3 MATRICES

KEY CONCEPT INVOLVED

1. **Matrices -** A system of mn numbers (real or complex) arranged in a rectangular array of m rows and n columns is called a matrix of order m × n. An m × n matrix (to be read as 'm by n' matrix)
An m × n matrix is written as

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \vdots & \vdots & \ldots\ldots & \vdots \\ \vdots & \vdots & \ldots\ldots & \vdots \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{bmatrix}$$

The numbers a_{11}, a_{12} etc are called the elements or entries of the matrix. If A is a matrix of order m × n, then we shall write $A = [a_{ij}]_{m \times n}$ where, a_{ij} represent the number in the *i*-th row and *j*-th column.

2. **Row Matrix -** A single row matrix is called a row matrix or a row vector. e.g. the matrix $[a_{11}, a_{12}, \ldots\ldots a_{1n}]$ is a row matrix.

3. **Column Matrix -** A single column matrix is called a column matrix or a column vector. e.g. the matrix $\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ is a m × 1 column matrix.

4. **Order of a Matrix -** A matrix having m rows and n columns is of the order m × n. i.e. consisting of m rows and n columns is denoted by $A = [a_{ij}]_{m \times n}$.

5. **Square Matrix -** If m = n, i.e. if the number of rows and columns of a matrix are equal, say n, then it is called a square matrix of order n.

6. **Null or Zero Matrix -** If all the elements of a matrix are equal to zero, then it is called a null matrix and is denoted by $O_{m \times n}$ or 0.

7. **Diagonal Matrix -** A square matrix, in which all its elements are zero except those in the leading diagonal is called a diagonal matrix, thus in a diagonal matrix, $a_{ij} = 0$, if $i \neq j$, e.g. the diagonal matrices of order 2 and 3 are $\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$, $\begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_3 \end{bmatrix}$

8. **Scalar Matrix -** A square matrix in which all the diagonal element are equal and all other elements equal to zero is called a scalar matrix.
i.e. in a scalar matrix $a_{ij} = k$ for $i = j$ and $a_{ij} = 0$ for $i \neq j$. Thus $\begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}$ is a scalar matrix.

9. **Unit Matrix or Identity Matrix -** A square matrix in which all its diagonal elements are equal to 1 and all other elements equal to zero is called a unit matrix or identity matrix.

e.g. a unit or identity matrix of order 2 and 3 are $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ respectively.

10. **Upper triangular Matrix -** A square matrix A whose elements $a_{ij} = 0$ for $i > j$ is called an upper triangular matrix.

11. **Lower triangular Matrix -** A square matrix A whose elements $a_{ij} = 0$ for $i < j$ is called a lower triangular matrix.

12. **Equal Matrices -** Two matrices A and B are said to be equal, written as A = B if
    (i) they are of the same order i.e. have the same number of rows and columns, and
    (ii) the elements in the corresponding places of the two matrices are the same.

13. **Transpose of a matrix -** Let A be a m × n matrix then the matrix of order n × m obtained by changing its rows into columns and columns into rows is called the transpose of A and is denoted by A' or A^T.

14. **Negative of Matrix -** Let $A = [a_{ij}]_{m \times n}$ be a matrix. Then the negative of the matrix A is defined as the matrix $[-a_{ij}]_{m \times n}$ and is denoted by –A.

15. **Symmetric Matrix -** a square matrix A is said to be symmetric if $A' = A$
Thus a square matrix $A = [a_{ij}]$ is symmetric if $A = [a_{ij}]$ is symmetric if $a_{ij} = -a_{ji}$ for all values of i and j.

16. **Skew-Symmetric Matrix -** A square matrix A is said to be skew-symmetric if $A' = -A$ Thus a square matrix $A = [a_{ij}]$ is skew-symmetric if $a_{ij} = -a_{ji}$ for all values of i and j.
In particular $a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii} = 0$ i.e. all diagonal elements of a skew-symmetric matrix are o.

17. For any square matrix A with real number entries, $A + A'$ is a symetric matrix and $A - A'$ is a skew symetric matrix.

18. Any square matrix can be expressed as the sum of a symetric and a skew symetric matrix.

If A be a square matrix, then we can write $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$, here $\frac{1}{2}(A + A')$ is symetric matrix and $\frac{1}{2}(A \quad A)$ is skew symetric matrix.

19. **Addition of Matrices -** Let there be two matrices A and B of the same order m × n. then the sum denoted by A + B is defined to be the matrix of order m × n obtained by adding the corresponding elements of A and B.
Thus if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$ then $A + B = [a_{ij} + b_{ij}]_{m \times n}$

20. **Scalar Multiplication of a Matrix -** Let $A = [a_{ij}]_{m \times n}$ be a matrix and K is a scalar. Then the matrix obtained by multiplying each element of matrix A by K is called the scalar multiplication of matrix A by K and is denoted by KA or AK.

