**

Q13 **ZERO**

Q14 **ZERO**

Q15 **ZERO**

Q16 **π /2**

Q17 **1/42**

Q18 **Л/8**

Q19 **x Sin x**

4 Mark Questions

Q1 $\frac{3}{\pi} + \frac{1}{\pi^2}$

Q2 **2π /3**

Q3 $\frac{-\pi^2}{2} \ln 2$

Q4 π

Q5 **$-e^x \cot(x/2)$**

Q6 $e^x \left(\frac{x-1}{x+1} \right) + c$

Q7 **Hint** $\int_0^{\pi/2} \frac{dx}{1+\frac{(\cos x)^3}{(\sin x)^3}}$

$\pi/4$

Q8 **Hint: Put x = tanϴ**

$\pi/8$ **(log 2)**

Q9 **4**

Q10 $\frac{\pi^2}{2ab}$

Q11 **0**

Q12 **Hint: put x=cos ^{2} t ,the question will be transformed to Q.11**

Q13 **(**$\pi/8$**) log 2**

Q14 **3/2**

Q15 $\frac{2}{\cos\frac{\pi}{8}+\sin\frac{\pi}{8}} - \sqrt{2}$

Q16 **a (**$\frac{\pi}{2}$ **- 1)**

Q17 $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$ **log (**$\sqrt{2}$ **- 1)**

Q18 **a**π

Q19 $\frac{\pi}{4\sqrt{5}}$

Q20 $\left(\frac{e^2}{2}\right)$ **- e**

Q21 **Hint.Take log x as the first function**

Q22 $\left(\frac{e^2}{2}\right)$ **- e**

Q23 **Hint: Take x^2 + x as first function and the other as second**

[57 - 5$\sqrt{5}$] / 5

Q24 π

Chapter 12
Linear Programming

4 /6Mark Questions

Q1 **Solve the following problem graphically:**

Minimise and Maximise Z= 3x +9y

Subject to the constraints: $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$

Q2 **Determine graphically the minimum value of the objective function.**

$Z = -50x + 20y$

Subject to constraints:

$2x - y \geq -5$

$3x + y \geq 3$

$2x - 3y \leq 12$

$x \geq 0, y \geq 0$

Q3 **Minimize Z= 3x + 2y**

Subject to constraints:

$x + y \geq 8$

$3x + 5y \leq 15$

$x \geq 0, y \geq 0$

Q4 Minimize $Z = x + 2y$

Subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x,y \geq 0$.

Show that the minimum of Z occurs at more than two points.

5. Minimize and Maximise $Z = 5x + 10y$

Q5 Minimize and Maximise $Z = 5x + 10y$

Subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x,y \geq 0$

Q6 Maximise $Z = -x + 2y$,

Subject to $x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$

Q7 Maximise $Z = x + y$

Subject to $x - y \leq -1$, $-x + y \leq 0$, $x, y \geq 0$

Q8 The corner points of a feasible region determined by the following system of linear

inequalities: $2x + y \leq 10$, $x + 3y \leq 15$, $x,y \geq 0$ are (0,0) , (5,0) , (3,4) and (0,5).

Let $Z = px + qy$,

where $p,q > 0$. Condition on p and q so that the maximum of Z occurs at both (3,4)

and (0,5) is

(A) $p = q$

(B) $p = 2q$

(C) $p = 3q$

(D) $q = 3p$

Q9 **If a young man drives his motorcycle at 25 km/hr he has to spend Rs.2 per km on petrol.If he rides at a faster speed of 40 km/hr the petrol cost increases to Rs.5 per km. He has Rs.100 to spend on petrol & wishes to find what is the max. distance he can travel within an hour.Express as LPP & solve it.**

Q10 **The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the inputs & outputs per production run,are as follows:**

	Input		Output	
Process	**Crude A**	**Crude B**	**Gasoline P**	**Gasoline Q**
1	**5**	**3**	**5**	**8**
2	**4**	**5**	**4**	**4**

The max.Crude A & B available are 200 & 150 units resp. Market requirementa are atleast 100 & 80 units P & Q resp..The profit froim process 1 & process 2 are Rs.300/- & Rs 400/- resp. Formulate LPP & solve.

