Knowledge... Everywhere

Physics

Units, Dimensions & Measurement

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1. Physical Quantity.

A quantity which can be measured and by which various physical happenings can be explained and expressed in form of laws is called a physical quantity. For example length, mass, time, force etc.

On the other hand various happenings in life e.g., happiness, sorrow etc. are not physical quantities because these cannot be measured.

Measurement is necessary to determine magnitude of a physical quantity, to compare two similar physical quantities and to prove physical laws or equations.

A physical quantity is represented completely by its magnitude and unit. For example, 10 meter means a length which is ten times the unit of length 1 kg. Here 10 represents the numerical value of the given quantity and meter represents the unit of quantity under consideration. Thus in expressing a physical quantity we choose a unit and then find that how many times that unit is contained in the given physical quantity, i.e.

Physical quantity (Q) = Magnitude × Unit = n × u

Where, n represents the numerical value and u represents the unit. Thus while expressing definite amount of physical quantity, it is clear that as the unit (u) changes, the magnitude (n) will also change but product ‘nu’ will remain same.

i.e. n u = constant, or $n_1u_1 = n_2u_2 = \text{constant}$; $\therefore \quad n \propto \frac{1}{u}$

i.e. magnitude of a physical quantity and units are inversely proportional to each other .Larger the unit, smaller will be the magnitude.

2. Types of Physical Quantity.

(1) Ratio (numerical value only): When a physical quantity is a ratio of two similar quantities, it has no unit.

e.g. Relative density = Density of object/Density of water at 4°C

Refractive index = Velocity of light in air / Velocity of light in medium

Strain = Change in dimension/Original dimension

Note: Angle is exceptional physical quantity, which though is a ratio of two similar physical quantities (angle = arc / radius) but still requires a unit (degrees or radians) to specify it along with its numerical value.

(2) Scalar (Magnitude only): These quantities do not have any direction e.g. Length, time, work, energy etc.

Magnitude of a physical quantity can be negative. In that case negative sign indicates that the numerical value of the quantity under consideration is negative. It does not specify the direction.

Scalar quantities can be added or subtracted with the help of following ordinary laws of addition or subtraction.

(3) Vector (magnitude and direction): e.g. displacement, velocity, acceleration, force etc.

Vector physical quantities can be added or subtracted according to vector laws of addition. These laws are different from laws of ordinary addition.

Note: There are certain physical quantities which behave neither as scalar nor as vector. For example, moment of inertia is not a vector as by changing the sense of rotation its value is not changed.

It is also not a scalar as it has different values in different directions (i.e. about different axes). Such physical quantities are called Tensors.

3. Fundamental and Derived Quantities.

(1) Fundamental quantities**:** Out of large number of physical quantities which exist in nature, there are only few quantities which are independent of all other quantities and do not require the help of any other physical quantity for their definition, therefore these are called absolute quantities. These quantities are also called fundamental or base quantities, as all other quantities are based upon and can be expressed in terms of these quantities.

(2) Derived quantities**:** All other physical quantities can be derived by suitable multiplication or division of different powers of fundamental quantities. These are therefore called derived quantities.

If length is defined as a fundamental quantity then area and volume are derived from length and are expressed in term of length with power 2 and 3 over the term of length.

Note: In mechanics Length, Mass and time are arbitrarily chosen as fundamental quantities. However this set of fundamental quantities is not a unique choice. In fact any three quantities in mechanics can be termed as fundamental as all other quantities in mechanics can be expressed in terms of these.

e.g. if speed and time are taken as fundamental quantities, length will become a derived quantity because then length will be expressed as Speed × Time. and if force and acceleration are taken as fundamental quantities, then mass will be defined as Force / acceleration and will be termed as a derived quantity.

4. Fundamental and Derived Units.

Normally each physical quantity requires a unit or standard for its specification so it appears that there must be as many units as there are physical quantities. However, it is not so. It has been found that if in mechanics we choose arbitrarily units of any three physical quantities we can express the units of all other physical quantities in mechanics in terms of these. Arbitrarily the physical quantities mass, length and time are chosen for this purpose. So any unit of mass, length and time in mechanics is called a **fundamental, absolute or base unit.** Other units which can be expressed in terms of fundamental units, are called derived units. For example light year or km is a fundamental units as it is a unit of length while s^{-1}, m^2 or kg/m are derived units as these are derived from units of time, mass and length respectively.

