9 DIFFERENTIAL EQUATIONS

KEY CONCEPT INVOLVED

1. **Differential Equation** – An equation containing an independent variable dependent variable and differential coefficient of dependent variable with respect to independent variable is called a differential equation.

   e.g. $\frac{dy}{dx} + 2xy = x^3$ and $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$

2. **Order of a differential Equation** – The order of a differential equation is the order of the highest order derivative appearing in the equation.
3. **Degree of a differential Equation** – The degree of a differential equation is the degree of the highest order derivative when differential coefficients are made free from radicals and fractions.
4. **Solution of a differential Equation** – The solution of a differential equation is a relation between the variables involved, not involving the differential coefficients, such that this relation and derivatives obtained form it satisfy the given differential equation.
5. **General Solution** – The solution which contains as many as orbirary constants as the order of the differential equation is called the general solution of the differential equation.
6. **Particular Solution** – Solution obtained by giving particular values to the arbitrary constants in the general solution of a differential equation is called a particular solution.
7. **Equations in variable separable form** – If the differential equation can be reduced to the form $f(x)\,dx = g(y)\,dy$ we say that the variables have been separated on integrating both sides of this reduced form, we get the general solution of the differential equation.

   $$\int f(x)\,dx = \int g(y)\,dy + c$$

8. **Equations Reducible to variable separable form** – Differential equations of the form $\frac{dy}{dx} = f(ax + by + c)$ can be reduced to variable separable form by the substitution $ax + by + c = v$
9. **Homogeneous Differential Equation** – A function f(x,y) is called a homogeneous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$ for any non zero constant λ.

   A differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero. To solve such ... a homogenous differential equation of the type

   $$\frac{dy}{dx} = F(x) = g\left(\frac{y}{x}\right) \qquad \text{...(i)}$$

   (i) Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in equation (i), we get reduces to the form $v + x\frac{dv}{dx} = g(v)$

   $$\Rightarrow \quad x \times \frac{dy}{dx} = g(v) - v$$

   Now, on separating the variables, we get

   $$\frac{dv}{g(x) - v} = \frac{dx}{x}$$

   Integrate both sides to obtain the solution in terms of v and x.

   Replace v by $\frac{y}{x}$ in the solution obtained to obtain the solution in terms of x and y.

If the homogeneous differential equation is in the form $\frac{dy}{dx} = F(x, y)$, where F (x, y) is homogeneous function of degree, then we make substitution $\frac{x}{y} = v$ i.e., $x = vy$ and the proceed further to find the general solution as discussed above by writting $\frac{dx}{dy} = F(x, y) = h\left(\frac{x}{y}\right)$

10. **Linear differential Equations** – A differential equation is known as first order linear differential equation, if the dependent variable y and its derivative are related as $\frac{dy}{dx} + Py = Q$, where P and Q are constant or functions of x.

Steps involved to solve first order linear differential equation:

(i) Write the given differential equation in the form $\frac{dy}{dx} + Py = Q$ and obtain P and Q.

(ii) Find integrating factor, I.F. = $e^{\int p dx}$

(iii) Multiply both sides of equation in (i) by I.F.

(iv) Integrate both sides of the equation obtained in (iii) w.r.t. x to obtain

$$y(\text{I.F.}) = \int Q.(\text{I.F.})\, dx + C$$

This gives the required solution.

In case, the first order linear differential equation is in the form $\frac{dx}{dy} + P_1 x = Q_1$, where , P_1 and Q_1 are constants or functions of y only. Then I.F. = $e^{\int P_1 dy}$ and the solution of the differential equation is given by $x \,.\, (\text{I. F}) = \int (Q_1 \cdot \text{I.F.})\, dy + C$

CONNECTING CONCEPTS

1. **Formation of Differential Equations** – Formation of a differential from a given equation representing a family of curves means finding a differential equation whose solution is the given equation. If an equation representing a family of curves, contains n arbitrary constants, then we differentiable the given equation n times to obtain n more equations. Using all these equations, we eliminate the constants. The equation so obtained is the differential equation of order n for the family of given curves.

2. **Methods of solving a differential equation of the type $\frac{dy}{dx} = f(x)$** – To solve this type of differential equations, first we write the differential equation as $dy = f(x)\, dx$

Then integrate boht sides with respect t x to obtain the solution

$$\int dy = \int f(x)\, dx + C$$

or $$y = \int f(x)\, dx + C$$

3. **Differential Equations of the type $\frac{dy}{dx} = f(y)$** – To solve this type of differential equations, first we write in the form of $dx = \frac{1}{f(y)}\, dy$ them integrate both sides to obtain the general solution

$$\Rightarrow \int dx = \int \frac{1}{f(y)}\, dy + c \text{ or } x = \int \frac{1}{f(y)}\, dy + c$$

4. Differential Equations of the type $\frac{d^2y}{dx^2} = f(x)$

(i) Integrate both sides of the differential equation in (i) with respect to x to obtain a first order first degree differential equation.

(ii) Integrate both sides of the first order differential equation obtained in (ii) with respect to x.

