## 10 <br> VECTOR ALGEBRA

## KEY CONCEPT INVOLVED

1. Vector - A vector is a quantity having both magnitude and direction, such as displacement, velocity, force and acceleration.
$A B$ is a directed line segment. It is a vector $\overrightarrow{A B}$ and its direction is from $A$ to $B$.

$$
\mathrm{A} \longrightarrow \mathrm{~B}
$$

Initial Points - The point A where from the vector $\overrightarrow{\mathrm{AB}}$ starts is known as initial point.
Terminal Point - The point B, where it ends is said to be the terminal point.
Magnitude - The distance between initial point and terminal point of a vector is the magnitude or length of the vector $\overrightarrow{\mathrm{AB}}$. It is denoted by $|\overrightarrow{\mathrm{AB}}|$ or AB .
2. Position Vector - Consider a point $p(x, y, z)$ in space. The vector $\overrightarrow{O P}$ with initial point, origin $O$ and terminal point P , is called the position vector of P .

3. Types of Vectors
(i) Zero Vector Or Null Vector - A vector whose initial and terminal points coincide is known as zero vector ( $\overrightarrow{\mathrm{O}}$ ).
(ii) Unit Vector - A vector whose magnitude is unity is said to be unit vector. It is denoted as â so that $|\hat{a}|=1$.
(iii) Co-initial Vectors - Two or more vectors having the same initial point are called co-initialvectors.
(iv) Collinear Vectors - If two or more vectors are parallel to the same line, such vectors are known as collinear vectors.
(v) Equal Vectors - If two vectors $\vec{a}$ and $\vec{b}$ have the same magnitude and direction regardless of the positions of their initial points, such vectors are said to be equal i.e., $\vec{a}=\vec{b}$.
(vi) Negative of a vector - A vector whose magnitude is same as that of a given vector $\overrightarrow{A B}$, but the direction is opposite to that of it, is known as negative of vector $\overrightarrow{\mathrm{AB}}$ i.e., $\overrightarrow{\mathrm{BA}}=-\overrightarrow{\mathrm{AB}}$

## 4. Sum of Vectors

(i) Sum of vectors $\vec{a}$ and $\vec{b}$ let the vectors $\vec{a}$ and $\vec{b}$ be so positioned that initial point of one coincides with terminal point of the other. If $\vec{a}=\overrightarrow{A B}, \vec{b}=\overrightarrow{B C}$. Then the vector $\vec{a}+\vec{b}$ is represented by the third side of $\Delta \mathrm{ABC}$. i.e., $\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}=\overrightarrow{\mathrm{AC}}$


This is known as the triangle law of vector addition.
Further $\overrightarrow{\mathrm{AC}}=-\overrightarrow{\mathrm{CA}}$

$$
\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}=-\overrightarrow{\mathrm{CA}} \quad \therefore \quad \overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}+\overrightarrow{\mathrm{CA}}=0
$$

when sides of a triangle ABC are taken in order i.e. initial and terminal points coincides. Then

$$
\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}+\overrightarrow{\mathrm{CA}}=0
$$

(ii) Parallelogram law of vector addition - If the two vectors $\vec{a}$ and $\vec{b}$ are represented by the two adjacent sides OA and OB of a parallelogram OACB, then their sum $\vec{a}+\vec{b}$ is represented in magnitude and direction by the diagonal OC of parallelogram through their common point O i.e., $\overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{OB}}=\overrightarrow{\mathrm{OC}}$

5. Multiplication of Vector by a Scalar - Let $\vec{a}$ be the given vector and $\lambda$ be a scalar, then product of $\lambda$ and $\vec{a}=\lambda \vec{a}$
(i) when $\lambda$ is +ve , then $\vec{a}$ and $\lambda \vec{a}$ are in the same direction.
(ii) when $\lambda$ is -ve. then $\vec{a}$ and $\lambda \vec{a}$ are in the opposite direction. Also $|\lambda \vec{a}|=|\lambda||\vec{a}|$.
6. Components of Vector - Let us take the points $\mathrm{A}(1,0,0), \mathrm{B}(0,1,0)$ and $\mathrm{C}(0,0,1)$ on the coordinate axes $\mathrm{OX}, \mathrm{OY}$ and OZ respectively. Now, $|\overrightarrow{\mathrm{OA}}|=1,|\overrightarrow{\mathrm{OB}}|=1$ and $|\overrightarrow{\mathrm{OC}}|=1$, Vectors $\overrightarrow{\mathrm{OA}}, \overrightarrow{\mathrm{OB}}$ and $\overrightarrow{\mathrm{OC}}$ each having magnitude 1 is known as unit vector. These are denoted by $\hat{\mathrm{i}}, \hat{\mathrm{j}}$ and $\hat{\mathrm{k}}$.


