# 6 APPLICATION OF DERIVATIVES

## KEY CONCEPTS INVOLVED

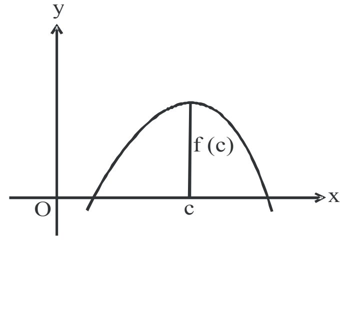
1. **Rate of change of Quantities –** Let $y = f(x)$ be a function. If the change in one quantity y varies with another quantity x, then $\frac{dy}{dx}$ or $f'(x)$ denotes the rate of change of y with respect to x. $\left.\frac{dy}{dx}\right]_{x = x_0}$ or $f'(x_0)$ represents the rate of change of y w.r.t. x at $x = x_0$.
2. **Increasing and Decreasing function at $x_0$** A function f is said to be
   (*a*) Increasing on an interval (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in (a, b)$
   Alternatively, if $f'(x) \geq 0$ for each x in (a, b)
   (*b*) Decreasing on (a, b) if $x_1 < x_2$ in (a, b) $\Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in (a, b)$ Alternatively, if $f'(x) \leq 0$ for each x in (a, b)
3. **Test : Increasing/decreasing/constant function –** Let f be a continuous on [a, b] and differentiable in an open interval (a, b), then.
   (i) f is increasing on [a, b], if $f'(x) > 0$ for each $x \in (a, b)$
   (ii) f is decreasing on [a, b], if $f'(x) < 0$ for each $x \in (a, b)$
   (iii) f is constant on [a, b], if $f'(x) = 0$ for each $x \in (a, b)$
4. **Tangent to a Curve –** Let $y = f(x)$ be the equation of a curve. The equation of the tangent at $(x_0, y_0)$ is $y - y_0 = m(x - x_0)$, where m = slope of the tangent $= \left.\frac{dy}{dx}\right]_{(x_0, y_0)}$ or $f'(x_0)$
5. **Normal to the Curve –** Let $y = f(x)$ be the equation of the curve Equation of the normal at $(x_0, y_0)$ is

$$y - y_0 = -\frac{1}{m}(x - x_0)$$

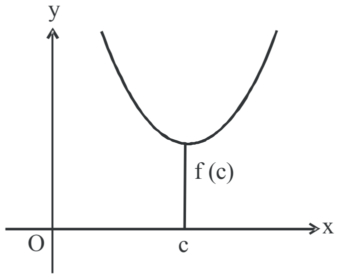
where $m$ = Slope of the tangent at $(x_0, y_0)$

$$= \left.\frac{dy}{dx}\right]_{(x_0, y_0)} \text{ or } f'(x_0)$$

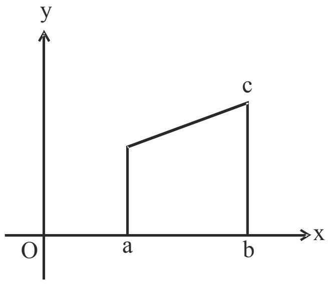
6. **Approximation –** Let $y = f(x)$, $\Delta x$ be a small increament in x and $\Delta y$ be the increament in y corrseponding to the increament in x, i.e., $\Delta y = f(x + \Delta x) - f(x)$. Then approximate value of $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$
7. **Maximum Value, Minimum value, Extreme Value –** Let f be a function defined in the interval I, then
   (i) **Maximum Value –** If there exists a point c in I such that $f(c) \geq f(x)$, for all $x \in I$ then f (c) is called maximum value of f in I. The point c is known as a point of maximum value of f in I.



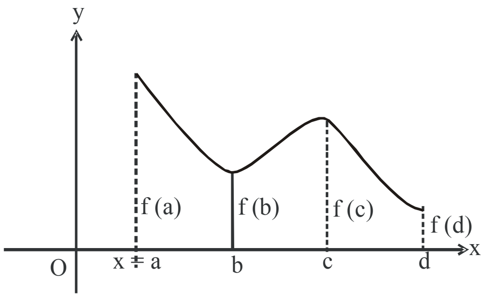
(ii) **Minimum Value –** If three exists a point c in I such that $f(c) \le f(x), \forall x \in I$, then f (x) is called the minimum value of f in I. The point c is called as a point of minimum value of f in I




(iii) **Extreme Value –** If there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I, then f (c) is the extreme value of f (x) in I.
The point c is said to be an extreme point.




