5 CONTINUITY AND DIFFERENTIABILITY

KEY CONCEPT INVOLVED

1. **Continuity -** A real valued function $f(x)$ of variable x defined on an interval I is said to be continuous at $x = a \in I$, $\lim\limits_{x \to a} f(x)$ exists, is finite and is equal to $f(a)$.

   $\therefore \lim\limits_{h \to 0} f(a+h) = \lim\limits_{h \to 0} f(a-h) = f(a)$, where 'h' is a very small +ve quantity.
2. A function $f(x)$ is said to be continuous in an interval I, if it is continuous at each point of the interval.
3. **Discontinuity -** A function said to be discontinuous at a point $x = a$, if it is not continuous at this point. This point $x = a$ where the function is not continuous is called the point of discontinuity.
4. Suppose f and g be two real functions continuous at a real number c, then
   (i) $f + g$ is continuous at $x = c$
   (ii) $f - g$ is continuous at $x = c$
   (iii) $f \cdot g$ is continuous at $x = c$
   (iv) $\frac{f}{g}$ is continuous at $x = c$, (provided $g(c) \neq 0$)
5. (i) If g is a continuous function, then $\frac{1}{g}$ is also continuous.
   (ii) Suppose f and g are real valued functions such that (fog) is defined at c. If f and g is continuous at c then (fog) is also continuous at c.
6. **Differentiability -** The concept of differentiability has been introduced in the lower class let f be a real function and c is a point in its domain. The derivative $f'(c)$ of f at c is defined as $\lim\limits_{h \to 0} \frac{f(c+h) - f(c)}{h}$, provided limit exists

   Thus, $f'(c) = \frac{d}{dx}[f(x)]_c$. $f'(x)$ is defined as $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

   Every differentiable function is continuous.
7. **Algebra of Derivatives -** Let u, v be the function of x.
   (i) $(u \pm v)' = u' \pm v'$
   (ii) $(uv)' = u'v + uv'$
   (iii) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, where $v \neq 0$.
8. **Chain Rule -** If f and g are differentiable functions in their domain, then $fog(x)$ or $f\,g(x)$ is also differentiable and $(fog)'(x) = f'\,g(x) \times g'(x)$

   More easily if $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

   If y is a function of u, u is a function of v and v is a function of x then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}$.
9. **Implicit functions -** An equation in the form $f(x, y) = 0$ in which y is not expressible in terms of x is called as an implicit function of x and y.

Both sides of equations are differentiated term wise with respect to x then from this equation $\frac{dy}{dx}$ is obtained. It may be noted that when a function of y occurs, then differentiate it w.r.t. y and multiply it by $\frac{dy}{dx}$.

Collect the terms containing $\frac{dy}{dx}$ at one side and find $\frac{dy}{dx}$

10. Exponential function - The exponential function with positive base $b > 1$, is the function $y = b^x$.

(i) The graph of $y = 10^x$ is

(ii) Domain = R

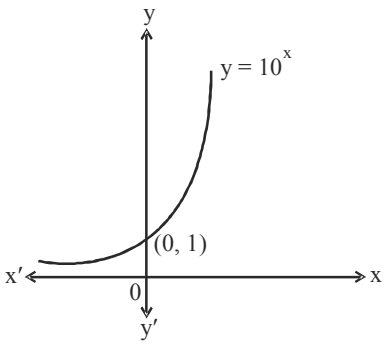
(iii) Range = R^+

(iv) The point (0, 1) always lies on the graph.

(v) It is an increasing function

(vi) As $x \to -\infty$ $y \to 0$

(vii) $\frac{d}{dx} a^x = a^x \log_e a$, $\frac{d}{dx} e^x = e^x$.



11. Logarithmic function - Let $b > 1$ be a real number. $b^x = a$ may be written as $\log_b a = x$.

(i) The graph of $y = \log_{10} x$ is

(ii) Domain = R^+

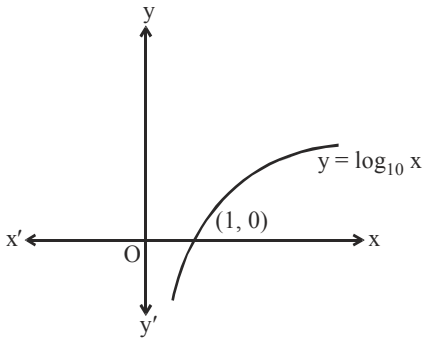
(iii) Range = R

(iv) It is an increasing function.

(v) As $x \to 0$, $y \to -\infty$.

(vi) The functiony = e^x and $y = \log_e x$ are the mirror images of each other

(vii) $\frac{d}{dx}(\log_a x) = \frac{1}{x}\log_a e$, $\frac{d}{dx}\log_e x = \frac{1}{x}$



12. Derivatives of functions in Parametric form - The set of equations $x = f(t)$, $y = g(t)$ is called the parametric form of an equation.