Consider the vector $\overrightarrow{\mathrm{OP}}$, where P is the point $(\mathrm{x}, \mathrm{y}, \mathrm{z})$. Now OQ , OR , OS are the projections of OP on coordinates axes.
$\therefore \quad \mathrm{OQ}=\mathrm{x}, \mathrm{OR}=\mathrm{y}, \mathrm{OS}=\mathrm{z}$
$\therefore \quad \overrightarrow{\mathrm{OQ}}=x \hat{\mathrm{i}}, \quad \overrightarrow{\mathrm{OR}}=y \hat{\mathrm{j}}, \quad \overrightarrow{\mathrm{OS}}=\mathrm{z} \hat{\mathrm{k}}$


$$
\Rightarrow \quad \overrightarrow{\mathrm{OP}}=x \hat{\mathrm{i}},+y \hat{\mathrm{j}},+\mathrm{z} \hat{\mathrm{k}}, \quad|\overrightarrow{\mathrm{OP}}|=\sqrt{\mathrm{x}^{2}+y^{2}+\mathrm{z}^{2}}=|\overrightarrow{\mathrm{r}}|
$$

$x, y, z$ are called the scalar components and $x \hat{i}, y \hat{j}, z \hat{k}$ are called the vector components of vector $\overrightarrow{\mathrm{OP}}$.
7. Vector joining two points - Let $\mathrm{P}_{1}\left(\mathrm{x}_{1}, \mathrm{y}_{1}, \mathrm{z}_{1}\right)$ and $\mathrm{P}_{2}\left(\mathrm{x}_{2}, \mathrm{y}_{2} \mathrm{z}_{2}\right)$ be the two points. Then vector joining the points $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ is $\overrightarrow{\mathrm{P}_{1} \mathrm{P}_{2}}$. Join $\mathrm{P}_{1}, \mathrm{P}_{2}$ with O . Now $\overrightarrow{\mathrm{OP}}_{2}=\overrightarrow{\mathrm{OP}}_{1}+\overrightarrow{\mathrm{P}_{1} \mathrm{P}_{2}}$ (by triangle law)


$$
\begin{array}{ll}
\therefore \quad & \overrightarrow{\mathrm{P}_{1} \mathrm{P}_{2}}=\overrightarrow{\mathrm{OP}}_{2}-\overrightarrow{\mathrm{OP}}_{1} \\
& =\left(x_{2} \hat{\mathrm{i}}+y_{2} \hat{\mathrm{j}}+z_{2} \hat{\mathrm{k}}\right)-\left(x_{1} \hat{\mathrm{i}}+y_{1} \hat{\mathrm{j}}+z_{1} \hat{\mathrm{k}}\right)=\left(x_{2}-x_{1}\right) \hat{\mathrm{i}}+\left(\mathrm{y}_{2}-y_{1}\right) \hat{\mathrm{j}}+\left(z_{2}-z_{1}\right) \hat{\mathrm{k}} \\
& \left|\overrightarrow{\mathrm{P}_{1} \mathrm{P}_{2}}\right|=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}}
\end{array}
$$

## 8. Section Formula

(i) A line segment PQ is divided by a point R in the ratio $\mathrm{m}: \mathrm{n}$ internally i.e., $\frac{P R}{R Q}=\frac{m}{n}$


If $\vec{a}$ and $\vec{b}$ are the position vectors of $P$ and $Q$ then the position vector $\vec{r}$ of $R$ is given by

$$
\overrightarrow{\mathrm{r}}=\frac{\mathrm{m} \overrightarrow{\mathrm{~b}}+\mathrm{na}}{\mathrm{~m}+\mathrm{n}}
$$

If $R$ be the mid-point of $P Q$, then $\vec{r}=\frac{\vec{a}+\vec{b}}{2}$
(ii) when $R$ divides PQ externally, i.e., $|\vec{a}||\cdot \vec{b}| \hat{n}$


Then $\overrightarrow{\mathrm{r}}=\frac{\mathrm{m} \overrightarrow{\mathrm{b}}-\mathrm{n} \overrightarrow{\mathrm{a}}}{\mathrm{m}-\mathrm{n}}$
9. Projection of vector along a directed line - Let the vector $\overrightarrow{\mathrm{AB}}$ makes an angle $\theta$ with directed line $\ell$.