**8. Absolute Maxima and Minima –** let f be a continuous function on an interval I = [a, b]. Then f has the absolute maximum value and f attains it at least once in I. Similarly, f has the absolute minimum value and attains at least once in I

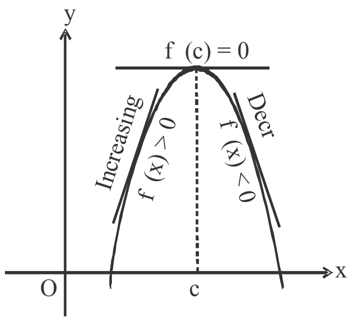


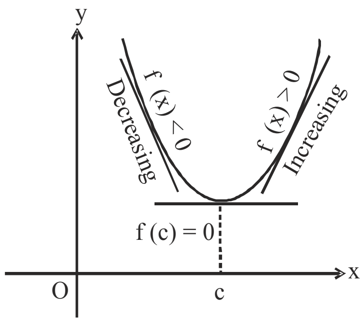

At x = b , there is a local minima
At x = c, there is a local maxima
At x = a, f (a) is the greatest value or absolute max. value.
At x = d, f (d) is the least value or absolute min. value.

**9. Local Maxima and Minima –** let f be a real valued function and c be an interior point in the domain of f, then

(a) **Local Maxima –** c is a point of local maxima if there is an h > 0, such that $f(c) \ge f(x)$ for all $x \in [c - h, c + h)$
The value f (c) is called local maximum value of f.

(b) **Local Minima –** c is a point of local minima if there is an h > 0, such that $f(c) \le f(x)$ for all $x \in (c - h\ c + h)$
The value of f (c) is known as the local minimum value of f.
**Geometrically –** If x = c is a point of local maxima of f, then






f is increasing (i.e., $f'(x) > 0$) in the interval $(c - h, c)$ and decreasing (i.e., $f'(x) < 0$) in the interval $(c, c + h)$

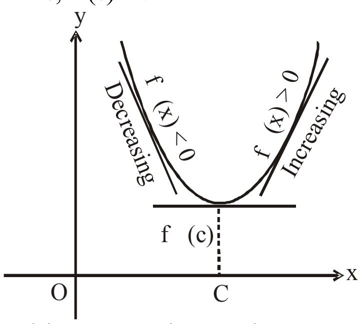
$\Rightarrow$ $f'(c) = 0$

Similarly, if $x = c$ is a point of local minima of f, then f is decreasing (i.e., $f'(x) < 0$) in the interval $(c - h, c)$ and increasing (i.e., $f'(x) > 0$) in the interval $(c, c + h)$.

$\Rightarrow$ $f'(c) = 0$

**10. Test of Local Maxima and Minima –**

(i) Let f be a differentiable function defined on an open interval I and $c \in I$ be any point. f has a local maxima or a local minima at $x = c$, $f'(c) = 0$




(ii) If $f'(x)$ changes sign from positive to negative as x increases from left to right through c i.e.,
(a) $f'(x) > 0$ at every point in $(c - h, c)$
(b) $f'(x) < 0$ at every point in $(c, c + h)$
Then c is called a point of local maxima of f and f(c) is local maximum value of f.

(iii) If $f'(x)$ changes sign from negative to positive as x increase from left to right through c i.e.,
(a) $f'(x) < 0$ at every point in $(c - h, c)$
(b) $f'(x) > 0$ at every point in $(c, c + h)$
Then c is called a point of local minima of f and f(c) is a local minimum value of f.

(iv) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Such a point is called point of inflection.

**11. Second Derivative Test of Local Maxima and Minima –** let f be a twice differentiable function defined on an interval I and $c \in I$ and f be differentiable at $c \in I$, then,

(i) $x = c$ is a local maxima,
if $f'(c) = 0$ and $f''(c) < 0$.
f(c) is the local maximum value of f

(ii) $x = c$ is a local minima, if $f'(c) = 0$ and $f''(c) > 0$
f(c) is the local minimum value of f.

(iii) Point of inflection If $f'(c) = 0$ and $f''(c) = 0$
Test fails. Then we apply first derivative test and find whether c is a point of local maxima, local minima or a point of inflexion.

12. **To find absolute maximum value or absolute minimum value –**

(i) Find all the critical points where $f'(x) = 0$

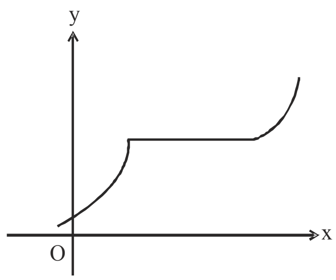
(ii) Consider the end point also.

