Physics formulas from Mechanics, Waves, Optics, Heat and Thermodynamics, Electricity and Magnetism and Modern Physics. Also includes the value of Physical Constants. Helps in quick revision for CBSE, NEET, JEE Mains, and Advanced.

## 0.1: Physical Constants

Speed of light Planck constant

Gravitation constant Boltzmann constant Molar gas constant Avogadro's number Charge of electron Permeability of vacuum
Permitivity of vacuum
Coulomb constant
Faraday constant Mass of electron
Mass of proton
Mass of neutron
Atomic mass unit
Atomic mass unit
Stefan-Boltzmann constant
Rydberg constant Bohr magneton
Bohr radius
Standard atmosphere
Wien displacement constant

## 1 MECHANICS

## 1.1: Vectors

Notation: $\vec{a}=a_{x} \hat{\imath}+a_{y} \hat{\jmath}+a_{z} \hat{k}$
Magnitude: $a=|\vec{a}|=\sqrt{a_{x}^{2}+a_{y}^{2}+a_{z}^{2}}$
Dot product: $\vec{a} \cdot \vec{b}=a_{x} b_{x}+a_{y} b_{y}+a_{z} b_{z}=a b \cos \theta$

## Cross product:



$$
\begin{gathered}
\vec{a} \times \vec{b}=\left(a_{y} b_{z}-a_{z} b_{y}\right) \hat{\imath}+\left(a_{z} b_{x}-a_{x} b_{z}\right) \hat{\jmath}+\left(a_{x} b_{y}-a_{y} b_{x}\right) \hat{k} \\
|\vec{a} \times \vec{b}|=a b \sin \theta
\end{gathered}
$$

## 1.2: Kinematics

Average and Instantaneous Vel. and Accel.:

$$
\begin{array}{ll}
\vec{v}_{\mathrm{av}}=\Delta \vec{r} / \Delta t, & \vec{v}_{\text {inst }}=d \vec{r} / d t \\
\vec{a}_{\mathrm{av}}=\Delta \vec{v} / \Delta t & \vec{a}_{\text {inst }}=d \vec{v} / d t
\end{array}
$$

Motion in a straight line with constant $a$ :

$$
v=u+a t, \quad s=u t+\frac{1}{2} a t^{2}, \quad v^{2}-u^{2}=2 a s
$$

Relative Velocity: $\vec{v}_{A / B}=\vec{v}_{A}-\vec{v}_{B}$

Projectile Motion:


$$
\begin{aligned}
& x=u t \cos \theta, \quad y=u t \sin \theta-\frac{1}{2} g t^{2} \\
& y=x \tan \theta-\frac{g}{2 u^{2} \cos ^{2} \theta} x^{2} \\
& T=\frac{2 u \sin \theta}{g}, \quad R=\frac{u^{2} \sin 2 \theta}{g}, \quad H=\frac{u^{2} \sin ^{2} \theta}{2 g}
\end{aligned}
$$

## 1.3: Newton's Laws and Friction

Linear momentum: $\vec{p}=m \vec{v}$
Newton's first law: inertial frame.
Newton's second law: $\vec{F}=\frac{\mathrm{d} \vec{p}}{\mathrm{~d} t}, \quad \vec{F}=m \vec{a}$
Newton's third law: $\vec{F}_{\mathrm{AB}}=-\vec{F}_{\mathrm{BA}}$
Frictional force: $f_{\text {static }, \max }=\mu_{s} N, \quad f_{\text {kinetic }}=\mu_{k} N$
Banking angle: $\frac{v^{2}}{r g}=\tan \theta, \frac{v^{2}}{r g}=\frac{\mu+\tan \theta}{1-\mu \tan \theta}$
Centripetal force: $F_{\mathrm{c}}=\frac{m v^{2}}{r}, \quad a_{\mathrm{c}}=\frac{v^{2}}{r}$
Pseudo force: $\vec{F}_{\text {pseudo }}=-m \vec{a}_{0}, \quad F_{\text {centrifugal }}=-\frac{m v^{2}}{r}$
Minimum speed to complete vertical circle:

$$
v_{\min , \text { bottom }}=\sqrt{5 g l}, \quad v_{\min , \text { top }}=\sqrt{g l}
$$

Conical pendulum: $T=2 \pi \sqrt{\frac{l \cos \theta}{g}}$


## 1.4: Work, Power and Energy

Work: $W=\vec{F} \cdot \vec{S}=F S \cos \theta, \quad W=\int \vec{F} \cdot \mathrm{~d} \vec{S}$
Kinetic energy: $K=\frac{1}{2} m v^{2}=\frac{p^{2}}{2 m}$
Potential energy: $F=-\partial U / \partial x$ for conservative forces.

$$
U_{\text {gravitational }}=m g h, \quad U_{\text {spring }}=\frac{1}{2} k x^{2}
$$

Work done by conservative forces is path independent and depends only on initial and final points: $\oint \vec{F}_{\text {conservative }} \cdot \mathrm{d} \vec{r}=0$.

Work-energy theorem: $W=\Delta K$

Mechanical energy: $E=U+K$. Conserved if forces are conservative in nature.

Power $P_{\mathrm{av}}=\frac{\Delta W}{\Delta t}, \quad P_{\text {inst }}=\vec{F} \cdot \vec{v}$

## 1.5: Centre of Mass and Collision

Centre of mass: $x_{\mathrm{cm}}=\frac{\sum x_{i} m_{i}}{\sum m_{i}}, \quad x_{\mathrm{cm}}=\frac{\int x \mathrm{~d} m}{\int \mathrm{~d} m}$

## CM of few useful configurations:

1. $m_{1}, m_{2}$ separated by $r$ :

2. Triangle (CM $\equiv$ Centroid) $y_{c}=\frac{h}{3}$

3. Semicircular ring: $y_{c}=\frac{2 r}{\pi}$
4. Semicircular disc: $y_{c}=\frac{4 r}{3 \pi}$


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6. Solid Hemisphere: $y_{c}=\frac{3 r}{8}$