System of units: A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units. The common systems are given below –

(1) CGS system**:** The system is also called Gaussian system of units. In it length, mass and time have been chosen as the fundamental quantities and corresponding fundamental units are centimeter (cm), gram (g) and second (s) respectively.

(2) MKS system**:** The system is also called Giorgi system. In this system also length, mass and time have been taken as fundamental quantities, and the corresponding fundamental units are meter, kilogram and second.

(3) FPS system**:** In this system foot, pound and second are used respectively for measurements of length, mass and time. In this system force is a derived quantity with unit poundal.

(4) S. I. system: It is known as International system of units, and is infact extended system of units applied to whole physics. There are seven fundamental quantities in this system. These quantities and their units are given in the following table

Quantity	Name of Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	Kelvin	K
Amount of Substance	mole	mol
Luminous Intensity	candela	cd

Besides the above seven fundamental units two supplementary units are also defined

–

Radian (rad) for plane angle and Steradian (sr) for solid angle.

Note: Apart from fundamental and derived units we also use very frequently practical units. These may be fundamental or derived units

e.g., light year is a practical unit (fundamental) of distance while horse power is a practical unit (derived) of power.

Practical units may or may not belong to a system but can be expressed in any system of units

e.g., 1 mile = 1.6 km = 1.6 × 10^3 m.

5. S.I. Prefixes.

In physics we have to deal from very small (micro) to very large (macro) magnitudes as one side we talk about the atom while on the other side of universe, e.g., the mass of an electron is 9.1×10^{-31} kg while that of the sun is 2×10^{30} kg. To express such large or small magnitudes simultaneously we use the following prefixes:

Power of 10	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deca	da
10^{-1}	deci	d
10^{-1}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

6. Standards of Length, Mass and Time.

(1) Length: Standard meter is defined in terms of wavelength of light and is called atomic standard of length.

The meter is the distance containing 1650763.73 wavelength in vacuum of the radiation corresponding to orange red light emitted by an atom of krypton-86.

Now a days meter is defined as length of the path travelled by light in vacuum in 1/299, 7792, 458 part of a second.

(2) Mass: The mass of a cylinder made of platinum-iridium alloy kept at International Bureau of Weights and Measures is defined as 1 kg.

On atomic scale, 1 kilogram is equivalent to the mass of 5.0188 × 10^{25} atoms of $_6C^{12}$ (an isotope of carbon).

(3) Time**:** 1 second is defined as the time interval of 9192631770 vibrations of radiation in Cs-133 atom. This radiation corresponds to the transition between two hyperfine levels of the ground state of Cs-133.

7. Practical Units.

(1) Length**:**

(i) 1 fermi = 1 fm = 10^{-15} m

(ii) 1 X-ray unit = 1XU = 10^{-13} m

(iii) 1 angstrom = 1Å = 10^{-10} m = 10^{-8} cm = 10^{-7} mm = 0.1 μmm

(iv) 1 micron = μm = 10^{-6} m

(v) 1 astronomical unit = 1 A.U. = 1. 49 × 10^{11} m ≈ 1.5 × 10^{11} m ≈ 10^{8} km

(vi) 1 Light year = 1 ly = 9.46 × 10^{15} m

(vii) 1 Parsec = 1pc = 3.26 light year

(2) Mass**:**

(i) Chandra Shekhar unit: 1 CSU = 1.4 times the mass of sun = 2.8 × 10^{30} kg

(ii) Metric tonne: 1 Metric tonne = 1000 kg

(iii) Quintal: 1 Quintal = 100 kg

(iv) Atomic mass unit (amu): amu = 1.67 × 10^{-27} kg mass of proton or neutron is of the order of 1 amu

(3) Time**:**

(i) Year: It is the time taken by earth to complete 1 revolution around the sun in its orbit.

(ii) Lunar month: It is the time taken by moon to complete 1 revolution around the earth in its orbit.

1 L.M. = 27.3 days

(iii) Solar day: It is the time taken by earth to complete one rotation about its axis with respect to sun. Since this time varies from day to day, average solar day is calculated by taking average of the duration of all the days in a year and this is called Average Solar day.

1 Solar year = 365.25 average solar day

Or

Average solar day $= \frac{1}{365.25}$ the part of solar year

(iv) Sedrial day: It is the time taken by earth to complete one rotation about its axis with respect to a distant star.

1 Solar year = 366.25 Sedrial day = 365.25 average solar day. Thus 1 Sedrial day is less than 1 solar day.

(v) Shake: It is an obsolete and practical unit of time.

