1 RELATIONS AND FUNCTIONS

KEY CONCEPT INVOLVED

1. **Relations -** Let A and B be two non-empty sets then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.
   Let $R \subseteq A \times B$ and $(a, b) \in R$. then we say that a is related to b by the relation R as aRb. If $(a, b) \notin R$ as $a \not{R} b$.
2. **Domain and Range of a Relation -** Let R be a relation from A to B, that is, let $R \subset A \times B$. then *Domain R* = {a : $a \in A, (a, b) \in R$ for some $b \in B$} i.e. dom. R is the set of all the first elements of the ordered pairs which belong to R. *Range R* = (b : $b \in B, (a, b) \in R$ for some $a \in A$} i.e. range R is the set of all the second elements of the ordered pairs which belong to R. Thus Dom. $R \subset A$, Range $R \subset B$.
3. **Inverse Relation -** Let $R \subset A \times B$ be a relation from A to B. Then inverse relation $R^{-1} \subset B \times A$ is defined by R^{-1} $\{(b, a) : (a, b) \in R\}$
   It is clear that
   (i) $aRb = bR^{-1} a$
   (ii) dom. R^{-1} = range R and range R^{-1} = dom R.
   (iii) $(R^{-1})^{-1} = R$.
4. **Composition of Relation -** Let $R \subset A \times B$, $S \subset B \times C$ be two relations. Then composition of the relations R and S is denoted by $SoR \subset A \times C$ and is defined by $(a, c) \in (SoR)$ iff $b \in B$ such that $(a, b) \in R$, $(b, c) \in S$.
5. **Relations in a set -** let $A (\neq \phi)$ be a set and $R \subset A \times A$ i.e. R is a relation in the set A.
6. **Reflexive Relations -** R is a reflexive relation if $(a, a) \in R$, $\forall$ $a \in R$ it should be noted that if for any $a \in A$ such that $a \not{R} a$. then R is not reflexive.
7. **Symmetric Relation -** R is called symmetric relation on A if $(x, y) \in R \Rightarrow (y, x) \in R$.
   i.e. if x is related to y, then y is also related to x.
   It should be noted that R is symmetric iff $R^{-1} = R$.
8. **Anti Symmetric Relations -** R is called an anti symmetric relation if $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$.
   Thus if $a \neq b$ then a may be related to b or b may be related to a but never both.
9. **Transitive Relations -** R is called a transitive relation if $(a, b) \in R$ $(b, c) \in R \Rightarrow (a, c) \in R$
10. **Identity Relations -** R is an identity relation if $(a, b) \in R$ iff $a = b$. i.e. every element of A is related to only itself and always identity relation is reflexive symmetric and transitive.
11. **Equivalence Relations -** a relation R in a set A is called an equivalence relation if
    (i) R is reflexive i.e. $(a, a) \in R$ $\forall$ $a \in A$
    (ii) R is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$
    (iii) R is transitive i.e. $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.
12. **Functions -** Suppose that to each element in a set A there is assigned, by some rule, an unique element of a set B. Such rules are called functions. If we let f denote these rules, then we write $f : A \to B$ as f is a function of A into B.
13. **Equal Functions -** If f and g are functions defined on the same domain A and if $f(a) = g(a)$ for every $a \in A$, then $f = g$.

14. **Constant Functions -** Let $f : A \to B$. If $f(a) = b$, a constant, for all $a \in A$, then f is called a constant function. Thus f is called a constant function if range f consists of only one element.
15. **Identity Functions -** A function f is such that $A \to A$ is called an identity function if $f(x) = x$, $\forall\ x \in A$ it is denoted by I_A.
16. **One-One Functions (Injective) -** Let $f : A \to B$ then f is called a one-one function. If no two different elements in A have the same image i.e. different elements in A have different elements in B.
Denoted by symbol f is one-one if
$$f(a) = f(a') \Rightarrow a = a'$$
i.e. $$a \neq a' \Rightarrow f(a) \neq f(a')$$
A mapping which is not one-one is called many one function.
17. **Onto functions (Surjective) -** In the mapping $f : A \to B$, if every member of B appears as the image of atleast one element of A, then we say "f is a function of A onto B or simply f is an onto functions" Thus f is onto iff $f(A) = B$
i.e. range = codomain
A function which is not onto is called into function.
18. **Inverse of a function -** Let $f : A \to B$ and $b \in B$ then the inverse of b i.e. $f^{-1}(b)$ consists of those elements in A which are mapped onto b i.e. $f^{-1}(b) = \{x\ ;\ x \in A, f(x) \in b\}$
$\therefore\ f^{-1}(b) \subset A$, $f^{-1}(b)$ may be a null set or a singleton.
19. **Inverse Functions -** Let $f : A \to B$ be a one-one onto-function from A onto B. Then for each $b \in B$. $f^{-1}(b) \in A$ and is unique. So, $f^{-1} : B \to A$ is a function defined by $f^{-1}(b) = a$, iff $f(a) = b$.
Then f^{-1} is called the inverse function of f. If f has inverse function, f is also called invertible or non-singular.
Thus f is invertible (non-singular) iff it is one-one onto (bijective) function.
20. **Composition Functions -** Let $f : A \to B$ and $g : B \to C$, be two functions,
Then composition of f and g denoted by $gof : A \to C$ is defined by $(gof)(a) = g\{f(a)\}$.
21. **Binary Operation -** A binary operation $*$ on a set A is a function $* : A \times A \to A$. We denote $*(a, b)$ by $a * b$
22. **Commutative Binary Operation -** A binary operation $*$ on the set A is commutative if for every $a, b \in A$, $a * b = b * a$.
23. **Associative Binary Operation -** A binary operation $*$ on the set A is associative if
$(a * b) * c = a * (b * c)$.
24. **An Identity Element e for Binary Operation -** Let $* :\ A \times A \to A$ be a binary operation. There exists an element $e \in A$ such that $a * e = a = e * a\ \forall\ a \in A$, then e is called an identity element for Binary Operation $*$.
25. **Inverse of an Element a -** Let $* : A \times A \to A$ be a binary operation with identity element e in A. an element $a \in A$ is invertible w.r.t. binary operation $*$, if there exists an element b in A such that $a * b = e = b * a$. and b is called the inverse of a and is denoted by a^{-1}.

