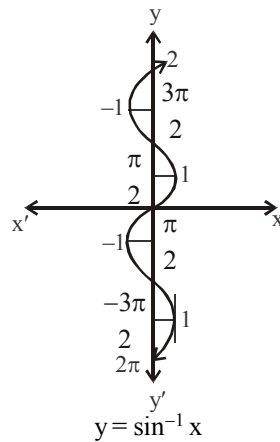


# 2

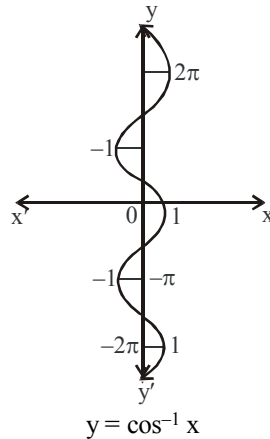
# INVERSE TRIGONOMETRIC FUNCTIONS

## KEY CONCEPT INVOLVED

- | 1.    | Functions | Domain   | Range                  |
|-------|-----------|--|------------------------|
| (i)   | sin       | $\mathbb{R}$   | $[-1, 1]$              |
| (ii)  | cos       | $\mathbb{R}$   | $[-1, 1]$              |
| (iii) | tan       | $\mathbb{R} - \{x : x = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}\}$ | $\mathbb{R}$           |
| (iv)  | cot       | $\mathbb{R} - \{x : x = n\pi, n \in \mathbb{Z}\}$                  | $\mathbb{R}$           |
| (v)   | sec       | $\mathbb{R} - \{x : x = (2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}\}$ | $\mathbb{R} - [-1, 1]$ |
| (vi)  | cosec     | $\mathbb{R} - \{x : x = n\pi, n \in \mathbb{Z}\}$                  | $\mathbb{R} - [-1, 1]$ |
2. **Inverse Function** - If  $f: X \rightarrow Y$  such that  $y = f(x)$  is one-one and onto, then we define another function  $g: Y \rightarrow X$  such that  $x = g(y)$ , where  $x \in X$  and  $y \in Y$  which is also one-one and onto. In such a case domain of  $g =$  Range of  $f$  and Range of  $g =$  domain of  $f$   
 $g$  is called inverse of  $f$  or  $g = f^{-1}$   
 Inverse of  $g = g^{-1} = (f^{-1})^{-1} = f$ .
3. **Principal value Branch of function  $\sin^{-1}$**  - It may be noted that for the domain  $[-1, 1]$  the range could be any one of the intervals  $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right], \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  or  $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$  corresponding to each interval we get a branch of the function  $\sin^{-1}$  the branch with range  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$  is called the principal value branch.  
 Thus  $\sin^{-1}: [-1, 1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

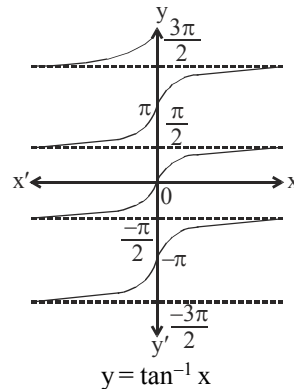


4. **Principal Value branch of function  $\cos^{-1}$**  - Domain of the function  $\cos^{-1}$  is  $[-1, 1]$ . Its range is one of the intervals  $(-\pi, 0)$ ,  $(0, \pi)$ ,  $(\pi, 2\pi)$ . etc. The branch with range  $(0, \pi)$  is called the principal value branch of the function  $\cos^{-1}$  thus  $\cos^{-1} : [-1, 1] \rightarrow [0, \pi]$



5. **Principal Value branch of function  $\tan^{-1}$**  - The function  $\tan^{-1}$  is defined whose domain is set of real numbers and range is one of the intervals  $\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right)$ ,  $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ ,  $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$  etc.

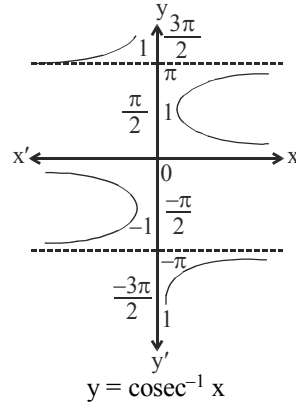
Graph of the function is as shown in the adjoining figure the branch with range  $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$  is called the principal value branch of function  $\tan^{-1}$ . Thus  $\tan^{-1} : \mathbb{R} \rightarrow \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ .



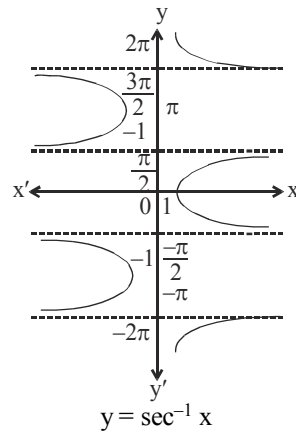
6. **Principal Value branch of function  $\operatorname{cosec}^{-1}$**  - The function  $\operatorname{cosec}^{-1}$  is defined on a function whose domain is  $\mathbb{R} - (-1, 1)$  and the range is any one of the interval  $\left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right] - \{\pi\}$ ,  $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ ,  $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$ , .....

The function corresponding to the range  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$  is called the principal value branch of  $\operatorname{cosec}^{-1}$

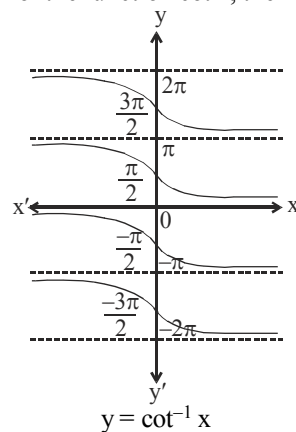
Thus,  $\operatorname{cosec}^{-1} : \mathbb{R} - (-1, 1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$



**7. Principal value branch of function  $\sec^{-1}$**  - The  $\sec^{-1}$  is defined as a function whose domain is  $\mathbb{R} - (-1, 1)$  and the range could be any of the intervals is .....,  $[-p, 0] - \left\{\frac{-\pi}{2}\right\}, [0, p] - \left\{\frac{\pi}{2}\right\}, [\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$ ..... etc. The branch corresponding to range  $[0, \pi] - \left\{\frac{\pi}{2}\right\}$  is known as the principal value branch of  $\sec^{-1}$ . Thus  $\sec^{-1} : \mathbb{R} - (-1, 1) \rightarrow [0, \pi] - \left\{\frac{\pi}{2}\right\}$ .



**8. Principal Value branch of function  $\cot^{-1}$**  - The  $\cot^{-1}$  function is defined as the function whose domain is  $\mathbb{R}$  and the range is any of the intervals.....  $(-\pi, 0), (0, \pi), (\pi, 2\pi)$  etc. The branch corresponding to  $(0, \pi)$  is called the principal value branch of the function  $\cot^{-1}$ , then  $\cot^{-1} : \mathbb{R} \rightarrow (0, \pi)$



9.	Inverse function	Domain	Principal Value branch
	$\sin^{-1}$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
	$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
	$\operatorname{cosec}^{-1}$	$\mathbb{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
	$\sec^{-1}$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
	$\tan^{-1}$	$\mathbb{R}$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
	$\cot^{-1}$	$\mathbb{R}$	$(0, \pi)$