1 Shake = 10^{-8} sec

8. Dimensions of a Physical Quantity.

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

To make it more clear, consider the physical quantity force

Force = mass × acceleration $= \frac{\text{mass} \times \text{velocity}}{\text{time}} = \frac{\text{mass} \times \text{length/time}}{\text{time}}$ = mass × length × $(\text{time})^{-2}$ .... (i)

Thus, the dimensions of force are 1 in mass, 1 in length and – 2 in time. Here the physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to indicate that the equation is among the dimensions and not among the magnitudes.

Thus equation (i) can be written as [force] = $[MLT^{-2}]$.

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional equation. If we consider only the R.H.S. of the equation, the expression is termed as dimensional formula.

Thus, dimensional formula for force is, $[MLT^{-2}]$.

9. Important Dimensions of Complete Physics

Mechanics

S. N.	**Quantity**	**Unit**	**Dimension**
(1)	Velocity or speed (v)	m/s	**$[M^0L^1T^{-1}]$**
(2)	Acceleration (a)	m/s^2	**$[M^0LT^{-2}]$**
(3)	Momentum (P)	kg-m/s	**$[M^1L^1T^{-1}]$**
(4)	Impulse (I)	Newton-sec or kg-m/s	$[M^1L^1T^{-1}]$
(5)	Force (F)	Newton	**$[M^1L^1T^{-2}]$**
(6)	Pressure (P)	Pascal	**$[M^1L^{-1}T^{-2}]$**
(7)	Kinetic energy (E_K)	Joule	**$[M^1L^2T^{-2}]$**
(8)	Power (P)	Watt or Joule/s	**$[M^1L^2T^{-3}]$**
(9)	Density (d)	kg/m^3	**$[M^1L^{-3}T^0]$**
(10)	Angular displacement (θ)	Radian (rad.)	**$[M^0L^0T^0]$**
(11)	Angular velocity (ω)	Radian/sec	**$[M^0L^0T^{-1}]$**
(12)	Angular acceleration (α)	$Radian/sec^2$	**$[M^0L^0T^{-2}]$**
(13)	Moment of inertia (I)	$kg\text{-}m^2$	**$[M^1L^2T^0]$**
(14)	Torque (τ)	Newton-meter	**$[M^1L^2T^{-2}]$**
(15)	Angular momentum (L)	Joule-sec	$[M^1L^2T^{-1}]$
(16)	Force constant or spring constant (k)	Newton/m	**$[M^1L^0T^{-2}]$**

S. N.	**Quantity**	**Unit**	**Dimension**
(17)	Gravitational constant (G)	N-m^2/kg^2	**$[M^{-1}L^3T^{-2}]$**
(18)	Intensity of gravitational field (E_g)	N/kg	**$[M^0L^1T^{-2}]$**
(19)	Gravitational potential (V_g)	Joule/kg	**$[M^0L^2T^{-2}]$**
(20)	Surface tension (T)	N/m or Joule/m^2	**$[M^1L^0T^{-2}]$**
(21)	Velocity gradient (V_g)	Second^{-1}	**$[M^0L^0T^{-1}]$**
(22)	Coefficient of viscosity (η)	kg/m-s	$[M^1L^{-1}T^{-1}]$
(23)	Stress	N/m^2	**$[M^1L^{-1}T^{-2}]$**
(24)	Strain	No unit	**$[M^0L^0T^0]$**
(25)	Modulus of elasticity (E)	N/m^2	**$[M^1L^{-1}T^{-2}]$**
(26)	Poisson Ratio (σ)	No unit	**$[M^0L^0T^0]$**
(27)	Time period (T)	Second	**$[M^0L^0T^1]$**
(28)	Frequency (n)	Hz	**$[M^0L^0T^{-1}]$**