7. Cone: the height of CM from the base is $h / 4$ for the solid cone and $h / 3$ for the hollow cone.

Motion of the CM: $M=\sum m_{i}$

$$
\vec{v}_{\mathrm{cm}}=\frac{\sum m_{i} \vec{v}_{i}}{M}, \quad \vec{p}_{\mathrm{cm}}=M \vec{v}_{\mathrm{cm}}, \quad \vec{a}_{\mathrm{cm}}=\frac{\vec{F}_{\mathrm{ext}}}{M}
$$

Impulse: $\vec{J}=\int \vec{F} \mathrm{~d} t=\Delta \vec{p}$

## Collision:



Momentum conservation: $m_{1} v_{1}+m_{2} v_{2}=m_{1} v_{1}^{\prime}+m_{2} v_{2}^{\prime}$ Elastic Collision: $\frac{1}{2} m_{1} v_{1}^{2}+\frac{1}{2} m_{2} v_{2}^{2}=\frac{1}{2} m_{1} v_{1}^{\prime 2}+\frac{1}{2} m_{2} v_{2}^{\prime 2}$ Coefficient of restitution:

$$
e=\frac{-\left(v_{1}^{\prime}-v_{2}^{\prime}\right)}{v_{1}-v_{2}}= \begin{cases}1, & \text { completely elastic } \\ 0, & \text { completely in-elastic }\end{cases}
$$

If $v_{2}=0$ and $m_{1} \ll m_{2}$ then $v_{1}^{\prime}=-v_{1}$.
If $v_{2}=0$ and $m_{1} \gg m_{2}$ then $v_{2}^{\prime}=2 v_{1}$.
Elastic collision with $m_{1}=m_{2}: v_{1}^{\prime}=v_{2}$ and $v_{2}^{\prime}=v_{1}$.
1.6: Rigid Body Dynamics

Angular velocity: $\omega_{\mathrm{av}}=\frac{\Delta \theta}{\Delta t}, \quad \omega=\frac{\mathrm{d} \theta}{\mathrm{d} t}, \quad \vec{v}=\vec{\omega} \times \vec{r}$
Angular Accel.: $\alpha_{\mathrm{av}}=\frac{\Delta \omega}{\Delta t}, \quad \alpha=\frac{\mathrm{d} \omega}{\mathrm{d} t}, \quad \vec{a}=\vec{\alpha} \times \vec{r}$

Rotation about an axis with constant $\alpha$ :

$$
\omega=\omega_{0}+\alpha t, \quad \theta=\omega t+\frac{1}{2} \alpha t^{2}, \quad \omega^{2}-\omega_{0}^{2}=2 \alpha \theta
$$

Moment of Inertia: $I=\sum_{i} m_{i} r_{i}{ }^{2}, \quad I=\int r^{2} \mathrm{~d} m$


Theorem of Parallel Axes: $I_{\|}=I_{\mathrm{cm}}+m d^{2}$

Theorem of Perp. Axes: $I_{z}=I_{x}+I_{y}$


Radius of Gyration: $k=\sqrt{I / m}$
Angular Momentum: $\vec{L}=\vec{r} \times \vec{p}, \quad \vec{L}=I \vec{\omega}$
Torque: $\vec{\tau}=\vec{r} \times \vec{F}, \quad \vec{\tau}=\frac{\mathrm{d} \vec{L}}{\mathrm{~d} t}, \quad \tau=I \alpha$


Conservation of $\vec{L}: \vec{\tau}_{\text {ext }}=0 \Longrightarrow \vec{L}=$ const.
Equilibrium condition: $\sum \vec{F}=\overrightarrow{0}, \quad \sum \vec{\tau}=\overrightarrow{0}$
Kinetic Energy: $K_{\text {rot }}=\frac{1}{2} I \omega^{2}$

## Dynamics:

$$
\begin{aligned}
& \vec{\tau}_{\mathrm{cm}}=I_{\mathrm{cm}} \vec{\alpha}, \quad \vec{F}_{\mathrm{ext}}=m \vec{a}_{\mathrm{cm}}, \quad \vec{p}_{\mathrm{cm}}=m \vec{v}_{\mathrm{cm}} \\
& K=\frac{1}{2} m v_{\mathrm{cm}}^{2}+\frac{1}{2} I_{\mathrm{cm}} \omega^{2}, \quad \vec{L}=I_{\mathrm{cm}} \vec{\omega}+\vec{r}_{\mathrm{cm}} \times m \vec{v}_{\mathrm{cm}}
\end{aligned}
$$

## 1.7: Gravitation

Gravitational force: $F=G \frac{m_{1} m_{2}}{r^{2}}$


Potential energy: $U=-\frac{G M m}{r}$
Gravitational acceleration: $g=\frac{G M}{R^{2}}$
Variation of g with depth: $g_{\text {inside }} \approx g\left(1-\frac{h}{R}\right)$
Variation of $\mathbf{g}$ with height: $g_{\text {outside }} \approx g\left(1-\frac{2 h}{R}\right)$
Effect of non-spherical earth shape on $g$ : $g_{\text {at pole }}>g_{\text {at equator }}\left(\because R_{\mathrm{e}}-R_{\mathrm{p}} \approx 21 \mathrm{~km}\right)$

Effect of earth rotation on apparent weight:

$$
m g_{\theta}^{\prime}=m g-m \omega^{2} R \cos ^{2} \theta
$$



Orbital velocity of satellite: $v_{o}=\sqrt{\frac{G M}{R}}$
Escape velocity: $v_{e}=\sqrt{\frac{2 G M}{R}}$

## Kepler's laws:



First: Elliptical orbit with sun at one of the focus.
Second: Areal velocity is constant. $(\because \mathrm{d} \vec{L} / \mathrm{d} t=0)$.
Third: $T^{2} \propto a^{3}$. In circular orbit $T^{2}=\frac{4 \pi^{2}}{G M} a^{3}$.

