Physics

Motion in One Dimension

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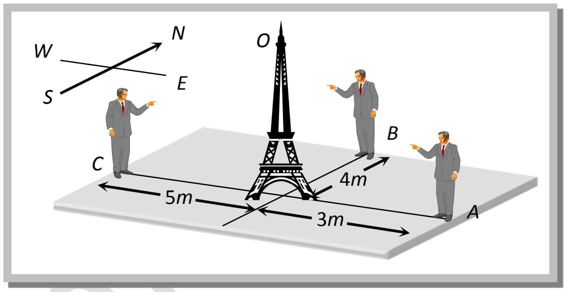
1. Position.

Any object is situated at point O and three observers from three different places are looking for same object, then all three observers will have different observations about the position of point O and no one will be wrong. Because they are observing the object from their different positions.



Observer 'A' says: Point O is 3 m away in west direction.

Observer 'B' says: Point O is 4 m away in south direction.

Observer 'C' says: Point O is 5 m away in east direction.

Therefore position of any point is completely expressed by two factors: Its distance from the observer and its direction with respect to observer.

That is why position is characterized by a vector known as position vector.

Let point P is in a xy plane and its coordinates are (x, y). Then position vector $(\vec{r})$ of point will be $x\hat{i} + y\hat{j}$ and if the point P is in a space and its coordinates are (x, y, z) then position vector can be expressed as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

2. Rest and Motion.

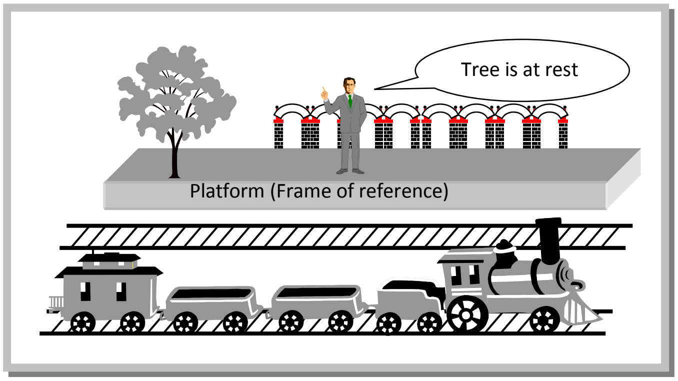
If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

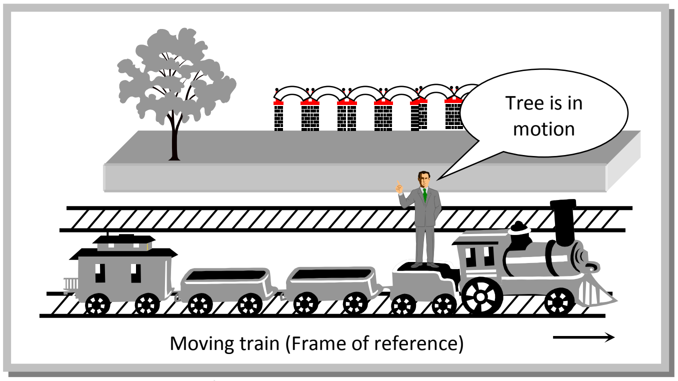
And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference: It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

A passenger standing on platform observes that tree on a platform is at rest. But when the same passenger is passing away in a train through station, observes that tree is in motion. In both conditions observer is right. But observations are different because in first situation observer stands on a platform, which reference frame at rest and in second situation observer is moving in train, which is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of references.





3. Types of Motion.

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
e.g.. Motion of car on a straight road. Motion of freely falling body.	e.g. Motion of car on a circular turn. Motion of billiards ball.	e.g.. Motion of flying kite. Motion of flying insect.

4. Particle or Point Mass.

The smallest part of matter with zero dimension which can be described by its mass and position is defined as a particle.

If the size of a body is negligible in comparison to its range of motion then that body is known as a particle.

A body (Group of particles) to be known as a particle depends upon types of motion. For example in a planetary motion around the sun the different planets can be presumed to be the particles.

In above consideration when we treat body as particle, all parts of the body undergo same displacement and have same velocity and acceleration.

5. Distance and Displacement.

(1) **Distance:** It is the actual path length covered by a moving particle in a given interval of time.

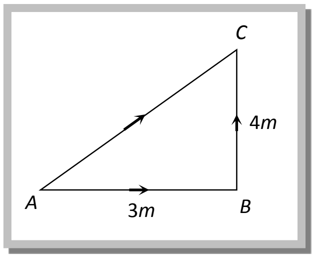
(i) If a particle starts from A and reach to C through point B as shown in the figure.