Q11

Answers: **Linear Programming**

Q1 **The max.value of Z on the feasible region occurs at the two corner points (15,15) & (0.20) and it is 180 in each case**

Q2 **No minimum value**

Q3 **No feasible region & hence no feasible solution**

Q4 **Minimum Z = 6 at all the points on the line segment joining points (6,0) and (0,3)**

Q5 **Minimum Z = 300 at (60,0)**

Maximum Z = 600 at all the points on the line segment joining the points (120, 0) and (60, 30)

Q6 **Z has no maximum value**

Q7 **No feasible region . Hence no maximum value of Z**

Q8 **q = 3p**

Q9 **Max. Z = 30 km when 50/3 Km travelled at 25 Km/hr & 40/3 Km travelled at**

40 Km /hr **(** $\frac{50}{3}, \frac{40}{3}$ **)**

Q10 **Max Z=** $\frac{Rs.1,80,000}{13}$ **, x =** $\frac{400}{13}$ **, y=** $\frac{150}{13}$

Chapter 3

Matrix

1 Mark Questions

Q1 **Write the number of possible matrices which can be made if it has 12 elements.**

Q2 **Let A = $[a_{ij}]$ be a matrix of order 2 x 3 and**

$$a_{ij} = \frac{i - j}{i + j}$$, **write the value of a_{23}.**

Q3 **If** $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ **find the relation between a and b.**

Q4 **If following information regarding the number of men and women workers in three factories I, II and III is written in the form of 3 x 2 matrix. What does the entry in third row and second column represent?**

	Men workers	Women workers
Factory I	**30**	**25**
Factory II	**25**	**31**
Factory III	**27**	**26**

Q5 **If ,** $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ **and** $B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$

Write the value of (i) $a_{22} + b_{21}$

(ii) $a_{11} b_{11} + a_{22} b_{22}$

Q6 **Is it possible to have the product of two matrices to be the null matrix** while neither of them is the null matrix? If it is so, give an example.

Q7 **Under what conditions is the matrix equation**

$A^2 - B^2 = (A-B)(A+B)$ is true.

Q8 **Write the order of matrix B if A is any matrix of order m x n such that**

AB and BA both are defined.

Q10 **If** $\mathbf{A} = \begin{bmatrix} -1 & 2 & -5 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}$

write the orders of AB and BA.

Q11 **Give an example of two non-zero matrices A and B such that**

$AB = 0$ but $BA \neq 0$.

Q12 If $A = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ find A^6.

Q13 **If** $\mathbf{A} = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ **and** $\mathbf{A^2 = I}$, find the value of $\alpha^2 + \beta\gamma$

Q14 **If** $\mathbf{A} = \begin{bmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{bmatrix}$

$\mathbf{0 < x < \frac{\pi}{2}}$ **and** $\mathbf{A + A' = I}$,

where I is unit matrix, find value of x.

Q15 **If the following matrix is skew symmetric, find the values of a, b, c.**

$$\mathbf{A} = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

Q16 **If A and B are symmetric matrices and AB = BA, prove that matrix X = AB is also symmetric.**

Q17 **If A and B are square matrices of same order and B is symmetric, show that $A'BA$ is also symmetric.**

Q18 **Give an example of a matrix which is both symmetric and skew symmetric**

Q19 $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ **= P + Q, where P is symmetric and Q is**

skew symmetric matrix, find the matrices P and Q.