Projection of AB on $\ell=|\overrightarrow{\mathrm{AB}}| \cos \theta=\overrightarrow{\mathrm{AC}}=\overrightarrow{\mathrm{p}}$.


The vector $\overrightarrow{\mathrm{p}}$ is called the projection vector. Its magnitudes is $\mid \overrightarrow{\mathrm{b}}$, which is known as projection of vector $\overrightarrow{\mathrm{AB}}$. The angle $\theta$ between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ is given by

$$
\begin{aligned}
\cos \theta & =\frac{\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{AC}}}{|\overrightarrow{\mathrm{AB}}||\overrightarrow{\mathrm{AC}}|}, \quad \text { Now projection } \mathrm{AC}=|\overrightarrow{\mathrm{AB}}| \cos \theta=\frac{\overrightarrow{\mathrm{AB}} \cdot \overrightarrow{\mathrm{AC}}}{|\overrightarrow{\mathrm{AC}}|} \\
& =\overrightarrow{\mathrm{AB}} \cdot\left(\frac{\overrightarrow{\mathrm{AC}}}{|\overrightarrow{\mathrm{AC}}|}\right), \quad \text { If } \overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{a}}, \text { then } \overrightarrow{\mathrm{AC}}=\overrightarrow{\mathrm{a}} \cdot\left(\frac{\overrightarrow{\mathrm{p}}}{|\overrightarrow{\mathrm{p}}|}\right)=\overrightarrow{\mathrm{a}} \cdot \hat{\mathrm{p}}
\end{aligned}
$$

Thus, the projection of $\vec{a}$ on $\vec{b}=\vec{a} \cdot\left(\frac{\vec{b}}{|\vec{b}|}\right)=\vec{a} \cdot \hat{b}$
10. Scalar Product of Two Vectors (Dot Product) - Scalar Product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a} \cdot \vec{b}=|\vec{a}||\vec{b}| \cos \theta$
Where $\theta$ is the angle between $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}(0 \leq \theta \leq \pi)$
(i) when $\theta=0$, then $\vec{a} \cdot \vec{b}=|\vec{a}||\vec{b}|=a b \quad$ Also $\vec{a} \cdot \vec{a}=|\vec{a}||\vec{a}|=a \cdot a=a^{2}$

$$
\therefore \quad \hat{\mathrm{i}} \cdot \hat{\mathrm{i}}=\hat{\mathrm{j}} \cdot \hat{\mathrm{j}}=\hat{\mathrm{k}} \cdot \hat{\mathrm{k}}=1
$$

(ii) when $\theta=\frac{\pi}{2}$, then $\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{b}}=|\overrightarrow{\mathrm{a}}||\overrightarrow{\mathrm{b}}| \cos \frac{\pi}{2}=0$

$$
\hat{\mathrm{i}} \cdot \hat{\mathrm{j}}=\hat{\mathrm{j}} \cdot \hat{\mathrm{k}}=\hat{\mathrm{k}} \cdot \hat{\mathrm{i}}=0
$$

11. Vector Product of two Vectors (Cross Product) - The vector product of two non-zero vectors $\vec{a}$ and $\vec{b}$, denoted by $\vec{a} \times \vec{b}$ is defined as
$\vec{a} \times \vec{b}=|\vec{a}||\vec{b}| \sin \theta \cdot \hat{n}$, where $\theta$ is the angle between $\vec{a}$ and $\vec{b}, 0 \leq \theta \leq \pi$.
Unit vector $\hat{n}$ is perpendicular to both vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a} \cdot \vec{b}$ and $\hat{n}$ form a right handed orthogonal system.
(i) If $\theta=0$, then $\vec{a} \times \vec{b}=0, \quad \therefore \vec{a} \times \vec{a}=0$
and $\therefore \hat{i} \times \hat{i}=\hat{j} \times \hat{j}=\hat{k} \times \hat{k}=0$
(ii) If $\theta=\Pi / 2$, then $\vec{a} \times \vec{b}=|\vec{a}||\vec{b}| \hat{n}$
$\hat{i} \times \hat{j}=\hat{k}, \hat{j} \times \hat{k}=\hat{i}, \hat{k} \times \hat{i}=\hat{j}$
Also, $\hat{\mathrm{j}} \times \hat{\mathrm{i}}=-\hat{\mathrm{k}}, \hat{\mathrm{k}} \times \hat{\mathrm{j}}=-\hat{\mathrm{i}}$ and $\hat{\mathrm{i}} \times \hat{\mathrm{k}}=\hat{\mathrm{j}}$