Now, $\frac{dx}{dt} = f'(t), \frac{dy}{dt} = g'(t)$, $\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ or $\frac{g'(t)}{f'(t)}$

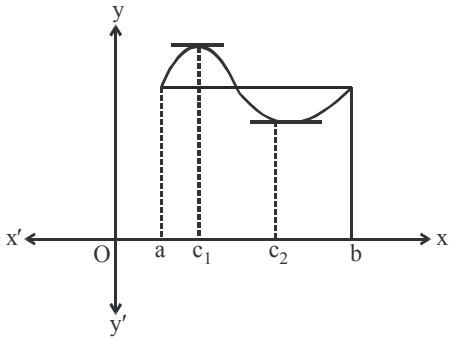
13. Second order derivative- let $y = f(x)$ then $\frac{dy}{dx} = f'(x)$

If $f'(x)$ is differentiable, then it is again differentiated and get

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) \text{ or } \frac{d^2y}{dx^2} = f''(x)$$

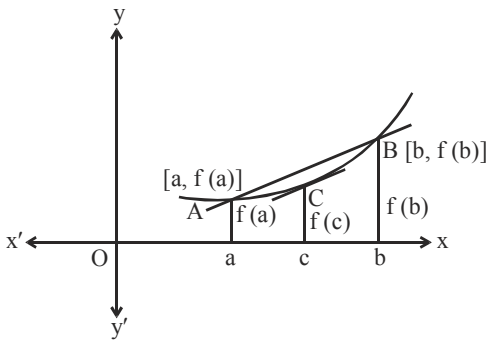
$\frac{d^2y}{dx^2}$ or $f''(x)$ is called the second derivative of y or $f(x)$ with respect to x.

14. Rolle's Theorem - Let $f : [a, b] \to R$ be continuous an closed interval [a, b] and differentiable an open interval (a, b) such that $f(a) = f(b)$ where a, b are real numbers, then there must exists at least one value $c \in (a, b)$ of x,such that $f'(c) = 0$.



We observe that $f(a) = f(b)$, There exists two point c_1 and $c_2 \in (a, b)$ such that $f'(c_1) = 0$ and $f'(c_2) = 0$, i.e. Tangent at c_1 and c_2 are parallel to x-axis.

15. Mean Value Theorem- Let $f : [a, b] \to R$ be a continuous function on the closed interval [a, b] and differentiable in the open interval (a, b), then there must exists at least one value $c \in (a, b)$ of x, such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.



Here, $\frac{f(b) - f(a)}{b - a}$ is the slope of secant drawn between A [a, f (a)] and B [b, f (b)]. There is at least one point $c \in (a, b)$ of x where slope of the tangent at x = c is parallel to chord AB.

CONNECTING CONCEPTS

Some common type functions as constant function, Identity function, implicit function, Modulus function, Exponential function, and logarithmic function are continuous in their domains.

1. Every polynomial function is differentiable at each $x \in R$.
2. The exponential function a^x, $a > 0$, is differentiable at each $x \in R$

3. Every constant function is differentiable at each $x \in R$.

4. The logarithmic function is differentiable at each point in its demain.

5. Trigonometric and inverse-trigenometric functions are differentiable in their domains.

6. The sum, difference, product and quotient of two differentiable functions is differentiable

7. The composition of differentiable function is differentiable function.

8. (i) $\log_b pq = \log_b p + \log_b q$

(ii) $\log_b \frac{p}{q} = \log_b p - \log_b q$

(iii) $\log_b p^x = x \log_b p$

(iv) $\log_a p = \frac{\log_b p}{\log_b a}$

9. Derivativs of Inverse Trigonometric Functions.

Functions	**Domain**	**Derivative**
$\sin^{-1} x$	$[-1, 1]$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$[-1,1]$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	R	$\frac{1}{1+x^2}$
$\cot^{-1} x$	R	$\frac{-1}{1+x^2}$
$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\frac{1}{x\sqrt{x^2-1}}$
$\text{cosec}^{-1} x$	$(-\infty, -1) \cup [1, \infty)$	$\frac{-1}{x\sqrt{x^2-1}}$