Q20 **If A is square matrix then write the value of A(AdjA)**

4 Mark Questions

Q1 **For what values of x and y are the following matrices equal**

$$\mathbf{A} = \begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Q2 **Find matrix A such that 2A-3B+5C = 0 where,**

$$\mathbf{B} = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

Q3 **Find the values of x and y for which the following matrix equation**

A-3B = C is satisfied, where

$$\mathbf{A} = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} x \\ 2y \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$$

Q4 **Let $f(x) = x^2 - 5x + 6$, find f(A)**

If, $\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

5. If, $\mathbf{A} = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ **and** $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ **find all those values of α for which**

A=B.

Q6 **. Using Principle of Mathematical Induction, prove that**

$$\mathbf{A^n} = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix} \qquad \textbf{Where,} \quad \mathbf{A} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

6 Mark Questions

Q1 **If** $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ **find x, y such that** $A^2 + xI = yA$

Hence find A^{-1}**.**

Q2 **. If** $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ **Prove that,** $A = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ **for every positive integer n.**

Q3 **The sum of three numbers is -1. If we multiply the second number by 2 , third number by 3 and add them we get 5. If we subtract the third number from the sum of first and second numbers we get -1. Represent it by a system of equations . Find the three numbers using inverse of a matrix .**

Q4 **If** $A(x_1,y_1)$, $B(x_2,y_2)$ **and** $C(x_3,y_3)$ **are the vertices of an equilateral triangle with each side equal to 'a' units, prove that,**

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = \sqrt{3}\ a^2$$

Answers: Matrix

1 Mark Questions

Q1 **6**

Q2 **-1/5**

Q3 **a=2b { a=4, b=2}**

Q4 **Number of women workers in factory III.**

Q5 **1, 20**

Q6 $A=\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$ $B=\begin{bmatrix} 1 & O \\ O & O \end{bmatrix}$ $AB=\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$

Q7 **AB = BA ie, if the matrices A & B commute with each other.**

Q8 **n x m**

Q9 **k = 4, a = -4, b = -10, c = 0**

Q10 **1 x 1, 3 x 3,**

Q10 **A =** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **B =** $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Q12 $A^6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q13 $\alpha^2 + \beta\gamma = 1$

Q14 $x = \pi/6$

Q15 **a = -2, b = 0, c = -3**

Q18 **Null Matrix**

Q19 $P = \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ $Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Q20 $|A|\ I$

4 Marks Questions

Q1 **x=2, y=2**

Q2 $A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$

Q3 **x=1, 2** $y = 3 \pm 3\sqrt{2}$

Q4 $f(A) = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

Q5 **No values of α can be found for which** $A^2 = B$ **is true.**

Q6 **order of A=2 x 3**

$$A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

6 Mark Questions

Q1 **x = 8 y = 8**

Q2 $\mathbf{A^{-1}} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

Q3 **Let numbers be x, y, z then**

$$x + y + z = -1$$

$$2y + 3z = 5$$

$$x + y - z = -1$$

Ans $x = -\frac{7}{2}$, $y = \frac{5}{2}$, $z = 0$

Chapter 13

PROBABILITY

1 Mark Questions

Q1 If A and B are two independent events, find P(B) when P(A U B) = 0.60 and P(A) = 0.35.

Q2 A card is drawn from a well shuffled pack of 52 cards. The outcome is noted and the pack is again reshuffled without replacing the card. Another card is then drawn. What is the probability that the first card is a spade and the second is a Black King.

Q3 Find the chance of drawing two white balls in succession from a bag containing 3 red and 5 white balls, the ball first drawn not replaced.

Q4 Given P(A) = 1/4, P(B) = 2/3 and P(A U B) = 3/4. Are the events independent ?

Q5 Given P(A) = 0.3, P(B) = 0.2. find P(B/A) if A and B are mutually exclusive events.

Q6 **Does the following table represents a probability distribution ?**

X	-2	-1	0	1	2
P(X)	0.1	0.2	-0.2	0.4	0.5

Q7 Father, mother and son line up at random for a family picture. If E is the event "Son on one end" and F is the event "Father in middle", Find P(F/E).

