4 DETERMINANTS

KEY CONCEPTS INVOLVED

1. **Determinant -** (i) A determinant is a particular type of expression written in a special concise form of rows and columns, equal in number.

   For example $\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is a determinant having 2 rows and 2 columns, hence it is of second order. The numbers a_1, b_1, a_2, b_2 are called the elements of the determinant. The value of the above determinant of third order is written as $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$. It has three rows and three columns.

   The number of elements = $3^2 = 9$. In general, the number of elements in a determinant of order $n = n^2$.

   (ii) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, is a matrix then determinant of matrix A is written as $|A|$ or det (A) $= \begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

   (iii) Only square matrices have determinants.
2. **Minors -** The determinant obtained by deleting the i-the row and j-th column passing through the element a_{ij} is called the minor of element a_{ij} and is denoted by M_{ij}.
3. **Cofactors -** The cofactor of element a_{ij} is $(-1)^{1+j}$ times the determinant obtained by deleting the i-th row and jth column passed through a_{ij} and is denoted by C_{ij} i.e. $C_{ij} = (-1)^{1+j} M_{ij}$
4. **Values of the determinant -** The sum of the products of elements of any row (column) by the corresponding co-factors is equal to the value of the determinant.

   let $\Delta = \begin{vmatrix} a_{11} & b_{12} & c_{13} \\ a_{21} & b_{22} & c_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, Then $\Delta = a_{11}\, c_{11} + a_{12}\, c_{12} + a_{13}\, c_{13}$
5. **Area of a Triangle -** The area of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is

   $$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

   (i) The area is positive, take only absolute value.

   (ii) If the three points are collinear, the area of triangle is zero.
6. $|AB| = |A|\,|B|$
7. A square matrix is invertible if and only if A is non-singular.
8. **Linear system of Equations -**

   Consistent System - The system of equation is said to be consistent if it has one or more then one solutions.

Inconsistent System - The system of equation is inconsistent if it has no solution

Consider the system of equation

$$a_1x + b_1y_+ c_1z = d_1$$
$$a_2x + b_2y_+ c_2z = d_2$$
$$a_3x + b_3y_+ c_3z = d_3$$

let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

The given system of equation can be written as

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

or $AX = B$

$\therefore$ $X = A^{-1} B.$

9. **Consistence/Inconsistence of system of Equations**

   (a) For a non-homogeneous system of equation $AX \neq 0$

   (i) If $|A| \neq 0$, $AX = B$ has a unique solution.

   (ii) If $|A| = 0$, and (adj A) $B \neq 0$ then the system of equation is inconsistent.

   (iii) If $|A| = 0$ and (adj A) $B = 0$, then the system of equation has infinitely many solutions.

   (b) For the homogeneous system of equation $AX = 0$

   (i) If $|A| \neq 0$, the solution is $x = 0, y = 0, z = 0$. This is called the trivial solution.

   (ii) If $|A| = 0$, the system has infinitely many solution. In such a case, we put one of the variables equal to k. let $z = k$, then we find the value of x and y in terms of k.

10. **Adjoint of a Determinant -** The adjoint of a square matrix is the transpose of matrix cofactors. If A_{ij} is the cofactor of a_{ij} of det A or $|a_{ij}|$, the

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

11. **Inverse of a matrix -** Inverse of a matrix A, $A^{-1} = \frac{1}{|A|}$ adj A ; if $|A| \neq 0$ i.e., matrix A is invertible or non-singular.

12. If A is a square matrix, then A (adj A) = (adj A) A = $|A| . I$

13. (i) $(AB)^{-1} = B^{-1} . A^{-1}$ (ii) $A^{-1} = (A^{-1})^T$ (iii) $(A^{-1})^{-1} = A$

CONNECTING CONCEPTS

1. The value of the determinant does not change when rows and columns are interchanged. The determinant obtained by interchanging the rows and columns is called the transpose of the determinant and is denoted by Δ^T. Thus $\Delta = \Delta^T$.
2. If all the elements of a row (column) are zero, then the value of the determinant is zero.
3. The interchange of any two rows of the determinant changes its sign.

   Thus if Δ^* is the new determinant obtained on interchanging any two rows (columns), then

$$\Delta = -\Delta^*$$

If i-th and j-th row are interchanged then this operation is denoted by $R_i \longleftrightarrow R_j$.

4. If all the elements of a row (column) of a determinant are multiplied by a non-zero constant, then the determinant gets multiplied by the same constant. Thus if we apply $R_i \to pR_i$, i.e, each element of i-th row is multiplied by p, then we get

$$\Delta^* = p\Delta \quad \text{or} \quad \Delta = \frac{1}{p}\Delta^* \ (P \neq 0)$$

5. If all the elements of a row (column) are proportional (identical) to the elements of some other row (column) then determinant is zero.