Heat

S. N.	**Quantity**	**Unit**	**Dimension**
(1)	Temperature (T)	Kelvin	**$[M^0L^0T^0\theta^1]$**
(2)	Heat (Q)	Joule	**$[ML^2T^{-2}]$**
(3)	Specific Heat (c)	Joule/kg-K	**$[M^0L^2T^{-2}\theta^{-1}]$**
(4)	Thermal capacity	Joule/K	**$[M^1L^2T^{-2}\theta^{-1}]$**
(5)	Latent heat (L)	Joule/kg	**$[M^0L^2T^{-2}]$**
(6)	Gas constant (R)	Joule/mol-K	**$[M^1L^2T^{-2}\theta^{-1}]$**
(7)	Boltzmann constant (k)	Joule/K	**$[M^1L^2T^{-2}\theta^{-1}]$**
(8)	Coefficient of thermal conductivity (K)	Joule/m-s-K	**$[M^1L^1T^{-3}\theta^{-1}]$**
(9)	Stefan's constant (σ)	Watt/m^2-K^4	**$[M^1L^0T^{-3}\theta^{-4}]$**
(10)	Wien's constant (b)	Meter-K	**$[M^0L^1T^o\theta^1]$**
(11)	Planck's constant (h)	Joule-s	**$[M^1L^2T^{-1}]$**

S. N.	Quantity	Unit	Dimension
(12)	Coefficient of Linear Expansion (□)	Kelvin^{-1}	**$[M^0L^0T^0\theta^{-1}]$**
(13)	Mechanical eq. of Heat (J)	Joule/Calorie	**$[M^0L^0T^0]$**
(14)	Vander wall's constant (a)	Newton-m^4	**$[ML^5T^{-2}]$**
(15)	Vander wall's constant (b)	m^3	**$[M^0L^3T^0]$**

Electricity

S. N.	Quantity	Unit	Dimension
(1)	Electric charge (q)	Coulomb	**$[M^0L^0T^1A^1]$**
(2)	Electric current (I)	Ampere	**$[M^0L^0T^0A^1]$**
(3)	Capacitance (C)	Coulomb/volt or Farad	**$[M^{-1}L^{-2}T^4A^2]$**
(4)	Electric potential (V)	Joule/coulomb	**$M^1L^2T^{-3}A^{-1}$**
(5)	Permittivity of free space (ε_0)	$\frac{Coulomb^2}{Newton\text{-}meter^2}$	**$[M^{-1}L^{-3}T^4A^2]$**
(6)	Dielectric constant (K)	Unitless	**$[M^0L^0T^0]$**
(7)	Resistance (R)	Volt/Ampere or ohm	**$[M^1L^2T^{-3}A^{-2}]$**
(8)	Resistivity or Specific resistance (ρ)	Ohm-meter	**$[M^1L^3T^{-3}A^{-2}]$**
(9)	Coefficient of Self-induction (L)	$\frac{volt-second}{ampere}$ or henery or ohm-second	**$[M^1L^2T^{-2}A^{-2}]$**
(10)	Magnetic flux (φ)	Volt-second or weber	**$[M^1L^2T^{-2}A^{-1}]$**
(11)	Magnetic induction (B)	$\frac{newton}{ampere-meter}$ $\frac{Joule}{ampere-meter^2}$ $\frac{volt-second}{meter^2}$ or Tesla	**$[M^1L^0T^{-2}A^{-1}]$**
(12)	Magnetic Intensity (H)	Ampere/meter	**$[M^0L^{-1}T^0A^1]$**
(13)	Magnetic Dipole Moment (M)	Ampere-meter^2	**$[M^0L^2T^0A^1]$**

S. N.	Quantity	Unit	Dimension
(14)	Permeability of Free Space (μ_0)	$\frac{Newton}{ampere^2}$ or $\frac{Joule}{ampere^2 - meter}$ or $\frac{Volt - second}{ampere - meter}$ or $\frac{Ohm - sec ond}{meter}$ or $\frac{henery}{meter}$	**$[M^1L^1T^{-2}A^{-2}]$**
(15)	Surface charge density (σ)	$Coulomb\ metre^{-2}$	**$[M^0L^{-2}T^1A^1]$**
(16)	Electric dipole moment (p)	$Coulomb - meter$	**$[M^0L^1T^1A^1]$**
(17)	Conductance (G) (1/R)	ohm^{-1}	**$[M^{-1}L^{-2}T^3A^2]$**
(18)	Conductivity (σ) (1/ρ)	$ohm^{-1} meter^{-1}$	**$[M^{-1}L^{-3}T^3A^2]$**
(19)	Current density (J)	Ampere/m^2	**$M^0L^{-2}T^0A^1$**
(20)	Intensity of electric field (E)	Volt/meter, Newton/coulomb	**$M^1L^1T^{-3}A^{-1}$**
(21)	Rydberg constant (R)	m^{-1}	**$M^0L^{-1}T^0$**