## CONNECTING CONCEPTS

1. Direction Cosines - Let $\mathrm{OX}, \mathrm{OY}, \mathrm{OZ}$ be the positive coordinate axes, $\mathrm{P}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ by any point in the space. Let $\overrightarrow{\mathrm{OP}}$ makes angles $\alpha, \beta, \gamma$ with coordinate, axes OX, OY, OZ. The angle $\alpha, \beta, \gamma$ are known as direction angles, cosine of theseangles i.e.,

$\cos \alpha, \cos \beta, \cos \gamma$ are called direction cosines of line OP. these direction cosines are denoted by $\ell, \mathrm{m}, \mathrm{n}$ i.e., $\ell=\cos \alpha, m=\cos \beta, n=\cos \gamma$
2. Relation Between, $1, m, n$ and Direction Ratios-

The perpendiculars $\mathrm{PA}, \mathrm{PB}, \mathrm{PC}$ are drawn on coordinate axes $\mathrm{OX}, \mathrm{OY}, \mathrm{OZ}$ reprectively. Let $|\overrightarrow{\mathrm{OP}}|=\mathrm{r}$
In $\triangle \mathrm{OAP}, \angle \mathrm{A}=90^{\circ}, \cos \alpha=\frac{\mathrm{x}}{\mathrm{r}}=\ell, \therefore \mathrm{x}=\ell \mathrm{r}, \quad$ In $\triangle \mathrm{OBP} . \angle \mathrm{B}=90^{\circ}, \cos \beta=\frac{\mathrm{y}}{\mathrm{r}}=\mathrm{m} \quad \therefore \mathrm{y}=\mathrm{mr}$
In $\triangle \mathrm{OCP}, \angle \mathrm{C}=90^{\circ}, \cos \gamma=\frac{\mathrm{z}}{\mathrm{r}}=\mathrm{n}, \quad \therefore \mathrm{z}=\mathrm{nr}$
Thus the coordinates of P may b expressed as ( $\ell \mathrm{r}, \mathrm{mr}, \mathrm{nr}$ )
Also, $\mathrm{OP}^{2}=\mathrm{x}^{2}+\mathrm{y}^{2}+\mathrm{z}^{2}, \mathrm{r}^{2}=(\mathrm{lr})^{2}+(\mathrm{mr})^{2}+(\mathrm{nr})^{2} \Rightarrow \ell^{2}+\mathrm{m}^{2}+\mathrm{n}^{2}=1$
Set of any there numbers, which are proportional to direction cosines are called direction ratio of the vactor. Direction ratio are denoted by $\mathrm{a}, \mathrm{b}$ and c .
The numbers $\ell \mathrm{r} \mathrm{mr}$ and nr , proportional to the direction cosines, hence, they are also direction ratios of vector $\overrightarrow{\mathrm{OP}}$.

## 3. Properties of Vector Addition -

1. For two vectors $\vec{a}, \vec{b}$ the sum is commutative i.e., $\vec{a}+\vec{b}=\vec{b}+\vec{a}$
2. For three vectors $\vec{a}, \vec{b}$ and $\vec{c}$, the sum of vectors is associative i.e.,