### CONNECTING CONCEPTS

1. (i)  $\sin^{-1} 1/x = \operatorname{cosec}^{-1} x, x \geq 1, x \leq -1$  (ii)  $\cos^{-1} 1/x = \sec^{-1} x, x \geq 1, x \leq -1$   
 (iii)  $\tan^{-1} 1/x = \cot^{-1} x, x > 0$  (iv)  $\operatorname{cosec}^{-1} 1/x = \sin^{-1} x, x \in [-1, 1]$   
 (v)  $\sec^{-1} 1/x = \cos^{-1} x, x \in [-1, 1]$  (vi)  $\cot^{-1} 1/x = \tan^{-1} x, x > 0$
2. (i)  $\sin^{-1}(-x) = -\sin^{-1} x, x \in [-1, 1]$   
 (ii)  $\tan^{-1}(-x) = -\tan^{-1} x, x \in \mathbb{R}$   
 (iii)  $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x, |x| \geq 1$   
 (iv)  $\cos^{-1}(-x) = \pi - \cos^{-1} x, x \in [-1, 1]$   
 (v)  $\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \geq 1$   
 (vi)  $\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in \mathbb{R}$
3. (i)  $\sin^{-1} x + \cos^{-1} x = \pi/2, x \in [-1, 1]$   
 (ii)  $\tan^{-1} x + \cot^{-1} x = \pi/2, x \in \mathbb{R}$   
 (iii)  $\operatorname{cosec}^{-1} x + \sec^{-1} x = \pi/2, |x| \geq 1$
4. (i)  $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}, xy < 1$   
 (ii)  $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}, xy > -1$   
 (iii)  $2 \sin^{-1} x = \sin^{-1} (2x\sqrt{1-x^2}), -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$   
 (iv)  $2 \cos^{-1} x = \cos^{-1} (2x^2 - 1), -\frac{1}{\sqrt{2}} \leq x \leq 1$   
 (v)  $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}, -1 < x < 1 = \sin^{-1} \frac{2x}{1+x^2}, |x| \leq 1 = \cos^{-1} \frac{1-x^2}{1+x^2}, x \geq 0$

# Class 12 Maths NCERT Solutions

NCERT Solutions	Important Questions	NCERT Exemplar
<a href="#">Chapter 1 Relations and Functions</a>	<b>Relations and Functions</b>	<a href="#">Chapter 1 Relations and Functions</a>
<a href="#">Chapter 2 Inverse Trigonometric Functions</a>	<a href="#">Concept of Relations and Functions</a>	<a href="#">Chapter 2 Inverse Trigonometric Functions</a>
<a href="#">Chapter 3 Matrices</a>	<a href="#">Binary Operations</a>	<a href="#">Chapter 3 Matrices</a>
<a href="#">Chapter 4 Determinants</a>	<a href="#">Inverse Trigonometric Functions</a>	<a href="#">Chapter 4 Determinants</a>
<a href="#">Chapter 5 Continuity and Differentiability</a>	<b>Matrices</b>	<a href="#">Chapter 5 Continuity and Differentiability</a>
<a href="#">Chapter 6 Application of Derivatives</a>	<a href="#">Matrix and Operations of Matrices</a>	<a href="#">Chapter 6 Application of Derivatives</a>
<a href="#">Chapter 7 Integrals Ex 7.1</a>	<a href="#">Transpose of a Matrix and Symmetric Matrix</a>	<a href="#">Chapter 7 Integrals</a>
<a href="#">Integrals Class 12 Ex 7.2</a>	<a href="#">Inverse of a Matrix by Elementary Operations</a>	<a href="#">Chapter 8 Applications of Integrals</a>
<a href="#">Integrals Class 12 Ex 7.3</a>	<b>Determinants</b>	<a href="#">Chapter 9 Differential Equations</a>
<a href="#">Integrals Class 12 Ex 7.4</a>	<a href="#">Expansion of Determinants</a>	<a href="#">Chapter 10 Vector Algebra</a>
<a href="#">Integrals Class 12 Ex 7.5</a>	<a href="#">Properties of Determinants</a>	<a href="#">Chapter 11 Three Dimensional Geometry</a>
<a href="#">Integrals Class 12 Ex 7.6</a>	<a href="#">Inverse of a Matrix and Application of Determinants and Matrix</a>	<a href="#">Chapter 12 Linear Programming</a>
<a href="#">Integrals Class 12 Ex 7.7</a>	<b>Continuity and Differentiability</b>	<a href="#">Chapter 13 Probability</a>
<a href="#">Integrals Class 12 Ex 7.8</a>	<a href="#">Continuity</a>	
<a href="#">Integrals Class 12 Ex 7.9</a>	<a href="#">Differentiability</a>	
<a href="#">Integrals Class 12 Ex 7.10</a>	<b>Application of Derivatives</b>	
<a href="#">Integrals Class 12 Ex 7.11</a>	<a href="#">Rate Measure Approximations and Increasing-Decreasing Functions</a>	
<a href="#">Integrals Class 12 Miscellaneous Exercise</a>	<a href="#">Tangents and Normals</a>	
<a href="#">Chapter 8 Application of Integrals</a>	<a href="#">Maxima and Minima</a>	
<a href="#">Chapter 9 Differential Equations</a>	<b>Integrals</b>	
<a href="#">Chapter 10 Vector Algebra</a>	<a href="#">Types of Integrals</a>	
<a href="#">Chapter 11 Three Dimensional Geometry</a>	<b>Differential Equation</b>	
<a href="#">Chapter 12 Linear Programming</a>	<a href="#">Formation of Differential Equations</a>	
<a href="#">Chapter 13 Probability Ex</a>	<a href="#">Solution of Different Types of Differential</a>	