6. If each element of any row (column) is sum of two numbers, the determinant can be expressed as the sum of two determinants of the same order eg.

$$\begin{vmatrix} a_1+\alpha_1 & b_1 & c_1 \\ a_2+\alpha_2 & b_2 & c_2 \\ a_3+\alpha_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix}$$

7. The value of a determinant remains unaltered under an operation of the form

$R_i \to R_i + pR_j \to$ similarly, for columns i.e., operation of the form $C_i \to C_i + pC_j + q\,C_k$; $j, k \neq i$

8. If a determinant $\Delta(x)$ becomes zero on putting $x = \alpha$, then $(x - \alpha)$ is a factor of $\Delta(x)$.

9. Determinant which have all elements equal to zero except the diagonal elements, is equal to the product of the diagonal elements.

$$\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = abc$$

Class 12 Maths NCERT Solutions

NCERT Solutions	Important Questions	NCERT Exemplar
Chapter 1 Relations and Functions	**Relations and Functions**	Chapter 1 Relations and Functions
Chapter 2 Inverse Trigonometric Functions	Concept of Relations and Functions	Chapter 2 Inverse Trigonometric Functions
Chapter 3 Matrices	Binary Operations	Chapter 3 Matrices
Chapter 4 Determinants	Inverse Trigonometric Functions	Chapter 4 Determinants
Chapter 5 Continuity and Differentiability	**Matrices**	Chapter 5 Continuity and Differentiability
Chapter 6 Application of Derivatives	Matrix and Operations of Matrices	Chapter 6 Application of Derivatives
Chapter 7 Integrals Ex 7.1	Transpose of a Matrix and Symmetric Matrix	Chapter 7 Integrals
Integrals Class 12 Ex 7.2	Inverse of a Matrix by Elementary Operations	Chapter 8 Applications of Integrals
Integrals Class 12 Ex 7.3	**Determinants**	Chapter 9 Differential Equations
Integrals Class 12 Ex 7.4	Expansion of Determinants	Chapter 10 Vector Algebra
Integrals Class 12 Ex 7.5	Properties of Determinants	Chapter 11 Three Dimensional Geometry
Integrals Class 12 Ex 7.6	Inverse of a Matrix and Application of Determinants and Matrix	Chapter 12 Linear Programming
Integrals Class 12 Ex 7.7	**Continuity and Differentiability**	Chapter 13 Probability
Integrals Class 12 Ex 7.8	Continuity	
Integrals Class 12 Ex 7.9	Differentiability	
Integrals Class 12 Ex 7.10	**Application of Derivatives**	
Integrals Class 12 Ex 7.11	Rate Measure Approximations and Increasing-Decreasing Functions	
Integrals Class 12 Miscellaneous Exercise	Tangents and Normals	
Chapter 8 Application of Integrals	Maxima and Minima	
Chapter 9 Differential Equations	**Integrals**	
Chapter 10 Vector Algebra	Types of Integrals	
Chapter 11 Three Dimensional Geometry	**Differential Equation**	
Chapter 12 Linear Programming	Formation of Differential Equations	
Chapter 13 Probability Ex	Solution of Different Types of Differential	

13.1	Equations	
Probability Solutions Ex 13.2	**Vector Algebra**	
Probability Solutions Ex 13.3	Algebra of Vectors	
Probability Solutions Ex 13.4	Dot and Cross Products of Two Vectors	
Probability Solutions Ex 13.5	**Three Dimensional Geometry**	
	Direction Cosines and Lines	
	Plane	
	Linear Programming	
	Probability	
	Conditional Probability and Independent Events	
	Baye's Theorem and Probability Distribution	

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Chapter 1: Relations	Chapter 12: Higher Order Derivatives	Chapter 23 Algebra of Vectors
Chapter 2: Functions	Chapter 13: Derivative as a Rate Measurer	Chapter 24: Scalar Or Dot Product
Chapter 3: Binary Operations	Chapter 14: Differentials, Errors and Approximations	Chapter 25: Vector or Cross Product
Chapter 4: Inverse Trigonometric Functions	Chapter 15: Mean Value Theorems	Chapter 26: Scalar Triple Product
Chapter 5: Algebra of Matrices	Chapter 16: Tangents and Normals	Chapter 27: Direction Cosines and Direction Ratios
Chapter 6: Determinants	Chapter 17: Increasing and Decreasing Functions	Chapter 28 Straight line in space
Chapter 7: Adjoint and Inverse of a Matrix	Chapter 18: Maxima and Minima	Chapter 29: The plane
Chapter 8: Solution of Simultaneous Linear Equations	Chapter 19: Indefinite Integrals	Chapter 30: Linear programming
Chapter 9: Continuity	Chapter 20: Definite Integrals	Chapter 31: Probability
Chapter 10: Differentiability	Chapter 21: Areas of Bounded Regions	Chapter 32: Mean and variance of a random variable
Chapter 11: Differentiation	Chapter 22: Differential Equations	Chapter 33: Binomial Distribution

JEE Main Maths Chapter wise Previous Year Questions

1. Relations, Functions and Reasoning
2. Complex Numbers
3. Quadratic Equations And Expressions
4. Matrices, Determinatnts and Solutions of Linear Equations
5. Permutations and Combinations
6. Binomial Theorem and Mathematical Induction
7. Sequences and Series
8. Limits,Continuity,Differentiability and Differentiation
9. Applications of Derivatives
10. Indefinite and Definite Integrals
11. Differential Equations and Areas
12. Cartesian System and Straight Lines
13. Circles and System of Circles
14. Conic Sections
15. Three Dimensional Geometry
16. Vectors
17. Statistics and Probability
18. Trignometry
19. Miscellaneous

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