Then distance travelled by particle $= AB + BC = 7$ m

(ii) Distance is a scalar quantity.

(iii) Dimension: [$M^0L^1T^0$]

(iv) Unit: meter (S.I.)



(2) **Displacement:** Displacement is the change in position vector i.e., a vector joining initial to final position.

(i) Displacement is a vector quantity

(ii) Dimension: [$M^0L^1T^0$]

(iii) Unit: meter (S.I.)

(iv) In the above figure the displacement of the particle $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$\Rightarrow |\ AC| = \sqrt{(AB)^2 + (BC)^2 + 2(AB)(BC)\cos 90^o} = 5\text{m}$$

(v) If $\vec{S}_1, \vec{S}_2, \vec{S}_3 .... \vec{S}_n$ are the displacements of a body then the total (net) displacement is the vector sum of the individuals. $\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3 + ... + \vec{S}_n$

(3) **Comparison between distance and displacement:**

(i) The magnitude of displacement is equal to minimum possible distance between two positions.

So distance $\geq$|Displacement|.

(ii) For a moving particle distance can never be negative or zero while displacement can be.

(Zero displacement means that body after motion has come back to initial position)

i.e., Distance > 0 but Displacement > = or < 0

(iii) For motion between two points displacement is single valued while distance depends on actual path and so can have many values.

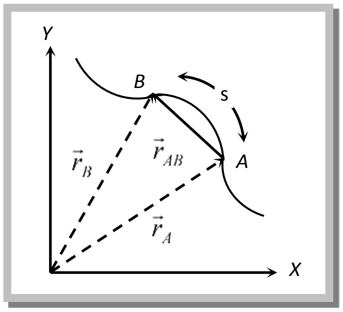
(iv) For a moving particle distance can never decrease with time while displacement can. Decrease in displacement with time means body is moving towards the initial position.

(v) In general magnitude of displacement is not equal to distance. However, it can be so if the motion is along a straight line without change in direction.

(vi) If $\vec{r}_A$ and $\vec{r}_B$ are the position vectors of particle initially and finally.

Then displacement of the particle

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$



And s is the distance travelled if the particle has gone through the path APB.

6. Speed and Velocity.

(1) **Speed:** Rate of distance covered with time is called speed.

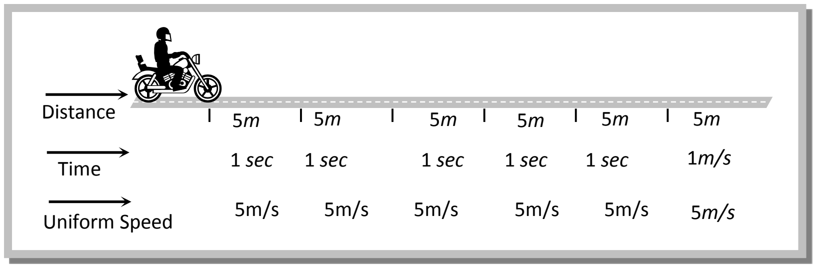
(i) It is a scalar quantity having symbol υ.

(ii) Dimension: $[M^0L^1T^{-1}]$

(iii) Unit: meter/second (S.I.), cm/second (C.G.S.)

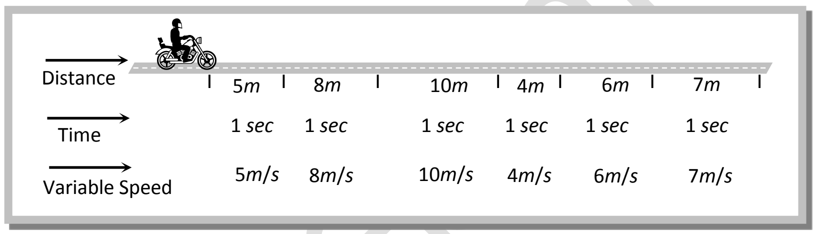
(iv) Types of speed:

(a) **Uniform speed:** When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= 5m) in each second. So we can say that particle is moving with uniform speed of 5 m/s.



(b) **Non-uniform (variable) speed:** In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels 5m in 1st second, 8m in 2nd second, 10m in 3rd second, 4m in 4th second etc.

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.



(c) **Average speed:** The average speed of a particle for a given 'Interval of time' is defined as the ratio of distance travelled to the time taken.