(iii) Calculate the functional values at all the points found in step (i) and (ii)

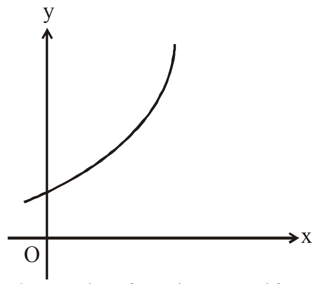
(iv) Identify the maximum and minimum values out of the values calculated in step (iii). These are absolute maximum and absolute minimum values.

## CONNECTING CONCEPTS

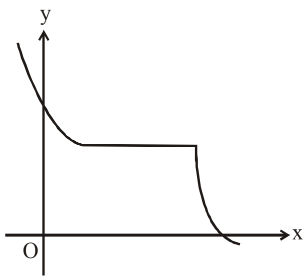
1. **Increasing Function –** f is said to be increasing on I, if $x_1 < x_2$ on I, then $f(x_1) \le f(x_2)$. for all $x_1, x_2 \in I$.



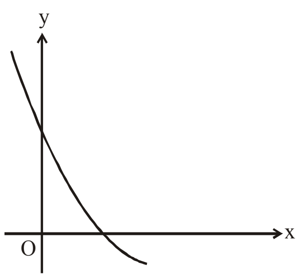

2. **Strictly Increasing function –** f is said to be strictly increasing on I, if $x_1 < x_2$ in I then $f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$.




3. **Decreasing function –** f is said to be decreasing function on I, if $x_1 < x_2$ in I, then $f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in I$.




4. **Strictly Decreasing function –** f is said to be strictly decreasing function on I, if $x_1 > x_2$ in I then $f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$.



**5. Particular case of tangent –** Let $m = \tan\theta$

If $\theta = 0, m = 0$

Equation of tangent is $y - y_0 = 0$ i.e., $y = y_0$

If $\quad \theta = \frac{\pi}{2}$ , m is not defined.

$\therefore \quad (x - x_0) = \frac{1}{m}(y - y_0)$

when $\quad \theta = \frac{\pi}{2}, \cot\frac{\pi}{2} = 0$

$\therefore$ Equation of tangent is $x - x_0 = 0$ or $x = x_0$

# Class 12 Maths NCERT Solutions

| NCERT Solutions | Important Questions | NCERT Exemplar |
|---|---|---|
| Chapter 1 Relations and Functions | **Relations and Functions** | Chapter 1 Relations and Functions |
| Chapter 2 Inverse Trigonometric Functions | Concept of Relations and Functions | Chapter 2 Inverse Trigonometric Functions |
| Chapter 3 Matrices | Binary Operations | Chapter 3 Matrices |
| Chapter 4 Determinants | Inverse Trigonometric Functions | Chapter 4 Determinants |
| Chapter 5 Continuity and Differentiability | **Matrices** | Chapter 5 Continuity and Differentiability |
| Chapter 6 Application of Derivatives | Matrix and Operations of Matrices | Chapter 6 Application of Derivatives |
| Chapter 7 Integrals Ex 7.1 | Transpose of a Matrix and Symmetric Matrix | Chapter 7 Integrals |
| Integrals Class 12 Ex 7.2 | Inverse of a Matrix by Elementary Operations | Chapter 8 Applications of Integrals |
| Integrals Class 12 Ex 7.3 | **Determinants** | Chapter 9 Differential Equations |
| Integrals Class 12 Ex 7.4 | Expansion of Determinants | Chapter 10 Vector Algebra |
| Integrals Class 12 Ex 7.5 | Properties of Determinants | Chapter 11 Three Dimensional Geometry |
| Integrals Class 12 Ex 7.6 | Inverse of a Matrix and Application of Determinants and Matrix | Chapter 12 Linear Programming |
| Integrals Class 12 Ex 7.7 | **Continuity and Differentiability** | Chapter 13 Probability |
| Integrals Class 12 Ex 7.8 | Continuity | |
| Integrals Class 12 Ex 7.9 | Differentiability | |
| Integrals Class 12 Ex 7.10 | **Application of Derivatives** | |
| Integrals Class 12 Ex 7.11 | Rate Measure Approximations and Increasing-Decreasing Functions | |
| Integrals Class 12 Miscellaneous Exercise | Tangents and Normals | |
| Chapter 8 Application of Integrals | Maxima and Minima | |
| Chapter 9 Differential Equations | **Integrals** | |
| Chapter 10 Vector Algebra | Types of Integrals | |
| Chapter 11 Three Dimensional Geometry | **Differential Equation** | |
| Chapter 12 Linear Programming | Formation of Differential Equations | |
| Chapter 13 Probability Ex | Solution of Different Types of Differential | |