## 1.8: Simple Harmonic Motion

Hooke's law: $F=-k x$ (for small elongation $x$.)
Acceleration: $a=\frac{\mathrm{d}^{2} x}{\mathrm{~d} t^{2}}=-\frac{k}{m} x=-\omega^{2} x$
Time period: $T=\frac{2 \pi}{\omega}=2 \pi \sqrt{\frac{m}{k}}$
Displacement: $x=A \sin (\omega t+\phi)$
Velocity: $v=A \omega \cos (\omega t+\phi)= \pm \omega \sqrt{A^{2}-x^{2}}$

Potential energy: $U=\frac{1}{2} k x^{2}$


Kinetic energy $K=\frac{1}{2} m v^{2}$


Total energy: $E=U+K=\frac{1}{2} m \omega^{2} A^{2}$

Simple pendulum: $T=2 \pi \sqrt{\frac{l}{g}}$


Physical Pendulum: $T=2 \pi \sqrt{\frac{I}{m g l}}$


Stoke's law: $F=6 \pi \eta r v$

Poiseuilli's equation: $\frac{\text { Volume flow }}{\text { time }}=\frac{\pi p r^{4}}{8 \eta l}$


Terminal velocity: $v_{t}=\frac{2 r^{2}(\rho-\sigma) g}{9 \eta}$

## 2 Waves

## 2.1: Waves Motion

General equation of wave: $\frac{\partial^{2} y}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} y}{\partial t^{2}}$.
Notation: Amplitude $A$, Frequency $\nu$, Wavelength $\lambda$, Pe$\operatorname{riod} T$, Angular Frequency $\omega$, Wave Number $k$,

$$
T=\frac{1}{\nu}=\frac{2 \pi}{\omega}, \quad v=\nu \lambda, \quad k=\frac{2 \pi}{\lambda}
$$

Progressive wave travelling with speed $v$ :

$$
y=f(t-x / v), \rightsquigarrow+x ; \quad y=f(t+x / v), \rightsquigarrow-x
$$

## Progressive sine wave:



$$
y=A \sin (k x-\omega t)=A \sin (2 \pi(x / \lambda-t / T))
$$

## 2.2: Waves on a String

Speed of waves on a string with mass per unit length $\mu$ and tension $T: v=\sqrt{T / \mu}$

Transmitted power: $P_{\mathrm{av}}=2 \pi^{2} \mu v A^{2} \nu^{2}$

## Interference:

$$
\begin{aligned}
& y_{1}=A_{1} \sin (k x-\omega t), \quad y_{2}=A_{2} \sin (k x-\omega t+\delta) \\
& y=y_{1}+y_{2}=A \sin (k x-\omega t+\epsilon) \\
& A=\sqrt{{A_{1}{ }^{2}+A_{2}^{2}+2 A_{1} A_{2} \cos \delta}} \\
& \tan \epsilon=\frac{A_{2} \sin \delta}{A_{1}+A_{2} \cos \delta} \\
& \delta= \begin{cases}2 n \pi, & \text { constructive } \\
(2 n+1) \pi, & \text { destructive. }\end{cases}
\end{aligned}
$$

## Standing Waves:



$$
\begin{aligned}
& y_{1}=A_{1} \sin (k x-\omega t), \quad y_{2}=A_{2} \sin (k x+\omega t) \\
& y=y_{1}+y_{2}=(2 A \cos k x) \sin \omega t \\
& x= \begin{cases}\left(n+\frac{1}{2}\right) \frac{\lambda}{2}, & \text { nodes; } \quad n=0,1,2, \ldots \\
n \frac{\lambda}{2}, & \text { antinodes. } \quad n=0,1,2, \ldots\end{cases}
\end{aligned}
$$

## String fixed at both ends:



1. Boundary conditions: $y=0$ at $x=0$ and at $x=L$
2. Allowed Freq.: $L=n \frac{\lambda}{2}, \nu=\frac{n}{2 L} \sqrt{\frac{T}{\mu}}, n=1,2,3, \ldots$.
3. Fundamental/1 $1^{\text {st }}$ harmonics: $\nu_{0}=\frac{1}{2 L} \sqrt{\frac{T}{\mu}}$

4. $1^{\text {st }}$ overtone $/ 2^{\text {nd }}$ harmonics: $\nu_{1}=\frac{2}{2 L} \sqrt{\frac{T}{\mu}}$
5. $2^{\text {nd }}$ overtone/3 ${ }^{\text {rd }}$ harmonics: $\nu_{2}=\frac{3}{2 L} \sqrt{\frac{T}{\mu}}$
$\cdots$
6. All harmonics are present.

String fixed at one end:


1. Boundary conditions: $y=0$ at $x=0$
2. Allowed Freq.: $L=(2 n+1) \frac{\lambda}{4}, \nu=\frac{2 n+1}{4 L} \sqrt{\frac{T}{\mu}}, n=$ $0,1,2, \ldots$.
3. Fundamental $/ 1^{\text {st }}$ harmonics: $\nu_{0}=\frac{1}{4 L} \sqrt{\frac{T}{\mu}}$
4. $1^{\text {st }}$ overtone $/ 3^{\text {rd }}$ harmonics: $\nu_{1}=\frac{3}{4 L} \sqrt{\frac{T}{\mu}}$

5. $2^{\text {nd }}$ overtone $/ 5^{\text {th }}$ harmonics: $\nu_{2}=\frac{5}{4 L} \sqrt{\frac{T}{\mu}}$

6. Only odd harmonics are present.

Sonometer: $\nu \propto \frac{1}{L}, \nu \propto \sqrt{T}, \nu \propto \frac{1}{\sqrt{\mu}} . \nu=\frac{n}{2 L} \sqrt{\frac{T}{\mu}}$

## 2.3: Sound Waves

Displacement wave: $s=s_{0} \sin \omega(t-x / v)$
Pressure wave: $p=p_{0} \cos \omega(t-x / v), p_{0}=(B \omega / v) s_{0}$
Speed of sound waves:

$$
v_{\text {liquid }}=\sqrt{\frac{B}{\rho}}, \quad v_{\text {solid }}=\sqrt{\frac{Y}{\rho}}, \quad v_{\text {gas }}=\sqrt{\frac{\gamma P}{\rho}}
$$

Intensity: $I=\frac{2 \pi^{2} B}{v} s_{0}{ }^{2} \nu^{2}=\frac{p_{0}{ }^{2} v}{2 B}=\frac{p_{0}{ }^{2}}{2 \rho v}$

## Standing longitudinal waves:

$$
\begin{aligned}
& p_{1}=p_{0} \sin \omega(t-x / v), \quad p_{2}=p_{0} \sin \omega(t+x / v) \\
& p=p_{1}+p_{2}=2 p_{0} \cos k x \sin \omega t
\end{aligned}
$$