Average speed $= \dfrac{\text{Distance travelled}}{\text{Time taken}}$; $v_{av} = \dfrac{\Delta s}{\Delta t}$

❑ Time average speed: When particle moves with different uniform speed $\upsilon_1,\ \upsilon_2,\ \upsilon_3$ ... etc. in different time intervals $t_1,\ t_2,\ t_3$, ... etc. respectively, its average speed over the total time of journey is given as

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + \ldots\ldots}{t_1 + t_2 + t_3 + \ldots\ldots} = \frac{\upsilon_1 t_1 + \upsilon_2 t_2 + \upsilon_3 t_3 + \ldots\ldots}{t_1 + t_2 + t_3 + \ldots\ldots}$$

Special case: When particle moves with speed v_1 up to half time of its total motion and in rest time it is moving with speed v_2 then $v_{av} = \dfrac{v_1 + v_2}{2}$

❑ Distance averaged speed : When a particle describes different distances $d_1,\ d_2,\ d_3$, ... with different time intervals $t_1,\ t_2,\ t_3$, ... with speeds $v_1, v_2, v_3 \ldots\ldots$ respectively then the speed of particle averaged over the total distance can be given as

$$\upsilon_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{d_1 + d_2 + d_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{d_1 + d_2 + d_3 + ...}{\frac{d_1}{\upsilon_1} + \frac{d_2}{\upsilon_2} + \frac{d_3}{\upsilon_3} + ...}$$

❑ When particle moves the first half of a distance at a speed of v_1 and second half of the distance at speed v_2 then

$$v_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

❑ When particle covers one-third distance at speed v_1, next one third at speed v_2 and last one third at speed v_3, then

$$v_{av} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

(d) **Instantaneous speed:** It is the speed of a particle at particular instant. When we say "speed", it usually means instantaneous speed.

The instantaneous speed is average speed for infinitesimally small time interval (i.e., $\Delta t \to 0$). Thus

Instantaneous speed $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

(2) **Velocity:** Rate of change of position i.e. rate of displacement with time is called velocity.

(i) It is a scalar quantity having symbol v .

(ii) Dimension: $[M^0L^1T^{-1}]$

(iii) Unit: meter/second (S.I.), cm/second (C.G.S.)

(iv) Types

(a) **Uniform velocity :** A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

(b) **Non-uniform velocity:** A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

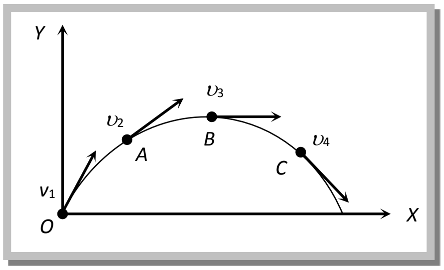
(c) **Average velocity**: It is defined as the ratio of displacement to time taken by the body

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}; \quad \vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

(d) **Instantaneous velocity:** Instantaneous velocity is defined as rate of change of position vector of particles with time at a certain instant of time.

Instantaneous velocity $\vec{v} = \lim_{t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(v) **Comparison between instantaneous speed and instantaneous velocity**



(a) Instantaneous velocity is always tangential to the path followed by the particle.

When a stone is thrown from point O then at point of projection the instantaneous velocity of stone is v_1, at point A the instantaneous velocity of stone is v_2, similarly at point B and C are v_3 and v_4 respectively.

Direction of these velocities can be found out by drawing a tangent on the trajectory at a given point.

(b) A particle may have constant instantaneous speed but variable instantaneous velocity.

Example: When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.

(c) The magnitude of instantaneous velocity is equal to the instantaneous speed.

(d) If a particle is moving with constant velocity then its average velocity and instantaneous velocity are always equal.

(e) If displacement is given as a function of time, then time derivative of displacement will give velocity.

Let displacement $\vec{x} = A_0 - A_1 t + A_2 t^2$

Instantaneous velocity $\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}(A_0 - A_1 t + A_2 t^2)$

$$\vec{v} = -A_1 + 2A_2 t$$

For the given value of t, we can find out the instantaneous velocity.

E.g. for $t = 0$,Instantaneous velocity $\vec{v} = -A_1$ and Instantaneous speed $|\vec{v}| = A_1$

(vi) **Comparison between average speed and average velocity**

(a) Average speed is scalar while average velocity is a vector both having same units (m/s) and dimensions $[LT^{-1}]$.

(b) Average speed or velocity depends on time interval over which it is defined.

(c) For a given time interval average velocity is single valued while average speed can have many values depending on path followed.