21. **Multiplication of Matrices -** Product of two matrices exists only if number of column of first matrix is equal to the number of rows of the second. Let A be m × n and B be n × p matrices. Then the product of matrices A and B denated by A.B is the matrix of order m × p whose (i, j)th element is obtained by adding the products of corresponding elements of *i*th row of A and *j*th column of B.

22. **Elementary Row Operations -** The operations known as elementary row operations on a matrix are-
    (i) The interchange of any two rows of a matrix. (The notations $R_i \leftrightarrow R_j$ is used for the interchange of the *i*-th and *j*-th rows.)
    (ii) The multiplication of every element of a row by a non-zero element (constant).
    (The notations $K.R_i$ is used for the multiplication of every element of *i*-th row by a constant K.
    (iii) The addition of the elements of a row, the product of the corresponding elements of any other row by any non-zero constant. (The notation $R_i + K.R_j$ is generally used for addition to the elements of *i*-th row to the element of *j*-th row multiplied by the constant K $(K \neq 0)$)

23. **Invertible matrices -** If A is a square matrix of order m, and if there exists another square matrix B of the same order m, such that AB = BA = I, then B is called the Inverse matrix of A and it is denoted by A^{-1}. In that care A is said to be invertible.

24. If A and B are invertible matrices of the same order, then $(AB)^{-1} = B^{-1}. A^{-1}$.

25. **Inverse of a matrix by elementry operations** - Let X, A and B be matrices of, the same order such that X = AB. In order to apply a sequence of elementry row operations on the matrix equation X = AB, we will apply these row operations simultaneously on X and on the first matrix A of the product AB on RHS.

Similarly, in order to apply a sequence of elementry column operations on the matrix equation X = AB, we will apply, these operations simultaneously on X and on the second matrix B of the product AB on RHS.

In view of the above discussion, we conclude that if A is a matrix such that A^{-1} exists, then to find A^{-1} using elementry row operations, write A = IA and apply a sequence of row operation on A = IA till we get, I =BA. The matrix B will be the inverse of A. Similarly, if we with to find A^{-1} using column operations, then, write A = AI on A = IA till we get, I = BA. The matrix and apply a sequence of column operations on A = AI till we get, I = AB.

Remark - In case, after applying one or more elementry row (column) operations on A = IA (A = AI). If we obtain all zero in one or more rows of the matrix A on L.H.S., that A^{-1} does not exist.

CONNECTING CONCEPTS

1. The elements a_{ij} of a matrix for which i = j are called the diagonal elements of a matrix and the line along which all these elements lie is called the principal diagonal or the diagonal of the matrix.

2. **Properties of transpose of the matrices-**

(i) $(A + B)' = A' + B'$

(ii) $(KA)' = KA'$, where K is constant

(iii) $(AB)' = B'A'$

(iv) $(A')' = A$

3. **Properties of Matrix addition-**

(i) Matrix Addition is Commutative - If A and B be two m × n matrices, then A + B = B + A

(ii) Matrix Addition is Associative - If A, B and C be three m × n matrices, then

$$(A + B) + C = A + (B + C)$$

4. **Properties of Multiplication of a Matrix by a Scalar-**

(i) If K_1 and K_2 are scalars and A be a matrix, then $(K_1 + K_2) A = K_1 A + K_2 A$.

(ii) If K_1 and K_2 are scalars and A be a matrix, then $K_1 (K_2 A) = (K_1 K_2) A$.

(iii) If A and B are two matrices of the same order and K, a scalar, then $K (A + B) = KA + KB$.

(iv) If K_1 and K_2 are two scalars and A is any matrix then $(K_1 + K_2) A = K_1 A + K_2 A$.

(v) If A is any matrix and K be a scalar.
then $(–K) A = – (KA) = K (–A)$.

5. **Properties of Matrix Multiplication -**

(i) Associative law for Multiplication - If A, B and C be three matrices of order m × n and n × p and p × q, respectively, then $(AB) C = A (BC)$.

(ii) Distributive Law - If A, B, C be three matrices of order m × n, n × p and n × q respectively.
then $A \cdot (B + C) = A \cdot B + A \cdot C$

(iii) Matrix Multiplication is not commutative.
i.e. $A \cdot B \neq B \cdot A$

(iv) The existence of multiplicative Identity : For every square matrix A, there exists an identity matrix of same order such that IA = AI = A.

6. If A be any n × n square matrix, then

$$A \cdot (\text{Adj } A) = (\text{Adj } A) \cdot A = |A|. I_n$$

where I_n is an n × n unit matrix

7. (i) Only square matrix can have inverse

(ii) The matrix $B = A^{-1}$, will also be a square matrix of same order A.

(iii) The square matrix A is said to be invertible if A^{-1} exists.