## Closed organ pipe:



1. Boundary condition: $y=0$ at $x=0$
2. Allowed freq.: $L=(2 n+1) \frac{\lambda}{4}, \nu=(2 n+1) \frac{v}{4 L}, n=$ $0,1,2, \ldots$
3. Fundamental/ $1^{\text {st }}$ harmonics: $\nu_{0}=\frac{v}{4 L}$
4. $1^{\text {st }}$ overtone $/ 3^{\text {rd }}$ harmonics: $\nu_{1}=3 \nu_{0}=\frac{3 v}{4 L}$

5. $2^{\text {nd }}$ overtone $/ 5^{\text {th }}$ harmonics: $\nu_{2}=5 \nu_{0}=\frac{5 v}{4 L}$

6. Only odd harmonics are present.

## Open organ pipe:



1. Boundary condition: $y=0$ at $x=0$

Allowed freq.: $L=n \frac{\lambda}{2}, \nu=n \frac{v}{4 L}, n=1,2, \ldots$
2. Fundamental/ $1^{\text {st }}$ harmonics: $\nu_{0}=\frac{v}{2 L}$
3. $1^{\text {st }}$ overtone $/ 2^{\text {nd }}$ harmonics: $\nu_{1}=2 \nu_{0}=\frac{2 v}{2 L}$
4. $2^{\text {nd }}$ overtone $/ 3^{\text {rd }}$ harmonics: $\nu_{2}=3 \nu_{0}=\frac{3 v}{2 L}$

5. All harmonics are present.

## Resonance column:



$$
l_{1}+d=\frac{\lambda}{2}, \quad l_{2}+d=\frac{3 \lambda}{4}, \quad v=2\left(l_{2}-l_{1}\right) \nu
$$

Beats: two waves of almost equal frequencies $\omega_{1} \approx \omega_{2}$

$$
\begin{aligned}
& p_{1}=p_{0} \sin \omega_{1}(t-x / v), \quad p_{2}=p_{0} \sin \omega_{2}(t-x / v) \\
& p=p_{1}+p_{2}=2 p_{0} \cos \Delta \omega(t-x / v) \sin \omega(t-x / v) \\
& \omega=\left(\omega_{1}+\omega_{2}\right) / 2, \quad \Delta \omega=\omega_{1}-\omega_{2} \quad \text { (beats freq.) }
\end{aligned}
$$

## Doppler Effect:

$$
\nu=\frac{v+u_{o}}{v-u_{s}} \nu_{0}
$$

where, $v$ is the speed of sound in the medium, $u_{0}$ is the speed of the observer w.r.t. the medium, considered positive when it moves towards the source and negative when it moves away from the source, and $u_{s}$ is the speed of the source w.r.t. the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

## 2.4: Light Waves

Plane Wave: $E=E_{0} \sin \omega\left(t-\frac{x}{v}\right), I=I_{0}$


Spherical Wave: $E=\frac{a E_{0}}{r} \sin \omega\left(t-\frac{r}{v}\right), I=\frac{I_{0}}{r^{2}}$


## Young's double slit experiment

Path difference: $\Delta x=\frac{d y}{D}$


Phase difference: $\delta=\frac{2 \pi}{\lambda} \Delta x$
Interference Conditions: for integer $n$,

$$
\begin{aligned}
& \delta= \begin{cases}2 n \pi, & \text { constructive; } \\
(2 n+1) \pi, & \text { destructive }\end{cases} \\
& \Delta x= \begin{cases}n \lambda, & \text { constructive } \\
\left(n+\frac{1}{2}\right) \lambda, & \text { destructive }\end{cases}
\end{aligned}
$$

## Intensity:

$$
\begin{aligned}
& I=I_{1}+I_{2}+2 \sqrt{I_{1} I_{2}} \cos \delta, \\
& I_{\max }=\left(\sqrt{I_{1}}+\sqrt{I_{2}}\right)^{2}, I_{\min }=\left(\sqrt{I_{1}}-\sqrt{I_{2}}\right)^{2} \\
& I_{1}=I_{2}: I=4 I_{0} \cos ^{2} \frac{\delta}{2}, I_{\max }=4 I_{0}, I_{\min }=0
\end{aligned}
$$

Fringe width: $w=\frac{\lambda D}{d}$
Optical path: $\Delta x^{\prime}=\mu \Delta x$

## Interference of waves transmitted through thin film:

$$
\Delta x=2 \mu d= \begin{cases}n \lambda, & \text { constructive } \\ \left(n+\frac{1}{2}\right) \lambda, & \text { destructive }\end{cases}
$$

Diffraction from a single slit:

For Minima: $n \lambda=b \sin \theta \approx b(y / D)$


Resolution: $\sin \theta=\frac{1.22 \lambda}{b}$
Law of Magus: $I=I_{0} \cos ^{2} \theta$


## 3 Optics

## 3.1: Reflection of Light

## Laws of reflection:

 incident Incident ray, reflected ray, and normal lie in the same plane (ii) $\angle i=\angle r$
## Plane mirror:


(i) the image and the object are equidistant from mirror (ii) virtual image of real object

## Spherical Mirror:



1. Focal length $f=R / 2$
2. Mirror equation: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$
3. Magnification: $m=-\frac{v}{u}$

## 3.2: Refraction of Light

Refractive index: $\mu=\frac{\text { speed of light in vacuum }}{\text { speed of light in medium }}=\frac{c}{v}$

Snell's Law: $\frac{\sin i}{\sin r}=\frac{\mu_{2}}{\mu_{1}}$


Apparent depth: $\mu=\frac{\text { real depth }}{\text { apparent depth }}=\frac{d}{d^{\prime}}$


Critical angle: $\theta_{c}=\sin ^{-1} \frac{1}{\mu}$


Deviation by a prism:


$$
\begin{aligned}
& \delta=i+i^{\prime}-A, \quad \text { general result } \\
& \mu=\frac{\sin \frac{A+\delta_{m}}{2}}{\sin \frac{A}{2}}, \quad i=i^{\prime} \text { for minimum deviation } \\
& \delta_{m}=(\mu-1) A, \quad \text { for small } A
\end{aligned}
$$

## Refraction at spherical surface:



$$
\frac{\mu_{2}}{v}-\frac{\mu_{1}}{u}=\frac{\mu_{2}-\mu_{1}}{R}, \quad m=\frac{\mu_{1} v}{\mu_{2} u}
$$

Lens maker's formula: $\frac{1}{f}=(\mu-1)\left[\frac{1}{R_{1}}-\frac{1}{R_{2}}\right]$

Lens formula: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}, \quad m=\frac{v}{u}$


Power of the lens: $P=\frac{1}{f}, P$ in diopter if $f$ in metre.
Two thin lenses separated by distance $d$ :

$$
\frac{1}{F}=\frac{1}{f_{1}}+\frac{1}{f_{2}}-\frac{d}{f_{1} f_{2}}
$$


$f_{1} \quad f_{2}$

## 3.3: Optical Instruments

Simple microscope: $m=D / f$ in normal adjustment.