Class 12 Maths NCERT Solutions

NCERT Solutions	Important Questions	NCERT Exemplar
Chapter 1 Relations and Functions	**Relations and Functions**	Chapter 1 Relations and Functions
Chapter 2 Inverse Trigonometric Functions	Concept of Relations and Functions	Chapter 2 Inverse Trigonometric Functions
Chapter 3 Matrices	Binary Operations	Chapter 3 Matrices
Chapter 4 Determinants	Inverse Trigonometric Functions	Chapter 4 Determinants
Chapter 5 Continuity and Differentiability	**Matrices**	Chapter 5 Continuity and Differentiability
Chapter 6 Application of Derivatives	Matrix and Operations of Matrices	Chapter 6 Application of Derivatives
Chapter 7 Integrals Ex 7.1	Transpose of a Matrix and Symmetric Matrix	Chapter 7 Integrals
Integrals Class 12 Ex 7.2	Inverse of a Matrix by Elementary Operations	Chapter 8 Applications of Integrals
Integrals Class 12 Ex 7.3	**Determinants**	Chapter 9 Differential Equations
Integrals Class 12 Ex 7.4	Expansion of Determinants	Chapter 10 Vector Algebra
Integrals Class 12 Ex 7.5	Properties of Determinants	Chapter 11 Three Dimensional Geometry
Integrals Class 12 Ex 7.6	Inverse of a Matrix and Application of Determinants and Matrix	Chapter 12 Linear Programming
Integrals Class 12 Ex 7.7	**Continuity and Differentiability**	Chapter 13 Probability
Integrals Class 12 Ex 7.8	Continuity	
Integrals Class 12 Ex 7.9	Differentiability	
Integrals Class 12 Ex 7.10	**Application of Derivatives**	
Integrals Class 12 Ex 7.11	Rate Measure Approximations and Increasing-Decreasing Functions	
Integrals Class 12 Miscellaneous Exercise	Tangents and Normals	
Chapter 8 Application of Integrals	Maxima and Minima	
Chapter 9 Differential Equations	**Integrals**	
Chapter 10 Vector Algebra	Types of Integrals	
Chapter 11 Three Dimensional Geometry	**Differential Equation**	
Chapter 12 Linear Programming	Formation of Differential Equations	
Chapter 13 Probability Ex	Solution of Different Types of Differential	

13.1	Equations	
Probability Solutions Ex 13.2	**Vector Algebra**	
Probability Solutions Ex 13.3	Algebra of Vectors	
Probability Solutions Ex 13.4	Dot and Cross Products of Two Vectors	
Probability Solutions Ex 13.5	**Three Dimensional Geometry**	
	Direction Cosines and Lines	
	Plane	
	Linear Programming	
	Probability	
	Conditional Probability and Independent Events	
	Baye's Theorem and Probability Distribution	

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Chapter 1: Relations	Chapter 12: Higher Order Derivatives	Chapter 23 Algebra of Vectors
Chapter 2: Functions	Chapter 13: Derivative as a Rate Measurer	Chapter 24: Scalar Or Dot Product
Chapter 3: Binary Operations	Chapter 14: Differentials, Errors and Approximations	Chapter 25: Vector or Cross Product
Chapter 4: Inverse Trigonometric Functions	Chapter 15: Mean Value Theorems	Chapter 26: Scalar Triple Product
Chapter 5: Algebra of Matrices	Chapter 16: Tangents and Normals	Chapter 27: Direction Cosines and Direction Ratios
Chapter 6: Determinants	Chapter 17: Increasing and Decreasing Functions	Chapter 28 Straight line in space
Chapter 7: Adjoint and Inverse of a Matrix	Chapter 18: Maxima and Minima	Chapter 29: The plane
Chapter 8: Solution of Simultaneous Linear Equations	Chapter 19: Indefinite Integrals	Chapter 30: Linear programming
Chapter 9: Continuity	Chapter 20: Definite Integrals	Chapter 31: Probability
Chapter 10: Differentiability	Chapter 21: Areas of Bounded Regions	Chapter 32: Mean and variance of a random variable
Chapter 11: Differentiation	Chapter 22: Differential Equations	Chapter 33: Binomial Distribution

JEE Main Maths Chapter wise Previous Year Questions

1. Relations, Functions and Reasoning
2. Complex Numbers
3. Quadratic Equations And Expressions
4. Matrices, Determinatnts and Solutions of Linear Equations
5. Permutations and Combinations
6. Binomial Theorem and Mathematical Induction
7. Sequences and Series
8. Limits,Continuity,Differentiability and Differentiation
9. Applications of Derivatives
10. Indefinite and Definite Integrals
11. Differential Equations and Areas
12. Cartesian System and Straight Lines
13. Circles and System of Circles
14. Conic Sections
15. Three Dimensional Geometry
16. Vectors
17. Statistics and Probability
18. Trignometry
19. Miscellaneous

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- NCERT Solutions for Class 12 Macro Economics
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- NCERT Solutions for Class 12 Computer Science (C++)
- NCERT Solutions for Class 12 Computer Science (Python)