$$
(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})
$$

4. Additive Inverse of Vector $\overrightarrow{\mathbf{a}}-$ If there exists vector $-\vec{a}$ such that $\vec{a}+(-\vec{a})=\vec{a}-\vec{a}=\overrightarrow{0}$ then $-\vec{a}$ is called the additure inverse of $\vec{a}$
5. Some Properties - Let $\vec{a}=a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}$ and $\vec{b}=b_{1} \hat{i}+b_{2} \hat{j}+b_{3} \hat{k}$
(i) $\vec{a}+\vec{b}=\left(a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}\right)+\left(b_{1} \hat{i}+b_{2} \hat{j}+b_{3} \hat{k}\right)=\left(a_{1}+b_{1}\right) \hat{i}+\left(a_{2}+b_{2}\right) \hat{j}+\left(a_{3}+b_{3}\right) \hat{k}$
(ii) $\vec{a}=\vec{b}$ or $\left(a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}\right)=\left(b_{1} \hat{i}+b_{2} \hat{j}+b_{3} \hat{k}\right) \quad \Rightarrow a_{1}=b_{1}, a_{2}=b_{2}, a_{3}=b_{3}$
(iii) $\lambda \vec{a}=\lambda\left(a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}\right)=\left(\lambda a_{1}\right) \hat{i}+\left(\lambda a_{2}\right) \hat{j}+\left(\lambda a_{3}\right) \hat{k}$
(iv) $\vec{a}$ and $\vec{b}$ are parallel, if and only if there exists a non zero scalar $\lambda$ such that $\vec{b}=\lambda \vec{a}$

$$
\begin{aligned}
& \text { i.e., } \quad \mathrm{b}_{1} \hat{\mathrm{i}}+\mathrm{b}_{2} \hat{\mathrm{j}}+\mathrm{b}_{3} \hat{\mathrm{k}}=\lambda\left(\mathrm{a}_{1} \hat{\mathrm{i}}+\mathrm{a}_{2} \hat{\mathrm{j}}+\mathrm{a}_{3} \hat{\mathrm{k}}\right)=\left(\lambda \mathrm{a}_{1}\right) \hat{\mathrm{i}}+\left(\lambda \mathrm{a}_{2}\right) \hat{\mathrm{j}}+\left(\lambda \mathrm{a}_{3}\right) \hat{\mathrm{k}} \\
& \therefore \quad \mathrm{~b}_{1}=\lambda \mathrm{a}_{1}, \quad, \mathrm{~b}_{2}=\lambda \mathrm{a}_{2}, \mathrm{~b}_{3}=\lambda \mathrm{a}_{3} \quad \therefore \frac{\mathrm{~b}_{1}}{\mathrm{a}_{1}}=\frac{\mathrm{b}_{2}}{\mathrm{a}_{2}}=\frac{\mathrm{b}_{3}}{\mathrm{a}_{3}}=\lambda
\end{aligned}
$$

6. Properties of scalar product of two vectors (Dot Product)
(i) $\cos \theta=\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

If $\vec{a}=a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}$ and $\vec{b}=b_{1} \hat{i}+b_{2} \hat{j}+b_{3} \hat{k}$
Then, $\vec{a} \cdot \vec{b}=\left(a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}\right) \cdot\left(b_{1} \hat{i}+b_{2} \hat{j}+b_{3} \hat{k}\right), \vec{a} \cdot \vec{b}=a_{1} b_{1}+a_{2} b_{2}+a_{3} b_{3}$

$$
|\overrightarrow{\mathrm{a}}|=\sqrt{\mathrm{a}_{1}^{2}+\mathrm{a}_{2}^{2}+\mathrm{a}_{3}^{2}},|\overrightarrow{\mathrm{~b}}|=\sqrt{\mathrm{b}_{1}^{2}+\mathrm{b}_{2}^{2}+\mathrm{b}_{3}^{2}} \quad \therefore \quad \cos \theta=\frac{\overrightarrow{\mathrm{a}} \cdot \overrightarrow{\mathrm{~b}}}{|\overrightarrow{\mathrm{a}}||\overrightarrow{\mathrm{b}}|}=\frac{\mathrm{a}_{1} \mathrm{~b}_{1}+\mathrm{a}_{2} \mathrm{~b}_{2}+\mathrm{a}_{3} \mathrm{~b}_{3}}{\sqrt{\mathrm{a}_{1}^{2}+\mathrm{a}_{2}^{2}+\mathrm{a}_{3}^{2} \cdot \sqrt{\mathrm{~b}_{1}^{2}+\mathrm{b}_{2}^{2}+\mathrm{b}_{3}^{2}}}}
$$

(ii) $\vec{a} \cdot \vec{b}$ is commutative i.e., $\vec{a} \cdot \vec{b}=\vec{b} \cdot \vec{a}$
(iii) If $\alpha$ is a scalar, then $(\alpha \vec{a}) \cdot \vec{b}=\alpha(\vec{a} \cdot \vec{b})=\vec{a} \cdot(\alpha \vec{b})$