Q8 A bag contains 5 brown and 4 white socks. A man pulls out 2 socks. Find the probability that these socks are of same colour.

Q10 Out of 30 consecutive integers, 2 are chosen at random. Find the probability that their sum is odd.

Q11 An urn contains 9 balls,two of which are red, 3 blue and 4 black. Three balls are drawn at random. Find the probability that they are of the same color.

Q12 **If X is a random variable with probability distribution as given below, find the value of k.**

X	0	1	2	3
P(X)	k	3k	3k	k

Q13 A random variable X takes the values 0,1,2,3 and its mean is 1.3. If P(X

=3)= 2P(X = 1) and P(X =2)= 0.3. then find P(X = 0).

Q14 If P(A) = 7/13, P(B) = 9/13 and P(A ∩ B) = 4/13. Find P(A'/B).

Q15 A couple has 2 children. Find the probability that both the children are boys, if it is known that at least one of the children is a boy.

Q16 A coin is tossed 7 times. Write the probability distribution of getting r heads.

Q17 If P(A)=0.4, P(B)= p and P(A U B)= 0.7, find the value of p if A and B are independent events.

Q18 **In two successive throws a pair of dice, find the probability of getting a total of 8 each time**

Q19 A policeman fires 4 bullets on a dacoit. The probability that the dacoit will not be killed by a bullet is 0.4. What is the probability that the dacoit is still alive ?

Q20 4 Cards are drawn from a well shuffled pack of 52 cards. Find the probability of drawing all the 4 cards of the same suit if a card is replaced after each other.

4 Mark Questions

Q1 'A' speaks truth in 60% cases and 'B' in 90% cases. In what % of cases are they likely to contradict each other in stating the same fact?

Q2 **The probability of student A passing an examination is 3/5 and of student B passing is 4/5. Assuming the two events : 'A passes', 'B passes', as independent find the probability of:**

a. **Both students passing the examination**
b. **Only A passing the examination**
c. **Only one of the two passing the examination**
d. **Neither of the two passing the examination**

Q3 A problem in Mathematics is given to three students whose chances of solving it are 1/3,1/5,1/6. What is the probability that at least one of them solves the problem.

Q4 A' speaks truth in 60% cases and 'B' in 90% cases. In what % of cases are they likely to contradict each other in stating the same fact?

Q5 A and B are two independent events. The probability that both A and B

occur is 1/6 and the probability that neither of them occurs is 1/3. Find the probability of occurrence of A.

Q6 A coin is tossed until a head appears or until it has been tossed three times. Given that 'head' does not occur on the first toss, what is the probability that the coin is tossed thrice?

Q7 A company has two plants to manufacture scooters. Plant-1 manufactures 70% of the scooters and Plant-2 manufactures 30%. At Plant-1, 80% of the scooters are rated of standard quality and at Plant-2, 90% of the scooters are rated of standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it has come from Plant-2.

Q8 Find the mean and standard deviation of the probability distribution of the numbers obtained when a card is drawn at random from a set of 7 cards numbered 1 to 7.

Q9 A pair of dice is rolled twice. Let X denote the number of times, ' a total of 9 is obtained'. Find the mean and variance of the random variable X.

Q10 **A box contains 12 bulbs of which 3 are defective. A sample of 3 bulbs is selected from the box. Let X denote the number of defective bulbs in the sample, find the probability distribution of X.**

Q11 A box contains 12 bulbs of which 3 are defective. A sample of 3 bulbs is selected from the box. Let X denote the number of defective bulbs in the sample, find the probability distribution of X.

Q12 Two dice are thrown six times. 'A total of 7' is considered as success. Find the probability of at least 4 successes.

Q13 **Four cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that**

i. **All the four cards are spades?**

ii. **Only three cards are spades?**

iii. None is a spade?

Q14 The sum of mean and variance of a binomial distribution is 35/16, for 5 trials. Find the distribution.