(d) If after motion body comes back to its initial position then $\vec{v}_{av} = \vec{0}$ (as $\Delta\vec{r} = 0$) but $v_{av} > \vec{0}$ and finite as $(\Delta s > 0)$.

(e) For a moving body average speed can never be negative or zero (unless $t \to \infty$) while average velocity can be i.e. $v_{av} > 0$ while $\vec{v}_{av}$ = or < 0.

7. Acceleration.

The time rate of change of velocity of an object is called acceleration of the object.

(1) It is a vector quantity. Its direction is same as that of change in velocity (Not of the velocity)

(2) There are three possible ways by which change in velocity may occur

When only direction of velocity changes	When only magnitude of velocity changes	When both magnitude and direction of velocity changes
Acceleration perpendicular to velocity	Acceleration parallel or anti-parallel to velocity	Acceleration has two components one is perpendicular to velocity and another parallel or anti-parallel to velocity
e.g. Uniform circular motion	e.g. Motion under gravity	e.g. Projectile motion

(3) Dimension: $[M^0L^1T^{-2}]$

(4) Unit: meter/second^2 (S.I.); cm/second^2 (C.G.S.)

(5) Types of acceleration:

(i) **Uniform acceleration:** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note: If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. e.g. Projectile motion.

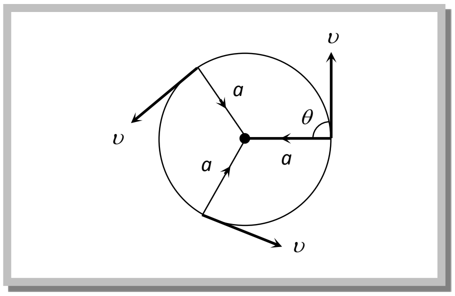
(ii) **Non-uniform acceleration:** A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

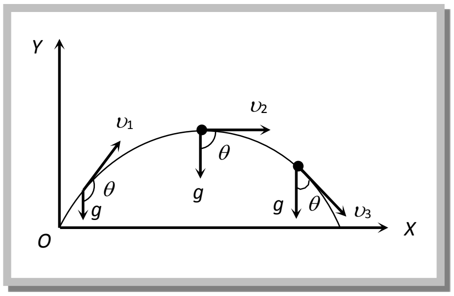
(iii) **Average acceleration:** $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$

(iv) **Instantaneous acceleration** = $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(v) For a moving body there is no relation between the direction of instantaneous velocity and direction of acceleration.





e.g. (a) In uniform circular motion θ = 90° always

(b) In a projectile motion θ is variable for every point of trajectory.

(vi) If a force $\vec{F}$ acts on a particle of mass m, by Newton's 2nd law, acceleration $\vec{a} = \frac{\vec{F}}{m}$

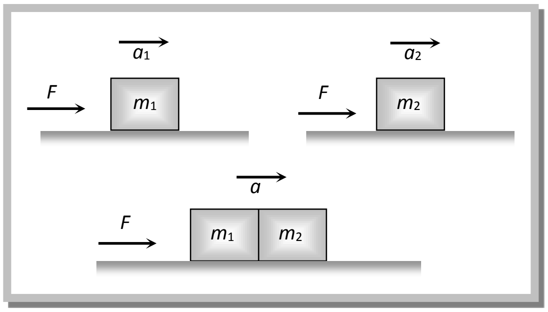
(vii) By definition $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2} \left[\text{As } \vec{v} = \frac{d\vec{x}}{dt} \right]$

i.e., if x is given as a function of time, second time derivative of displacement gives acceleration

(viii) If velocity is given as a function of position, then by chain rule $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v.\frac{d\upsilon}{dx} \left[\text{as } v = \frac{dx}{dt} \right]$

(ix) If a particle is accelerated for a time t_1 by acceleration a_1 and for time t_2 by acceleration a_2 then average acceleration is $a_{a\upsilon} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$

(x) If same force is applied on two bodies of different masses m_1 and m_2 separately then it produces accelerations a_1 and a_2 respectively. Now these bodies are attached together and form a combined system and same force is applied on that system so that a be the acceleration of the combined system, then



$$F = (m_1 + m_2)a \Rightarrow \frac{F}{a} = \frac{F}{a_1} + \frac{F}{a_2}$$

So, $\frac{1}{a} = \frac{1}{a_1} + \frac{1}{a_2} \Rightarrow a = \frac{a_1 a_2}{a_1 + a_2}$

(xi) Acceleration can be positive, zero or negative. Positive acceleration means velocity increasing with time, zero acceleration means velocity is uniform constant while negative acceleration (retardation) means velocity is decreasing with time.