## 7. Properties of Vector Product of two Vectors (Cross Product) -

(i) (a) If $\vec{a}=0$ or $\vec{b}=0$, then $\vec{a} \times \vec{b}=0$
(b) If $\vec{a} \| \vec{b}$, then $\vec{a} \times \vec{b}=0$
(ii) $\vec{a} \times \vec{b}$ is not commutative
i.e. $\vec{a} \times \vec{b}=\vec{b} \times \vec{a}$, but $\vec{a} \times \vec{b}=-\vec{b} \times \vec{a}$
(iii) If $\vec{a}$ and $\vec{b}$ represent adjacent sides of a parallelogram, then its area $|\vec{a} \times \vec{b}|$
(iv) If $\vec{a}$, $\vec{b}$ represent the adjacent sides of a triangle, then its area $=\frac{1}{2}|\vec{a} \times \vec{b}|$
(v) Distributive property $\vec{a} \times(\vec{b}+\vec{c})=\vec{a} \times \vec{b}+\vec{a} \times \vec{c}$
(a) If $\alpha$ be a scalar, then $\quad \alpha(\vec{a} \times \vec{b})=(\alpha \vec{a}) \times \vec{b}=\vec{a} \times(\alpha \vec{b})$
(b) If $\vec{a}=a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}$, and $\vec{b}=b_{1} \hat{i}+b_{2} \hat{j}+b_{3} \hat{k}$

$$
\text { Then, } \vec{a} \times \vec{b}=\left|\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
a_{1} & a_{2} & a_{3} \\
b_{1} & b_{2} & b_{3}
\end{array}\right|
$$

8. If $\alpha_{1} \beta_{1} \gamma$ are the direction angles of the vector $\vec{a}=\left(a_{1} \hat{i}+a_{2} \hat{j}+a_{3} \hat{k}\right)$. Then direction cosines of $\vec{a}$ are given as

$$
\cos \alpha=\frac{a_{1}}{|\vec{a}|}, \quad \cos \beta=\frac{a_{2}}{|\vec{a}|}, \quad \cos \gamma=\frac{a_{3}}{|\vec{a}|}
$$

9. Scalar Product of Two Vectors (Dot Product) - Scalar Product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a} \cdot \vec{b}=|\vec{a}||\vec{b}| \cos \theta$
where $\theta$ is the angle between $\overrightarrow{\mathrm{a}}$ and $\overrightarrow{\mathrm{b}}\left(0 \leq \theta<\frac{\pi}{2}\right)$
(i) When $\theta=0$, then $\vec{a} \cdot \vec{b}=|\vec{a}||\vec{b}|$. Also $\vec{a} \cdot \vec{a} a \cdot a=a^{2}$

$$
\therefore \quad \hat{\mathrm{i}} \cdot \hat{\mathrm{i}}=\hat{\mathrm{j}} \cdot \hat{\mathrm{j}}=\hat{\mathrm{k}} \cdot \hat{\mathrm{k}}=1
$$

(ii) When $\theta=\frac{\pi}{2}, \vec{a} \cdot \vec{b}=|\vec{a}||\vec{b}| \cos \frac{\pi}{2}=0$

## Class 12 Maths NCERT Solutions

| NCERT Solutions | Important Questions | NCERT Exemplar |
| :---: | :---: | :---: |
| Chapter 1 Relations and Functions | Relations and Functions | Chapter 1 Relations and Functions |
| Chapter 2 Inverse <br> Trigonometric Functions | Concept of Relations and Functions | Chapter 2 Inverse <br> Trigonometric Functions |
| Chapter 3 Matrices | Binary Operations | Chapter 3 Matrices |
| Chapter 4 Determinants | Inverse Trigonometric Functions | Chapter 4 Determinants |
| Chapter 5 Continuity and Differentiability | Matrices | Chapter 5 Continuity and Differentiability |
| Chapter 6 Application of Derivatives | Matrix and Operations of Matrices | Chapter 6 Application of Derivatives |
| Chapter 7 Integrals Ex 7.1 | Transpose of a Matrix and Symmetric Matrix | Chapter 7 Integrals |
| Integrals Class 12 Ex 7.2 | Inverse of a Matrix by Elementary Operations | Chapter 8 Applications of Integrals |
| Integrals Class 12 Ex 7.3 | Determinants | Chapter 9 Differential Equations |
| Integrals Class 12 Ex 7.4 | Expansion of Determinants | Chapter 10 Vector Algebra |
| Integrals Class 12 Ex 7.5 | Properties of Determinants | Chapter 11 Three Dimensional Geometry |
| Integrals Class 12 Ex 7.6 | Inverse of a Matrix and Application of Determinants and Matrix | Chapter 12 Linear Programming |
| Integrals Class 12 Ex 7.7 | Continuity and Differentiability | Chapter 13 Probability |
| Integrals Class 12 Ex 7.8 | Continuity |  |
| Integrals Class 12 Ex 7.9 | Differentiability |  |
| Integrals Class 12 Ex 7.10 | Application of Derivatives |  |
| Integrals Class 12 Ex 7.11 | Rate Measure Approximations and Increasing-Decreasing Functions |  |
| Integrals Class 12 <br> Miscellaneous Exercise | Tangents and Normals |  |
| Chapter 8 Application of Integrals | Maxima and Minima |  |
| Chapter 9 Differential Equations | Integrals |  |
| Chapter 10 Vector Algebra | Types of Integrals |  |
| Chapter 11 Three Dimensional Geometry | Differential Equation |  |
| Chapter 12 Linear <br> Programming | Formation of Differential Equations |  |
| Chapter 13 Probability Ex | Solution of Different Types of Differential |  |