Q15 A man takes a step forward with probability 0.4 and backwards with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.

Q16 **10% of the tools produced by a machine are defective. Find the probability**

distribution of the number of defective tools in a sample of 3 drawn at random.

Q17 3 defective bulbs are mixed with 7 good ones, 3 bulbs are drawn at random. Find the probability distribution of the defective bulbs.

Q18 **The probability distribution of a random variable X is given by**

X	0	1	2
P(X)	$3c^3$	$4c-10c^2$	5c - 1

where c > 1 then find each of the following :

i. **c**

ii. **P(X< 2)**

iii. **P(1 < X≤2)**

Q19 **A letter is known have to come from TATANAGAR or CALCUTTA. On the envelope just two consecutive letters TA are visible. Find the probability that the letter has come from**

i. **Calcutta**

ii. **Tatanagar**

Q20 **Fatima and John appear in an interview for two vacancies on the same post. The probability of Fatima's selection is 1/7 and that of John's selection is 1/5. What is the probability that:**

i. **Both of them will be selected.**

ii. **Only one of them is selected.**

iii. **None of them will be selected.**

<u>6 Mark Questions</u>

Q1 **A pair of dice is thrown 7 times . If getting a total of 7 is considered a successes, what is the probability of**

I) No Success II) 6 Success III) At least 6 Success IV) At most 6 Successes.

Q2 **Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the value x has the following form where k is some unknown constant:**

P(X=x) = {0.1 if x= 0

kx if x =1 or 2

k(5-x) if x =3 or 4

0,otherwise }

(a) Find the value of k

(b) What is the probability that you study

(i) At least 2 hours ?

(ii) Exactly two hours ?

(iii) At most two hours ?

Q3 **Three urns contain respectively 3 green and 2 white balls , 5green and 6 white balls and 2 green, 4white balls. One ball in drawn from each urn. Find the mean and variance of the probability distribution of the discrete random variable ,"Number of white balls drawn".**

Q4 **A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, what is the probability that the missing card is black.**

Answers PROBABILITY

1 Mark Questions

Q1 1) 5/13

Q2 25/2652

Q3 5/14

Q4 Yes

Q5 **0**

Q6 **No**

Q7 1/2

Q8 48/108

Q9 15/29

Q10 11/16

Q11 5/84

Q12 1/8

Q13 0.4

Q14 5/9

Q15 1/3

Q16 $P(r)={}^{7}c_{r}(1/2)^{7}$, r=0,1,2....7

Q17 0.5

Q18 25/1296

Q19 0.0256

Q20 1/64

4 Mark Questions

Q1 5 2/77

Q2 1 2/25,3/25,11/25,
2/25

Q3 5/9

Q4 42%

Q5 ½ OR 1/3

Q6 1/2

Q7 27/83

Q8 4,2

Q9 2/9,16/81

Q10

X	0	1	2	3
P(x)	84/220	108/220	27/220	1/220

Q11

X	0	1	2
P(x)	9/16	3/8	1/16

Q12 $(406/6^6)$

Q13 1/256, 3/64, 81/256

Q14 $(3/4+1/4)^5$

Q15 $^{11}C_5(3/5)^5(2/5)^5$

Q16

X	0	1	2	3
P(x)	727/1000	243/1000	27/1000	1/1000

Q17

X	0	1	2	3
P(x)	7/24	21/40	7/40	1/120

Q18 1/3, 1/3, 2/3

Q19 4/11

Q20 1/35, 2/7, 24/35

6 Mark Questions

Q1 $(5/6)^7, 35(1/6)^7, (1/6)^5, 1-(1/6)^7$

Q2 (a) 0.15 (b) 0.75 ,0.3,0.55

Q3 532/330,0.71

Q4 2/3

Q5

X	0	1	2	3	4
P(x)	625/1296	500/1296	150/1296	20/1296	1/1296