<a href="#">13.1</a>	<a href="#">Equations</a>	
<a href="#">Probability Solutions Ex 13.2</a>	<b>Vector Algebra</b>	
<a href="#">Probability Solutions Ex 13.3</a>	<a href="#">Algebra of Vectors</a>	
<a href="#">Probability Solutions Ex 13.4</a>	<a href="#">Dot and Cross Products of Two Vectors</a>	
<a href="#">Probability Solutions Ex 13.5</a>	<b>Three Dimensional Geometry</b>	
	<a href="#">Direction Cosines and Lines</a>	
	<a href="#">Plane</a>	
	<a href="#">Linear Programming</a>	
	<b>Probability</b>	
	<a href="#">Conditional Probability and Independent Events</a>	
	<a href="#">Baye's Theorem and Probability Distribution</a>	

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<a href="#">Chapter 1: Relations</a>	<a href="#">Chapter 12: Higher Order Derivatives</a>	<a href="#">Chapter 23 Algebra of Vectors</a>
<a href="#">Chapter 2: Functions</a>	<a href="#">Chapter 13: Derivative as a Rate Measurer</a>	<a href="#">Chapter 24: Scalar Or Dot Product</a>
<a href="#">Chapter 3: Binary Operations</a>	<a href="#">Chapter 14: Differentials, Errors and Approximations</a>	<a href="#">Chapter 25: Vector or Cross Product</a>
<a href="#">Chapter 4: Inverse Trigonometric Functions</a>	<a href="#">Chapter 15: Mean Value Theorems</a>	<a href="#">Chapter 26: Scalar Triple Product</a>
<a href="#">Chapter 5: Algebra of Matrices</a>	<a href="#">Chapter 16: Tangents and Normals</a>	<a href="#">Chapter 27: Direction Cosines and Direction Ratios</a>
<a href="#">Chapter 6: Determinants</a>	<a href="#">Chapter 17: Increasing and Decreasing Functions</a>	<a href="#">Chapter 28 Straight line in space</a>
<a href="#">Chapter 7: Adjoint and Inverse of a Matrix</a>	<a href="#">Chapter 18: Maxima and Minima</a>	<a href="#">Chapter 29: The plane</a>
<a href="#">Chapter 8: Solution of Simultaneous Linear Equations</a>	<a href="#">Chapter 19: Indefinite Integrals</a>	<a href="#">Chapter 30: Linear programming</a>
<a href="#">Chapter 9: Continuity</a>	<a href="#">Chapter 20: Definite Integrals</a>	<a href="#">Chapter 31: Probability</a>
<a href="#">Chapter 10: Differentiability</a>	<a href="#">Chapter 21: Areas of Bounded Regions</a>	<a href="#">Chapter 32: Mean and variance of a random variable</a>
<a href="#">Chapter 11: Differentiation</a>	<a href="#">Chapter 22: Differential Equations</a>	<a href="#">Chapter 33: Binomial Distribution</a>

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1. [Relations, Functions and Reasoning](#)
2. [Complex Numbers](#)
3. [Quadratic Equations And Expressions](#)
4. [Matrices, Determinants and Solutions of Linear Equations](#)
5. [Permutations and Combinations](#)
6. [Binomial Theorem and Mathematical Induction](#)
7. [Sequences and Series](#)
8. [Limits, Continuity, Differentiability and Differentiation](#)
9. [Applications of Derivatives](#)
10. [Indefinite and Definite Integrals](#)
11. [Differential Equations and Areas](#)
12. [Cartesian System and Straight Lines](#)
13. [Circles and System of Circles](#)
14. [Conic Sections](#)
15. [Three Dimensional Geometry](#)
16. [Vectors](#)
17. [Statistics and Probability](#)
18. [Trigonometry](#)
19. [Miscellaneous](#)

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