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|---|---|---|
| 13.1 | Equations | |
| Probability Solutions Ex 13.2 | **Vector Algebra** | |
| Probability Solutions Ex 13.3 | Algebra of Vectors | |
| Probability Solutions Ex 13.4 | Dot and Cross Products of Two Vectors | |
| Probability Solutions Ex 13.5 | **Three Dimensional Geometry** | |
| | Direction Cosines and Lines | |
| | Plane | |
| | Linear Programming | |
| | **Probability** | |
| | Conditional Probability and Independent Events | |
| | Baye's Theorem and Probability Distribution | |

## RD Sharma Class 12 Solutions

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|---|---|---|
| Chapter 1: Relations | Chapter 12: Higher Order Derivatives | Chapter 23 Algebra of Vectors |
| Chapter 2: Functions | Chapter 13: Derivative as a Rate Measurer | Chapter 24: Scalar Or Dot Product |
| Chapter 3: Binary Operations | Chapter 14: Differentials, Errors and Approximations | Chapter 25: Vector or Cross Product |
| Chapter 4: Inverse Trigonometric Functions | Chapter 15: Mean Value Theorems | Chapter 26: Scalar Triple Product |
| Chapter 5: Algebra of Matrices | Chapter 16: Tangents and Normals | Chapter 27: Direction Cosines and Direction Ratios |
| Chapter 6: Determinants | Chapter 17: Increasing and Decreasing Functions | Chapter 28 Straight line in space |
| Chapter 7: Adjoint and Inverse of a Matrix | Chapter 18: Maxima and Minima | Chapter 29: The plane |
| Chapter 8: Solution of Simultaneous Linear Equations | Chapter 19: Indefinite Integrals | Chapter 30: Linear programming |
| Chapter 9: Continuity | Chapter 20: Definite Integrals | Chapter 31: Probability |
| Chapter 10: Differentiability | Chapter 21: Areas of Bounded Regions | Chapter 32: Mean and variance of a random variable |
| Chapter 11: Differentiation | Chapter 22: Differential Equations | Chapter 33: Binomial Distribution |

## JEE Main Maths Chapter wise Previous Year Questions

1. Relations, Functions and Reasoning
2. Complex Numbers
3. Quadratic Equations And Expressions
4. Matrices, Determinatnts and Solutions of Linear Equations
5. Permutations and Combinations
6. Binomial Theorem and Mathematical Induction
7. Sequences and Series
8. Limits,Continuity,Differentiability and Differentiation
9. Applications of Derivatives
10. Indefinite and Definite Integrals
11. Differential Equations and Areas
12. Cartesian System and Straight Lines
13. Circles and System of Circles
14. Conic Sections
15. Three Dimensional Geometry
16. Vectors
17. Statistics and Probability
18. Trignometry
19. Miscellaneous

## NCERT Solutions for Class 12

- NCERT Solutions for Class 12 Maths
- NCERT Solutions for Class 12 Physics
- NCERT Solutions for Class 12 Chemistry
- NCERT Solutions for Class 12 Biology
- NCERT Solutions for Class 12 English
- NCERT Solutions for Class 12 English Vistas
- NCERT Solutions for Class 12 English Flamingo
- NCERT Solutions for Class 12 Hindi
- NCERT Solutions for Class 12 Hindi Aroh (आरोह भाग 2)
- NCERT Solutions for Class 12 Hindi Vitan (वितान भाग 2)
- NCERT Solutions for Class 12 Business Studies
- NCERT Solutions for Class 12 Accountancy
- NCERT Solutions for Class 12 Psychology
- NCERT Solutions for Class 12 Sociology
- NCERT Solutions for Class 12 History
- NCERT Solutions for Class 12 Entrepreneurship
- NCERT Solutions for Class 12 Political Science
- NCERT Solutions for Class 12 Economics
- NCERT Solutions for Class 12 Macro Economics
- NCERT Solutions for Class 12 Micro Economics
- NCERT Solutions for Class 12 Computer Science (C++)
- NCERT Solutions for Class 12 Computer Science (Python)