10. Quantities Having Same Dimensions.

S. N.	Dimension	Quantity
(1)	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
(2)	$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
(3)	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
(4)	$[M^1L^1T^{-1}]$	Momentum, impulse
(5)	$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity
(6)	$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
(7)	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
(8)	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)

(9)	$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
(10)	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
(11)	$[M^0L^2T^{-2}\theta^{-1}]$	Thermal capacity, gas constant, Boltzmann constant and entropy
(12)	$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g}$, where l = length g = acceleration due to gravity, m = mass, k = spring constant
(13)	$[M^0L^0T^1]$	L/R, $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance
(14)	$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t,\ VIt,\ qV,\ LI^2,\ \frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

11. Application of Dimensional Analysis.

(1) **To find the unit of a physical quantity in a given system of units:** Writing the definition or formula for the physical quantity we find its dimensions. Now in the dimensional formula replacing M, L and T by the fundamental units of the required system we get the unit of physical quantity. However, sometimes to this unit we further assign a specific name, e.g., Work = Force × Displacement

So $[W] = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

So its units in C.G.S. system will be g cm^2/s^2 which is called erg while in M.K.S. system will be kg m^2/s^2 which is called joule.

(2) **To find dimensions of physical constant or coefficients** : As dimensions of a physical quantity are unique, we write any formula or equation incorporating the given constant and then by substituting the dimensional formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(i) Gravitational constant: According to Newton’s law of gravitation

$$F = G\frac{m_1 m_2}{r^2} \text{ or } G = \frac{Fr^2}{m_1 m_2}$$

Substituting the dimensions of all physical quantities

$$[G] = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}L^3T^{-2}]$$

(ii) Plank constant: According to Planck $E = h\nu$ or $h = \frac{E}{\nu}$

Substituting the dimensions of all physical quantities

$$[h] = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

(iii) Coefficient of viscosity: According to Poiseuille’s formula $\frac{dV}{dt} = \frac{\pi pr^4}{8\eta l}$ or

$$\eta = \frac{\pi pr^4}{8l(dV/dt)}$$

Substituting the dimensions of all physical quantities

$$[\eta] = \frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3/T]} = [ML^{-1}T^{-1}]$$

(3) **To convert a physical quantity from one system to the other:** The measure of a physical quantity is nu = constant

If a physical quantity X has dimensional formula [$M^aL^bT^c$] and if (derived) units of that physical quantity in two systems are $[M_1^a L_1^b T_1^c]$ and $[M_2^a L_2^b T_2^c]$ respectively and n_1 and n_2 be the numerical values in the two systems respectively, then $n_1[u_1] = n_2[u_2]$

$$\Rightarrow n_1[M_1^a L_1^b T_1^c] = n_2[M_2^a L_2^b T_2^c]$$

$$\Rightarrow n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Where M_1, L_1 and T_1 are fundamental units of mass, length and time in the first (known) system and M_2, L_2 and T_2 are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

Example: (1) conversion of Newton into Dyne.

The Newton is the S.I. unit of force and has dimensional formula $[MLT^{-2}]$.

So 1 N = 1 kg-m/ sec^2

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 1\left[\frac{kg}{gm}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2}$

$$= 1\left[\frac{10^3\,gm}{gm}\right]^1\left[\frac{10^2\,cm}{cm}\right]^1\left[\frac{sec}{sec}\right]^{-2} = 10^5$$

∴ 1 N = 10^5 Dyne

(2) Conversion of gravitational constant (G) from C.G.S. to M.K.S. system

The value of G in C.G.S. system is 6.67×10^{-8} C.G.S. units while its dimensional formula is $[M^{-1}L^3T^{-2}]$

So G = 6.67×10^{-8} $cm^3/g\ s^2$

By using $n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c = 6.67\times10^{-8}\left[\frac{gm}{kg}\right]^{-1}\left[\frac{cm}{m}\right]^3\left[\frac{sec}{sec}\right]^{-2}$

$$= 6.67\times10^{-8}\left[\frac{gm}{10^3\,gm}\right]^{-1}\left[\frac{cm}{10^2\,cm}\right]^3\left[\frac{sec}{sec}\right]^{-2} = 6.67\times10^{-11}$$

∴ G = 6.67×10^{-11} M.K.S. units

(4) **To check the dimensional correctness of a given physical relation :** This is based on the 'principle of homogeneity'. According to this principle the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$,

Then according to principle of homogeneity [X] = [A] = $[(BC)^2]$ $= [\sqrt{DEF}]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