## Compound microscope:



1. Magnification in normal adjustment: $m=\frac{v}{u} \frac{D}{f_{e}}$
2. Resolving power: $R=\frac{1}{\Delta d}=\frac{2 \mu \sin \theta}{\lambda}$

## Astronomical telescope:



1. In normal adjustment: $m=-\frac{f_{o}}{f_{e}}, L=f_{o}+f_{e}$
2. Resolving power: $R=\frac{1}{\Delta \theta}=\frac{1}{1.22 \lambda}$

## 3.4: Dispersion

Cauchy's equation: $\mu=\mu_{0}+\frac{A}{\lambda^{2}}, \quad A>0$
Dispersion by prism with small $A$ and $i$ :

1. Mean deviation: $\delta_{y}=\left(\mu_{y}-1\right) A$
2. Angular dispersion: $\theta=\left(\mu_{v}-\mu_{r}\right) A$

Dispersive power: $\omega=\frac{\mu_{v}-\mu_{r}}{\mu_{y}-1} \approx \frac{\theta}{\delta_{y}}$ (if $A$ and $i$ small $)$

Dispersion without deviation:


$$
\left(\mu_{y}-1\right) A+\left(\mu_{y}^{\prime}-1\right) A^{\prime}=0
$$

Deviation without dispersion:

$$
\left(\mu_{v}-\mu_{r}\right) A=\left(\mu_{v}^{\prime}-\mu_{r}^{\prime}\right) A^{\prime}
$$

## 4 Heat and Thermodynamics

## 4.1: Heat and Temperature

Temp. scales: $F=32+\frac{9}{5} C, \quad K=C+273.16$
Ideal gas equation: $p V=n R T, \quad n$ : number of moles van der Waals equation: $\left(p+\frac{a}{V^{2}}\right)(V-b)=n R T$

Thermal expansion: $L=L_{0}(1+\alpha \Delta T)$,

$$
A=A_{0}(1+\beta \Delta T), V=V_{0}(1+\gamma \Delta T), \gamma=2 \beta=3 \alpha
$$

Thermal stress of a material: $\frac{F}{A}=Y \frac{\Delta l}{l}$

## 4.2: Kinetic Theory of Gases

General: $M=m N_{A}, k=R / N_{A}$

Maxwell distribution of speed:


RMS speed: $v_{r m s}=\sqrt{\frac{3 k T}{m}}=\sqrt{\frac{3 R T}{M}}$
Average speed: $\bar{v}=\sqrt{\frac{8 k T}{\pi m}}=\sqrt{\frac{8 R T}{\pi M}}$
Most probable speed: $v_{p}=\sqrt{\frac{2 k T}{m}}$
Pressure: $p=\frac{1}{3} \rho v_{r m s}^{2}$
Equipartition of energy: $K=\frac{1}{2} k T$ for each degree of freedom. Thus, $K=\frac{f}{2} k T$ for molecule having $f$ degrees of freedoms.
Internal energy of $n$ moles of an ideal gas is $U=\frac{f}{2} n R T$.

## 4.3: Specific Heat

Specific heat: $s=\frac{Q}{m \Delta T}$
Latent heat: $L=Q / m$
Specific heat at constant volume: $C_{v}=\left.\frac{\Delta Q}{n \Delta T}\right|_{V}$
Specific heat at constant pressure: $C_{p}=\left.\frac{\Delta Q}{n \Delta T}\right|_{p}$
Relation between $C_{p}$ and $C_{v}: C_{p}-C_{v}=R$
Ratio of specific heats: $\gamma=C_{p} / C_{v}$
Relation between $U$ and $C_{v}: \Delta U=n C_{v} \Delta T$
Specific heat of gas mixture:

$$
C_{v}=\frac{n_{1} C_{v 1}+n_{2} C_{v 2}}{n_{1}+n_{2}}, \quad \gamma=\frac{n_{1} C_{p 1}+n_{2} C_{p 2}}{n_{1} C_{v 1}+n_{2} C_{v 2}}
$$

Molar internal energy of an ideal gas: $U=\frac{f}{2} R T$, $f=3$ for monatomic and $f=5$ for diatomic gas.

## 4.4: Theromodynamic Processes

First law of thermodynamics: $\Delta Q=\Delta U+\Delta W$
Work done by the gas:

$$
\begin{aligned}
& \Delta W=p \Delta V, \quad W=\int_{V_{1}}^{V_{2}} p \mathrm{~d} V \\
& W_{\text {isothermal }}=n R T \ln \left(\frac{V_{2}}{V_{1}}\right) \\
& W_{\text {isobaric }}=p\left(V_{2}-V_{1}\right) \\
& W_{\text {adiabatic }}=\frac{p_{1} V_{1}-p_{2} V_{2}}{\gamma-1} \\
& W_{\text {isochoric }}=0
\end{aligned}
$$

Efficiency of the heat engine:


$$
\begin{aligned}
& \eta=\frac{\text { work done by the engine }}{\text { heat supplied to it }}=\frac{Q_{1}-Q_{2}}{Q_{1}} \\
& \eta_{\text {carnot }}=1-\frac{Q_{2}}{Q_{1}}=1-\frac{T_{2}}{T_{1}}
\end{aligned}
$$