Class 12 Maths NCERT Solutions

NCERT Solutions	Important Questions	NCERT Exemplar
Chapter 1 Relations and Functions	**Relations and Functions**	Chapter 1 Relations and Functions
Chapter 2 Inverse Trigonometric Functions	Concept of Relations and Functions	Chapter 2 Inverse Trigonometric Functions
Chapter 3 Matrices	Binary Operations	Chapter 3 Matrices
Chapter 4 Determinants	Inverse Trigonometric Functions	Chapter 4 Determinants
Chapter 5 Continuity and Differentiability	**Matrices**	Chapter 5 Continuity and Differentiability
Chapter 6 Application of Derivatives	Matrix and Operations of Matrices	Chapter 6 Application of Derivatives
Chapter 7 Integrals Ex 7.1	Transpose of a Matrix and Symmetric Matrix	Chapter 7 Integrals
Integrals Class 12 Ex 7.2	Inverse of a Matrix by Elementary Operations	Chapter 8 Applications of Integrals
Integrals Class 12 Ex 7.3	**Determinants**	Chapter 9 Differential Equations
Integrals Class 12 Ex 7.4	Expansion of Determinants	Chapter 10 Vector Algebra
Integrals Class 12 Ex 7.5	Properties of Determinants	Chapter 11 Three Dimensional Geometry
Integrals Class 12 Ex 7.6	Inverse of a Matrix and Application of Determinants and Matrix	Chapter 12 Linear Programming
Integrals Class 12 Ex 7.7	**Continuity and Differentiability**	Chapter 13 Probability
Integrals Class 12 Ex 7.8	Continuity	
Integrals Class 12 Ex 7.9	Differentiability	
Integrals Class 12 Ex 7.10	**Application of Derivatives**	
Integrals Class 12 Ex 7.11	Rate Measure Approximations and Increasing-Decreasing Functions	
Integrals Class 12 Miscellaneous Exercise	Tangents and Normals	
Chapter 8 Application of Integrals	Maxima and Minima	
Chapter 9 Differential Equations	**Integrals**	
Chapter 10 Vector Algebra	Types of Integrals	
Chapter 11 Three Dimensional Geometry	**Differential Equation**	
Chapter 12 Linear Programming	Formation of Differential Equations	
Chapter 13 Probability Ex	Solution of Different Types of Differential	

13.1	Equations	
Probability Solutions Ex 13.2	**Vector Algebra**	
Probability Solutions Ex 13.3	Algebra of Vectors	
Probability Solutions Ex 13.4	Dot and Cross Products of Two Vectors	
Probability Solutions Ex 13.5	**Three Dimensional Geometry**	
	Direction Cosines and Lines	
	Plane	
	Linear Programming	
	Probability	
	Conditional Probability and Independent Events	
	Baye's Theorem and Probability Distribution	

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Chapter 1: Relations	Chapter 12: Higher Order Derivatives	Chapter 23 Algebra of Vectors
Chapter 2: Functions	Chapter 13: Derivative as a Rate Measurer	Chapter 24: Scalar Or Dot Product
Chapter 3: Binary Operations	Chapter 14: Differentials, Errors and Approximations	Chapter 25: Vector or Cross Product
Chapter 4: Inverse Trigonometric Functions	Chapter 15: Mean Value Theorems	Chapter 26: Scalar Triple Product
Chapter 5: Algebra of Matrices	Chapter 16: Tangents and Normals	Chapter 27: Direction Cosines and Direction Ratios
Chapter 6: Determinants	Chapter 17: Increasing and Decreasing Functions	Chapter 28 Straight line in space
Chapter 7: Adjoint and Inverse of a Matrix	Chapter 18: Maxima and Minima	Chapter 29: The plane
Chapter 8: Solution of Simultaneous Linear Equations	Chapter 19: Indefinite Integrals	Chapter 30: Linear programming
Chapter 9: Continuity	Chapter 20: Definite Integrals	Chapter 31: Probability
Chapter 10: Differentiability	Chapter 21: Areas of Bounded Regions	Chapter 32: Mean and variance of a random variable
Chapter 11: Differentiation	Chapter 22: Differential Equations	Chapter 33: Binomial Distribution

JEE Main Maths Chapter wise Previous Year Questions

1. Relations, Functions and Reasoning
2. Complex Numbers
3. Quadratic Equations And Expressions
4. Matrices, Determinatnts and Solutions of Linear Equations
5. Permutations and Combinations
6. Binomial Theorem and Mathematical Induction
7. Sequences and Series
8. Limits,Continuity,Differentiability and Differentiation
9. Applications of Derivatives
10. Indefinite and Definite Integrals
11. Differential Equations and Areas
12. Cartesian System and Straight Lines
13. Circles and System of Circles
14. Conic Sections
15. Three Dimensional Geometry
16. Vectors
17. Statistics and Probability
18. Trignometry
19. Miscellaneous

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- NCERT Solutions for Class 12 Macro Economics
- NCERT Solutions for Class 12 Micro Economics
- NCERT Solutions for Class 12 Computer Science (C++)
- NCERT Solutions for Class 12 Computer Science (Python)