Example: (1) $F = mv^2 / r^2$

By substituting dimension of the physical quantities in the above relation –

$$[MLT^{-2}] = [M][LT^{-1}]^2 / [L]^2$$

i.e. $[MLT^{-2}] = [MT^{-2}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(2) $s = ut - (1/2)at^2$

By substituting dimension of the physical quantities in the above relation –

$$[L] = [LT^{-1}][T] - [LT^{-2}][T^2]$$

i.e. $[L] = [L] - [L]$

As in the above equation dimensions of each term on both sides are same, so this equation is dimensionally correct. However, from equations of motion we know that $s = ut + (1/2)at^2$

(5) **As a research tool to derive new relations:** If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

Example: (i) Time period of a simple pendulum.

Let time period of a simple pendulum is a function of mass of the bob (m), effective length (l), acceleration due to gravity (g) then assuming the function to be product of power function of m, l and g

i.e., $T = Km^x l^y g^z$; where K = dimensionless constant

If the above relation is dimensionally correct then by substituting the dimensions of quantities –

$$[T] = [M]^x [L]^y [LT^{-2}]^z$$

or $[M^0L^0T^1] = [M^xL^{y+z}T^{-2z}]$

Equating the exponents of similar quantities x = 0, y = 1/2 and z = – 1/2

So the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$

The value of dimensionless constant is found (2π) through experiments so

$$T = 2\pi\sqrt{\frac{l}{g}}$$

(ii) Stoke's law : When a small sphere moves at low speed through a fluid, the viscous force F, opposing the motion, is found experimentally to depend on the radius r, the velocity of the sphere v and the viscosity η of the fluid.

So $F = f(\eta, r, v)$

If the function is product of power functions of η, r and v, $F = K\eta^x r^y v^z$; where K is dimensionless constant.

If the above relation is dimensionally correct $[MLT^{-2}] = [ML^{-1}T^{-1}]^x[L]^y[LT^{-1}]^z$

or $[MLT^{-2}] = [M^xL^{-x+y+z}T^{-x-z}]$

Equating the exponents of similar quantities x = 1; – x + y + z = 1 and – x – z = – 2

Solving these for x, y and z, we get x = y = z = 1

So eq^n (i) becomes $F = K\eta rv$

On experimental grounds, $K = 6\pi$; so $F = 6\pi\eta rv$

This is the famous Stoke's law.

12. Limitations of Dimensional Analysis.

Although dimensional analysis is very useful it cannot lead us too far as,

(1) If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$it may be work or energy or torque.

(2) Numerical constant having no dimensions [K] such as (1/2), 1 or 2π etc. cannot be deduced by the methods of dimensions.

(3) The method of dimensions cannot be used to derive relations other than product of power functions.

For example: $s = ut + (1/2)at^2$ or $y = a\sin\omega t$

Cannot be derived by using this theory (try if you can). However, the dimensional correctness of these can be checked.

(4) The method of dimensions cannot be applied to derive formula if in mechanics a physical quantity depends on more than 3 physical quantities as then there will be less number (= 3) of equations than the unknowns (>3). However still we can check correctness of the given equation dimensionally. For example $T = 2\pi\sqrt{1/mgl}$ cannot be derived by theory of dimensions but its dimensional correctness can be checked.

(5) Even if a physical quantity depends on 3 physical quantities, out of which two have same dimensions, the formula cannot be derived by theory of dimensions, e.g., formula for the frequency of a tuning fork $f = (d/L^2)v$ cannot be derived by theory of dimensions but can be checked.

13. Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

(1) All non-zero digits are significant.

Example : 42.3 has three significant figures.

243.4 has four significant figures.

24.123 has five significant figures.

(2) A zero becomes significant figure if it appears between to non-zero digits.

Example: 5.03 has three significant figures.

5.604 has four significant figures.

4.004 has four significant figures.

(3) Leading zeroes or the zeroes placed to the left of the number are never significant.

Example: 0.543 has three significant figures.

0.045 has two significant figures.

0.006 has one significant figures.

(4) Trailing zeroes or the zeroes placed to the right of the number are significant.

Example: 4.330 has four significant figures.

433.00 has five significant figures.

343.000 has six significant figures.

(5) In exponential notation, the numerical portion gives the number of significant figures.

Example: 1.32×10^{-2} has three significant figures.
1.32×10^{4} has three significant figures.