CONNECTING CONCEPTS

1. In general $gof \neq fog$.
2. $f : A \to B$, be one-one, onto then
$f^{-1} of = I_A$ and $fof^{-1} = I_B$
3. $f : A \to B$, $g : B \to C$, $h : C \to D$
then $(hog)\ of = ho\ (gof)$.
4. $f : A \to B$, $g : B \to C$ be one-one and onto then $gof : A \to C$ is also one-one onto and $(gof)^{-1} = f^{-1} o\ g^{-1}$.
5. Let : $A \to B$, then I_B of $= f$ and $foI_A = f$. It should be noted that foI_B is not defined since for $(foI_B)(x) = fo\ \{I_B(x)\} = f(x)$
$I_B(x)$ exist when $x \in B$ and $f(x)$ exist when $x \in A$
6. $f : A \to B$, $g : B \to C$ are both one-one, then $gof : A \to C$ is also one-one it should be noted that for gof to be one-one f must be one-one.
7. If $f : A \to B$ $g : B \to C$ are both onto then gof must be onto. However, the converse is not true. But for gof to be onto g must be onto.

8. The domain of the functions

$$(f+g)(x) = f(x)+g(x)$$
$$(f-g)(x) = f(x)-g(x)$$
$$(fg)(x) = f(x)\,g(x)$$

is given by (dom. f) $\cap$ (dom g) while domain of the function $(f/g)(x) = \dfrac{f(x)}{g(x)}$ is given by.

(dom f) $\cap$ (dom. g) $- \{x : g(x) = 0\}$

9. If O (A) = m, O (B) = n, then total number of mappings from A to B is n^m.

10. If A and B are finite sets and O (A) = m, O (B) = n, $m \le n$.

Then number of injection (one-one) from A to B is $^nP_m = \dfrac{n!}{(n-m)!}$

11. If $f : A \to B$ is injective (one-one), then $O(A) \le O(B)$.

12. If $f : A \to B$ is surjective (onto), then $O(A) \ge O(B)$.

13. If $f : A \to B$ is bijective (one-one onto), then O (A) = O (B).

14. Let $f : A \to B$ and O (A) = O (B), then f is one-one $\Leftrightarrow$ it is onto.

15. Let $f : A \to B$ and $X_1, X_2 \subseteq A$, then f is one-one iff $f(X_1 \cap X_2) = f(X_1) \cap f(X_2)$

16. Let $f : A \to B$ and $X \subseteq A$, $Y \subseteq B$, then in general $f^{-1}(f(x)) \subseteq X$, $f(f^{-1}(y)) \subseteq Y$

If f is one-one onto $f^{-1}(f(x)) = x$, $f(f^{-1}(y)) = Y$.

Class 12 Maths NCERT Solutions

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Integrals Class 12 Ex 7.8	Continuity	
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Chapter 9 Differential Equations	**Integrals**	
Chapter 10 Vector Algebra	Types of Integrals	
Chapter 11 Three Dimensional Geometry	**Differential Equation**	
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13.1	Equations	
Probability Solutions Ex 13.2	**Vector Algebra**	
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Chapter 7: Adjoint and Inverse of a Matrix	Chapter 18: Maxima and Minima	Chapter 29: The plane
Chapter 8: Solution of Simultaneous Linear Equations	Chapter 19: Indefinite Integrals	Chapter 30: Linear programming
Chapter 9: Continuity	Chapter 20: Definite Integrals	Chapter 31: Probability
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Chapter 11: Differentiation	Chapter 22: Differential Equations	Chapter 33: Binomial Distribution

JEE Main Maths Chapter wise Previous Year Questions

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