Coeff. of performance of refrigerator:


$$
\mathrm{COP}=\frac{Q_{2}}{W}=\frac{Q_{2}}{Q_{1}-Q_{2}}
$$

Entropy: $\Delta S=\frac{\Delta Q}{T}, S_{f}-S_{i}=\int_{i}^{f} \frac{\Delta Q}{T}$

$$
\text { Const. } T: \Delta S=\frac{Q}{T}, \quad \text { Varying } T: \Delta S=m s \ln \frac{T_{f}}{T_{i}}
$$

Adiabatic process: $\Delta Q=0, p V^{\gamma}=$ constant

## 4.5: Heat Transfer

Conduction: $\frac{\Delta Q}{\Delta t}=-K A \frac{\Delta T}{x}$
Thermal resistance: $R=\frac{x}{K A}$

$$
\begin{aligned}
& R_{\text {series }}=R_{1}+R_{2}=\frac{1}{A}\left(\frac{x_{1}}{K_{1}}+\frac{x_{2}}{K_{2}}\right) \\
& \xrightarrow[|c| c \mid]{\begin{array}{|l|l|}
\hline K_{1} & K_{2} \\
\Vdash_{x_{1}} & { }^{*} \\
x_{2}
\end{array}} A \\
& \frac{1}{R_{\text {parallel }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}=\frac{1}{x}\left(K_{1} A_{1}+K_{2} A_{2}\right) \\
& \underset{\substack{\mid K_{2} \\
\hline K_{1} \\
\underset{x}{\mid c}}}{A_{2}} A_{1}
\end{aligned}
$$

Kirchhoff's Law: $\frac{\text { emissive power }}{\text { absorptive power }}=\frac{E_{\text {body }}}{a_{\text {body }}}=E_{\text {blackbody }}$

Wien's displacement law: $\lambda_{m} T=b$


Stefan-Boltzmann law: $\frac{\Delta Q}{\Delta t}=\sigma e A T^{4}$
Newton's law of cooling: $\frac{\mathrm{d} T}{\mathrm{~d} t}=-b A\left(T-T_{0}\right)$

## 5 Electricity and Magnetism

## 5.1: Electrostatics

Coulomb's law: $\vec{F}=\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}} \hat{r}$


Electric field: $\vec{E}(\vec{r})=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r^{2}} \hat{r}$


Electrostatic energy: $U=-\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r}$
Electrostatic potential: $V=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r}$

$$
\mathrm{d} V=-\vec{E} \cdot \vec{r}, \quad V(\vec{r})=-\int_{\infty}^{\vec{r}} \vec{E} \cdot \mathrm{~d} \vec{r}
$$

Electric dipole moment: $\vec{p}=q \vec{d}$

$$
-q \stackrel{\rightharpoonup}{\xrightarrow{\vec{p}}} \multimap+q
$$

Potential of a dipole: $V=\frac{1}{4 \pi \epsilon_{0}} \frac{p \cos \theta}{r^{2}}$


Field of a dipole:

$$
E_{r}=\frac{1}{4 \pi \epsilon_{0}} \frac{2 p \cos \theta}{r^{3}}, \quad E_{\theta}=\frac{1}{4 \pi \epsilon_{0}} \frac{p \sin \theta}{r^{3}}
$$



Torque on a dipole placed in $\vec{E}: \vec{\tau}=\vec{p} \times \vec{E}$
Pot. energy of a dipole placed in $\vec{E}: U=-\vec{p} \cdot \vec{E}$

## 5.2: Gauss's Law and its Applications

Electric flux: $\phi=\oint \vec{E} \cdot \mathrm{~d} \vec{S}$
Gauss's law: $\oint \vec{E} \cdot \mathrm{~d} \vec{S}=q_{\text {in }} / \epsilon_{0}$
Field of a uniformly charged ring on its axis:

$$
E_{P}=\frac{1}{4 \pi \epsilon_{0}} \frac{q x}{\left(a^{2}+x^{2}\right)^{3 / 2}}
$$


$E$ and $V$ of a uniformly charged sphere:

$$
\begin{aligned}
& E=\left\{\begin{array}{lll}
\frac{1}{4 \pi \epsilon_{0}} \frac{Q r}{R^{3}}, & \text { for } r<R & \\
\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}}, & \text { for } r \geq R
\end{array}\right. \\
& V= \begin{cases}\frac{Q}{8 \pi \epsilon_{0} R}\left(3-\frac{r^{2}}{R^{2}}\right), & \text { for } r<R \\
\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r}, & \text { for } r \geq R\end{cases}
\end{aligned}
$$

$E$ and $V$ of a uniformly charged spherical shell:

$$
\begin{aligned}
& E= \begin{cases}0, & \text { for } r<R \\
\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r^{2}}, & \text { for } r \geq R\end{cases} \\
& V= \begin{cases}\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{R}, & \text { for } r<R \\
\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r}, & \text { for } r \geq R\end{cases}
\end{aligned}
$$

Field of a line charge: $E=\frac{\lambda}{2 \pi \epsilon_{0} r}$
Field of an infinite sheet: $E=\frac{\sigma}{2 \epsilon_{0}}$
Field in the vicinity of conducting surface: $E=\frac{\sigma}{\epsilon_{0}}$

## 5.3: Capacitors

Capacitance: $C=q / V$
Parallel plate capacitor: $C=\epsilon_{0} A / d$

Spherical capacitor: $C=\frac{4 \pi \epsilon_{0} r_{1} r_{2}}{r_{2}-r_{1}}$


Capacitors in parallel: $C_{\text {eq }}=C_{1}+C_{2}$


Capacitors in series: $\frac{1}{C_{\mathrm{eq}}}=\frac{1}{C_{1}}+\frac{1}{C_{2}}$


Force between plates of a parallel plate capacitor:

$$
F=\frac{Q^{2}}{2 A \epsilon_{0}}
$$

Energy stored in capacitor: $U=\frac{1}{2} C V^{2}=\frac{Q^{2}}{2 C}=\frac{1}{2} Q V$
Energy density in electric field $E: U / V=\frac{1}{2} \epsilon_{0} E^{2}$
Capacitor with dielectric: $C=\frac{\epsilon_{0} K A}{d}$

## 5.4: Current electricity

Current density: $j=i / A=\sigma E$
Drift speed: $v_{d}=\frac{1}{2} \frac{e E}{m} \tau=\frac{i}{n e A}$
Resistance of a wire: $R=\rho l / A$, where $\rho=1 / \sigma$
Temp. dependence of resistance: $R=R_{0}(1+\alpha \Delta T)$
Ohm's law: $V=i R$
Kirchhoff's Laws: (i) The Junction Law: The algebraic sum of all the currents directed towards a node is zero i.e., $\Sigma_{\text {node }} I_{i}=0$. (ii) The Loop Law: The algebraic sum of all the potential differences along a closed loop in a circuit is zero i.e., $\Sigma_{\text {loop }} \Delta V_{i}=0$.