14. Rounding Off

While rounding off measurements, we use the following rules by convention:

(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

Example: $x = 7.82$ is rounded off to 7.8, again $x = 3.94$ is rounded off to 3.9.

(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example: x = 6.87 is rounded off to 6.9, again x = 12.78 is rounded off to 12.8.

(3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.

Example: x = 16.351 is rounded off to 16.4, again x = 6.758 is rounded off to 6.8.

(4) If digit to be dropped is 5 or 5 followed by zeroes, then preceding digit is left unchanged, if it is even.

Example: x = 3.250 becomes 3.2 on rounding off, again x = 12.650 becomes 12.6 on rounding off.

(5) If digit to be dropped is 5 or 5 followed by zeroes, then the preceding digit is raised by one, if it is odd.

Example: x = 3.750 is rounded off to 3.8, again x = 16.150 is rounded off to 16.2.

15. Significant Figures in Calculation.

In most of the experiments, the observations of various measurements are to be combined mathematically, i.e., added, subtracted, multiplied or divided as to achieve the final result. Since, all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.

(1) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places. The rule is illustrated by the following examples:

(i) 33.3 ← (has only one decimal place)
3.11
+ 0.313
36.723 ← (answer should be reported to one decimal place)
Answer = 36.7

(ii) 3.1421
0.241
+ 0.09 ← (has 2 decimal places)
3.4731 ← (answer should be reported to 2 decimal places)
Answer = 3.47

(iii) 62.831 ← (has 3 decimal places)
– 24.5492
38.2818 ← (answer should be reported to 3 decimal places after rounding off)
Answer = 38.282

(2) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation. The rule is illustrated by the following examples :

(i)

$$\begin{array}{r} 142.06 \\ \times\ 0.23 \\ \hline 32.6738 \\ \hline \end{array}$$

← (two significant figures)

← (answer should have two significant figures)

Answer = 33

(ii)

$$\begin{array}{r} 51.028 \\ \times\ 1.31 \\ \hline 66.84668 \\ \hline \end{array}$$

← (three significant figures)

Answer = 66.8

(iii) $\frac{0.90}{4.26} = 0.2112676$

Answer = 0.21

16. Order of Magnitude.

In scientific notation the numbers are expressed as, Number $= M \times 10^x$. Where M is a number lies between 1 and 10 and x is integer. Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

(1) Speed of light in vacuum $= 3 \times 10^8 ms^{-1} \approx 10^8 m/s$ (ignoring 3 < 5)

(2) Mass of electron $= 9.1 \times 10^{-31} kg \approx 10^{-30} kg$ (as 9.1 > 5).

17. Errors of Measurement.

The measuring process is essentially a process of comparison. Inspite of our best efforts, the measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

(1) **Absolute error:** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be a_1, a_2, a_3, ... a_n. The arithmetic mean of these value is $a_m = \frac{a_1 + a_2 + ...a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured values of the quantity are

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
....
$$\Delta a_n = a_m - a_n$$

The absolute errors may be positive in certain cases and negative in certain other cases.

(2) **Mean absolute error:** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + ...|\Delta a_n|}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$.

(3) **Relative error or Fractional error:** The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured.

Thus Relative error or Fractional error $= \dfrac{\text{mean absolute error}}{\text{mean value}} = \dfrac{\overline{\Delta a}}{a_m}$

4) **Percentage error:** When the relative/fractional error is expressed in percentage, we call it percentage error. Thus

Percentage error $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

18. Propagation of Errors.

(1) **Error in sum of the quantities:** Suppose x = a + b

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x i.e. sum of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \dfrac{(\Delta a + \Delta b)}{a + b} \times 100\%$

(2) **Error in difference of the quantities:** Suppose x = a – b

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x i.e. difference of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of $x = \dfrac{(\Delta a + \Delta b)}{a - b} \times 100\%$

(3) **Error in product of quantities:** Suppose x = a × b

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x i.e. product of a and b.

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

(4) **Error in division of quantities:** Suppose $x = \frac{a}{b}$

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x i.e. division of a and b.

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x = (Percentage error in value of a) + (Percentage error in value of b)

(5) **Error in quantity raised to some power:** Suppose $x = \frac{a^n}{b^m}$

Let Δa = absolute error in measurement of a,

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$

Percentage error in the value of x = n (Percentage error in value of a) + m (Percentage error in value of b)

Note: The quantity which have maximum power must be measured carefully because its contribution to error is maximum.