8. Every invertible matrix possesses a unique inverse.

Class 12 Maths NCERT Solutions

NCERT Solutions	Important Questions	NCERT Exemplar
Chapter 1 Relations and Functions	**Relations and Functions**	Chapter 1 Relations and Functions
Chapter 2 Inverse Trigonometric Functions	Concept of Relations and Functions	Chapter 2 Inverse Trigonometric Functions
Chapter 3 Matrices	Binary Operations	Chapter 3 Matrices
Chapter 4 Determinants	Inverse Trigonometric Functions	Chapter 4 Determinants
Chapter 5 Continuity and Differentiability	**Matrices**	Chapter 5 Continuity and Differentiability
Chapter 6 Application of Derivatives	Matrix and Operations of Matrices	Chapter 6 Application of Derivatives
Chapter 7 Integrals Ex 7.1	Transpose of a Matrix and Symmetric Matrix	Chapter 7 Integrals
Integrals Class 12 Ex 7.2	Inverse of a Matrix by Elementary Operations	Chapter 8 Applications of Integrals
Integrals Class 12 Ex 7.3	**Determinants**	Chapter 9 Differential Equations
Integrals Class 12 Ex 7.4	Expansion of Determinants	Chapter 10 Vector Algebra
Integrals Class 12 Ex 7.5	Properties of Determinants	Chapter 11 Three Dimensional Geometry
Integrals Class 12 Ex 7.6	Inverse of a Matrix and Application of Determinants and Matrix	Chapter 12 Linear Programming
Integrals Class 12 Ex 7.7	**Continuity and Differentiability**	Chapter 13 Probability
Integrals Class 12 Ex 7.8	Continuity	
Integrals Class 12 Ex 7.9	Differentiability	
Integrals Class 12 Ex 7.10	**Application of Derivatives**	
Integrals Class 12 Ex 7.11	Rate Measure Approximations and Increasing-Decreasing Functions	
Integrals Class 12 Miscellaneous Exercise	Tangents and Normals	
Chapter 8 Application of Integrals	Maxima and Minima	
Chapter 9 Differential Equations	**Integrals**	
Chapter 10 Vector Algebra	Types of Integrals	
Chapter 11 Three Dimensional Geometry	**Differential Equation**	
Chapter 12 Linear Programming	Formation of Differential Equations	
Chapter 13 Probability Ex	Solution of Different Types of Differential	

13.1	Equations	
Probability Solutions Ex 13.2	**Vector Algebra**	
Probability Solutions Ex 13.3	Algebra of Vectors	
Probability Solutions Ex 13.4	Dot and Cross Products of Two Vectors	
Probability Solutions Ex 13.5	**Three Dimensional Geometry**	
	Direction Cosines and Lines	
	Plane	
	Linear Programming	
	Probability	
	Conditional Probability and Independent Events	
	Baye's Theorem and Probability Distribution	

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Chapter 1: Relations	Chapter 12: Higher Order Derivatives	Chapter 23 Algebra of Vectors
Chapter 2: Functions	Chapter 13: Derivative as a Rate Measurer	Chapter 24: Scalar Or Dot Product
Chapter 3: Binary Operations	Chapter 14: Differentials, Errors and Approximations	Chapter 25: Vector or Cross Product
Chapter 4: Inverse Trigonometric Functions	Chapter 15: Mean Value Theorems	Chapter 26: Scalar Triple Product
Chapter 5: Algebra of Matrices	Chapter 16: Tangents and Normals	Chapter 27: Direction Cosines and Direction Ratios
Chapter 6: Determinants	Chapter 17: Increasing and Decreasing Functions	Chapter 28 Straight line in space
Chapter 7: Adjoint and Inverse of a Matrix	Chapter 18: Maxima and Minima	Chapter 29: The plane
Chapter 8: Solution of Simultaneous Linear Equations	Chapter 19: Indefinite Integrals	Chapter 30: Linear programming
Chapter 9: Continuity	Chapter 20: Definite Integrals	Chapter 31: Probability
Chapter 10: Differentiability	Chapter 21: Areas of Bounded Regions	Chapter 32: Mean and variance of a random variable
Chapter 11: Differentiation	Chapter 22: Differential Equations	Chapter 33: Binomial Distribution

JEE Main Maths Chapter wise Previous Year Questions

1. Relations, Functions and Reasoning
2. Complex Numbers
3. Quadratic Equations And Expressions
4. Matrices, Determinatnts and Solutions of Linear Equations
5. Permutations and Combinations
6. Binomial Theorem and Mathematical Induction
7. Sequences and Series
8. Limits,Continuity,Differentiability and Differentiation
9. Applications of Derivatives
10. Indefinite and Definite Integrals
11. Differential Equations and Areas
12. Cartesian System and Straight Lines
13. Circles and System of Circles
14. Conic Sections
15. Three Dimensional Geometry
16. Vectors
17. Statistics and Probability
18. Trignometry
19. Miscellaneous

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- NCERT Solutions for Class 12 Macro Economics
- NCERT Solutions for Class 12 Micro Economics
- NCERT Solutions for Class 12 Computer Science (C++)
- NCERT Solutions for Class 12 Computer Science (Python)