| 13.1 | Equations |  |
| :--- | :--- | :--- |
| Probability Solutions Ex 13.2 | Vector Algebra |  |
| Probability Solutions Ex 13.3 | Algebra of Vectors |  |
| Probability Solutions Ex 13.4 | Dot and Cross Products of Two Vectors |  |
| Probability Solutions Ex 13.5 | Three Dimensional Geometry |  |
|  | Direction Cosines and Lines |  |
|  | Plane |  |
|  | Linear Programming |  |
|  | Probability |  |
|  | Conditional Probability and Independent |  |
| Events |  |  |
|  | Baye's Theorem and Probability |  |
|  | Distribution |  |

## RD Sharma Class 12 Solutions

| Chapter 1: Relations | Chapter 12: Higher Order <br> Derivatives | Chapter 23 Algebra of Vectors |
| :--- | :--- | :--- |
| Chapter 2: Functions | Chapter 13: Derivative as a Rate <br> Measurer | Chapter 24: Scalar Or Dot <br> Product |
| Chapter 3: Binary Operations | Chapter 14: Differentials, Errors <br> and Approximations | Chapter 25: Vector or Cross <br> Product |
| Chapter 4: Inverse Trigonometric | Chapter 15: Mean Value Theorems | Chapter 26: Scalar Triple Product |
| Functions | Chapter 16: Tangents and Normals | Chapter 27: Direction Cosines <br> and Direction Ratios |
| Chapter 5: Algebra of Matrices | Chapter 17: Increasing and | Chapter 28 Straight line in space |
| Chapter 6: Determinants | Decreasing Functions | Chapter 18: Maxima and Minima |

## JEE Main Maths Chapter wise Previous Year Questions

1. Relations, Functions and Reasoning
2. Complex Numbers
3. Quadratic Equations And Expressions
4. Matrices, Determinatnts and Solutions of Linear Equations
5. Permutations and Combinations
6. Binomial Theorem and Mathematical Induction
7. Sequences and Series
8. Limits,Continuity,Differentiability and Differentiation
9. Applications of Derivatives
10. Indefinite and Definite Integrals
11. Differential Equations and Areas
12. Cartesian System and Straight Lines
13. Circles and System of Circles
14. Conic Sections
15. Three Dimensional Geometry
16. Vectors
17. Statistics and Probability
18. Trignometry
19. Miscellaneous

## NCERT Solutions for Class 12

- NCERT Solutions for Class 12 Maths
- NCERT Solutions for Class 12 Physics
- NCERT Solutions for Class 12 Chemistry
- NCERT Solutions for Class 12 Biology
- NCERT Solutions for Class 12 English
- NCERT Solutions for Class 12 English Vistas
- NCERT Solutions for Class 12 English Flamingo
- NCERT Solutions for Class 12 Hindi
- NCERT Solutions for Class 12 Hindi Aroh (आरोह भाग 2)
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- NCERT Solutions for Class 12 Political Science
- NCERT Solutions for Class 12 Economics
- NCERT Solutions for Class 12 Macro Economics
- NCERT Solutions for Class 12 Micro Economics
- NCERT Solutions for Class 12 Computer Science (C++)
- NCERT Solutions for Class 12 Computer Science (Python)