Resistors in parallel: $\frac{1}{R_{\text {eq }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$


Resistors in series: $R_{\text {eq }}=R_{1}+R_{2}$


Wheatstone bridge:


Balanced if $R_{1} / R_{2}=R_{3} / R_{4}$.
Electric Power: $P=V^{2} / R=I^{2} R=I V$

Galvanometer as an Ammeter:


$$
i_{g} G=\left(i-i_{g}\right) S
$$

Galvanometer as a Voltmeter:

$$
V_{\mathrm{AB}}=i_{g}(R+G)
$$

Charging of capacitors:



$$
q(t)=C V\left[1-e^{-\frac{t}{R C}}\right]
$$

Discharging of capacitors: $q(t)=q_{0} e^{-\frac{t}{R C}}$


Time constant in RC circuit: $\tau=R C$

Peltier effect: emf $e=\frac{\Delta H}{\Delta Q}=\frac{\text { Peltier heat }}{\text { charge transferred }}$.
Seeback effect:


1. Thermo-emf: $e=a T+\frac{1}{2} b T^{2}$
2. Thermoelectric power: $\mathrm{d} e / \mathrm{d} t=a+b T$.
3. Neutral temp.: $T_{n}=-a / b$.
4. Inversion temp.: $T_{i}=-2 a / b$.

Thomson effect: emf $e=\frac{\Delta H}{\Delta Q}=\frac{\text { Thomson heat }}{\text { charge transferred }}=\sigma \Delta T$.
Faraday's law of electrolysis: The mass deposited is

$$
m=Z i t=\frac{1}{F} E i t
$$

where $i$ is current, $t$ is time, $Z$ is electrochemical equivalent, $E$ is chemical equivalent, and $F=96485 \mathrm{C} / \mathrm{g}$ is Faraday constant.

## 5.5: Magnetism

Lorentz force on a moving charge: $\vec{F}=q \vec{v} \times \vec{B}+q \vec{E}$
Charged particle in a uniform magnetic field:

$$
\underset{\vec{B} \otimes r}{v} r=\frac{m v}{q B}, T=\frac{2 \pi m}{q B}
$$

Force on a current carrying wire:


$$
\vec{F}=i \vec{l} \times \vec{B}
$$

Magnetic moment of a current loop (dipole):

$$
\underbrace{\vec{\mu} \vec{A}}_{i} \vec{\mu}=i \vec{A}
$$

Torque on a magnetic dipole placed in $\vec{B}: \vec{\tau}=\vec{\mu} \times \vec{B}$

Energy of a magnetic dipole placed in $\vec{B}$ :

$$
U=-\vec{\mu} \cdot \vec{B}
$$

Hall effect: $V_{w}=\frac{B i}{n e d}$


## 5.6: Magnetic Field due to Current

Biot-Savart law: $\mathrm{d} \vec{B}=\frac{\mu_{0}}{4 \pi} \frac{i \mathrm{~d} \vec{l} \times \vec{r}}{r^{3}}$


Field due to a straight conductor:


$$
B=\frac{\mu_{0} i}{4 \pi d}\left(\cos \theta_{1}-\cos \theta_{2}\right)
$$

Field due to an infinite straight wire: $B=\frac{\mu_{0} i}{2 \pi d}$


Field on the axis of a ring:


$$
B_{P}=\frac{\mu_{0} i a^{2}}{2\left(a^{2}+d^{2}\right)^{3 / 2}}
$$

Field at the centre of an arc: $\left.B=\frac{\mu_{0} i \theta}{4 \pi a} \quad \begin{array}{cc}\vec{B} \odot)_{\theta} \\ a\end{array}\right)_{i}$
Field at the centre of a ring: $B=\frac{\mu_{0} i}{2 a}$
Ampere's law: $\oint \vec{B} \cdot \mathrm{~d} \vec{l}=\mu_{0} I_{\text {in }}$
Field inside a solenoid: $B=\mu_{0} n i, n=\frac{N}{l}$


Field inside a toroid: $B=\frac{\mu_{0} N i}{2 \pi r}$


Field of a bar magnet:


$$
B_{1}=\frac{\mu_{0}}{4 \pi} \frac{2 M}{d^{3}}, \quad B_{2}=\frac{\mu_{0}}{4 \pi} \frac{M}{d^{3}}
$$

Angle of dip: $B_{h}=B \cos \delta$


Tangent galvanometer: $B_{h} \tan \theta=\frac{\mu_{0} n i}{2 r}, \quad i=K \tan \theta$
Moving coil galvanometer: $n i A B=k \theta, \quad i=\frac{k}{n A B} \theta$
Time period of magnetometer: $T=2 \pi \sqrt{\frac{I}{M B_{h}}}$
Permeability: $\vec{B}=\mu \vec{H}$

## 5.7: Electromagnetic Induction

Magnetic flux: $\phi=\oint \vec{B} \cdot \mathrm{~d} \vec{S}$
Faraday's law: $e=-\frac{\mathrm{d} \phi}{\mathrm{d} t}$
Lenz's Law: Induced current create a $B$-field that opposes the change in magnetic flux.

Motional emf: $e=B l v$


Self inductance: $\phi=L i, \quad e=-L \frac{\mathrm{~d} i}{\mathrm{~d} t}$
Self inductance of a solenoid: $L=\mu_{0} n^{2}\left(\pi r^{2} l\right)$
Growth of current in LR circuit: $i=\frac{e}{R}\left[1-e^{-\frac{t}{L / R}}\right]$



Decay of current in LR circuit: $i=i_{0} e^{-\frac{t}{L / R}}$


Time constant of LR circuit: $\tau=L / R$
Energy stored in an inductor: $U=\frac{1}{2} L i^{2}$
Energy density of $B$ field: $u=\frac{U}{V}=\frac{B^{2}}{2 \mu_{0}}$
Mutual inductance: $\phi=M i, \quad e=-M \frac{\mathrm{~d} i}{\mathrm{~d} t}$

EMF induced in a rotating coil: $e=N A B \omega \sin \omega t$

## Alternating current:



$$
i=i_{0} \sin (\omega t+\phi), \quad T=2 \pi / \omega
$$

Average current in AC: $\bar{i}=\frac{1}{T} \int_{0}^{T} i \mathrm{~d} t=0$

Energy: $E=i_{\mathrm{rms}}{ }^{2} R T$
Capacitive reactance: $X_{c}=\frac{1}{\omega C}$
Inductive reactance: $X_{L}=\omega L$
Imepedance: $Z=e_{0} / i_{0}$

RC circuit:


$$
Z=\sqrt{R^{2}+(1 / \omega C)^{2}}, \quad \tan \phi=\frac{1}{\omega C R}
$$

LR circuit:


$$
Z=\sqrt{R^{2}+\omega^{2} L^{2}}, \quad \tan \phi=\frac{\omega L}{R}
$$

LCR Circuit:


$$
\begin{aligned}
& Z=\sqrt{R^{2}+\left(\frac{1}{\omega C}-\omega L\right)^{2}}, \quad \tan \phi=\frac{\frac{1}{\omega C}-\omega L}{R} \\
& \nu_{\text {resonance }}=\frac{1}{2 \pi} \sqrt{\frac{1}{L C}}
\end{aligned}
$$

Power factor: $P=e_{r m s} i_{r m s} \cos \phi$

Transformer: $\frac{N_{1}}{N_{2}}=\frac{e_{1}}{e_{2}}, e_{1} i_{1}=e_{2} i_{2}$


Speed of the EM waves in vacuum: $c=1 / \sqrt{\mu_{0} \epsilon_{0}}$

## 6 Modern Physics

## 6.1: Photo-electric effect

Photon's energy: $E=h \nu=h c / \lambda$
Photon's momentum: $p=h / \lambda=E / c$
Max. KE of ejected photo-electron: $K_{\max }=h \nu-\phi$
Threshold freq. in photo-electric effect: $\nu_{0}=\phi / h$

Stopping potential: $V_{o}=\frac{h c}{e}\left(\frac{1}{\lambda}\right)-\frac{\phi}{e}$

de Broglie wavelength: $\lambda=h / p$

## 6.2: The Atom

Energy in $n$th Bohr's orbit:

$$
E_{n}=-\frac{m Z^{2} e^{4}}{8 \epsilon_{0}^{2} h^{2} n^{2}}, \quad E_{n}=-\frac{13.6 Z^{2}}{n^{2}} \mathrm{eV}
$$

Radius of the $n$th Bohr's orbit:

$$
r_{n}=\frac{\epsilon_{0} h^{2} n^{2}}{\pi m Z e^{2}}, \quad r_{n}=\frac{n^{2} a_{0}}{Z}, \quad a_{0}=0.529 \AA
$$

Quantization of the angular momentum: $l=\frac{n h}{2 \pi}$
Photon energy in state transition: $E_{2}-E_{1}=h \nu$

Wavelength of emitted radiation: for a transition from $n$th to $m$ th state:

$$
\frac{1}{\lambda}=R Z^{2}\left[\frac{1}{n^{2}}-\frac{1}{m^{2}}\right]
$$

X-ray spectrum: $\lambda_{\text {min }}=\frac{h c}{e V}$


Moseley's law: $\sqrt{\nu}=a(Z-b)$
X-ray diffraction: $2 d \sin \theta=n \lambda$
Heisenberg uncertainity principle:
$\Delta p \Delta x \geq h /(2 \pi), \quad \Delta E \Delta t \geq h /(2 \pi)$

## 6.3: The Nucleus

Nuclear radius: $R=R_{0} A^{1 / 3}, \quad R_{0} \approx 1.1 \times 10^{-15} \mathrm{~m}$
Decay rate: $\frac{\mathrm{d} N}{\mathrm{~d} t}=-\lambda N$

Population at time $t: N=N_{0} e^{-\lambda t}$


Half life: $t_{1 / 2}=0.693 / \lambda$
Average life: $t_{\mathrm{av}}=1 / \lambda$
Population after $n$ half lives: $N=N_{0} / 2^{n}$.
Mass defect: $\Delta m=\left[Z m_{p}+(A-Z) m_{n}\right]-M$
Binding energy: $B=\left[Z m_{p}+(A-Z) m_{n}-M\right] c^{2}$
$Q$-value: $Q=U_{i}-U_{f}$
Energy released in nuclear reaction: $\Delta E=\Delta m c^{2}$ where $\Delta m=m_{\text {reactants }}-m_{\text {products }}$.

## 6.4: Vacuum tubes and Semiconductors

Half Wave Rectifier:


Full Wave Rectifier:


Triode Valve:


Plate resistance of a triode: $r_{p}=\left.\frac{\Delta V_{p}}{\Delta i_{p}}\right|_{\Delta V_{g}=0}$
Transconductance of a triode: $g_{m}=\left.\frac{\Delta i_{p}}{\Delta V_{g}}\right|_{\Delta V_{p}=0}$
Amplification by a triode: $\mu=-\left.\frac{\Delta V_{p}}{\Delta V_{g}}\right|_{\Delta i_{p}=0}$
Relation between $r_{p}, \mu$, and $g_{m}: \mu=r_{p} \times g_{m}$

Current in a transistor: $I_{e}=I_{b}+I_{c}$

$\alpha$ and $\beta$ parameters of a transistor: $\alpha=\frac{I_{c}}{I_{e}}, \quad \beta=$ $\frac{I_{c}}{I_{b}}, \beta=\frac{\alpha}{1-\alpha}$

Transconductance: $g_{m}=\frac{\Delta I_{c}}{\Delta V_{b e}}$
Logic Gates:

|  |  | AND |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | AB | OR | $\mathrm{A}+\mathrm{B}$ | NAND |  |
| $\overline{\mathrm{AB}}$ | NOR | $\mathrm{A}+\mathrm{B}$ | $\mathrm{A} \overline{\mathrm{B}}+\overline{\mathrm{A}} \mathrm{B}$ |  |  |  |
| 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |