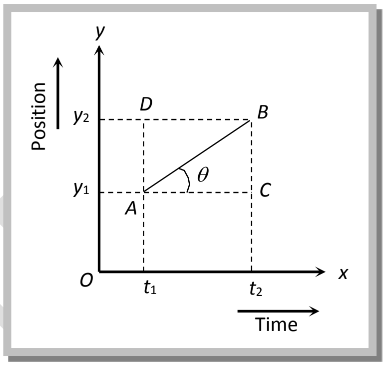
(xii) For motion of a body under gravity, acceleration will be equal to "g", where g is the acceleration due to gravity. Its normal value is 9.8 m/s^2 or 980 cm/s^2 or 32 feet/s^2.

8. Position Time Graph.

During motion of the particle its parameters of kinematical analysis (u, v, a, r) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.

Let AB is a position-time graph for any moving particle



As Velocity = $\dfrac{\text{Change in position}}{\text{Time taken}} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(i)

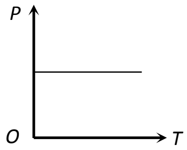
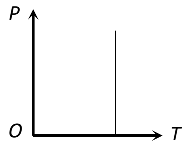
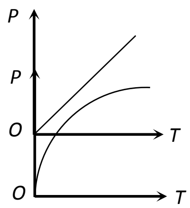
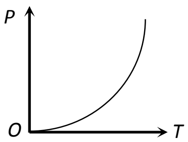
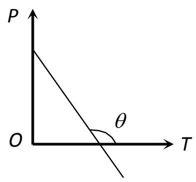
From triangle ABC $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ....(ii)

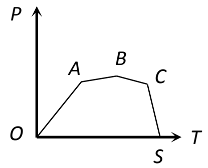
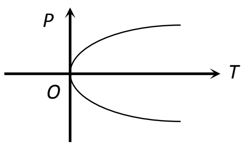
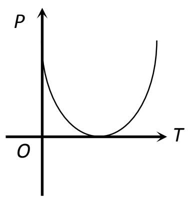
By comparing (i) and (ii) Velocity = $\tan\theta$

$v = \tan\theta$

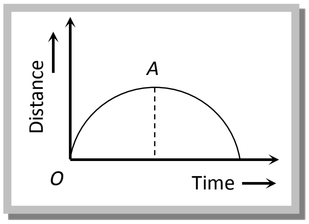
It is clear that slope of position-time graph represents the velocity of the particle.

Various position – time graphs and their interpretation

	θ = 0° so v = 0 i.e., line parallel to time axis represents that the particle is at rest.
	θ = 90° so v = ∞ i.e., line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity. Practically this is not possible.
	θ = constant so v = constant, a = 0 i.e., line with constant slope represents uniform velocity of the particle.
	θ is increasing so v is increasing, a is positive. i.e., line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.
	θ is decreasing so v is decreasing, a is negative i.e., line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.
	θ constant but > 90° so v will be constant but negative i.e., line with negative slope represent that particle returns towards the point of reference. (Negative displacement).

	Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.
	This graph shows that at one instant the particle has two positions. Which is not possible.
	The graph shows that particle coming towards origin initially and after that it is moving away from origin.

Note: If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A it is not valid as shown in the figure.

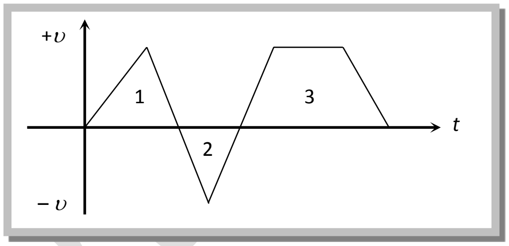


For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities v1 and v_2 respectively then $\frac{\upsilon_1}{\upsilon_2} = \frac{\tan\theta_1}{\tan\theta_2}$

9. Velocity Time Graph.

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

Distance and displacement: The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.



Then Total distance $= |A_1| + |A_2| + |A_3|$

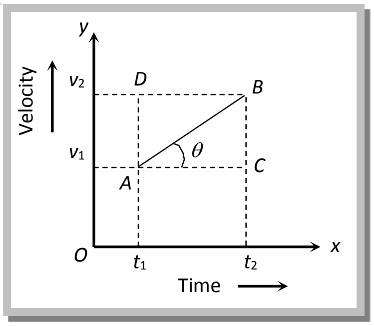
= Addition of modulus of different area. i.e. $s = \int |\upsilon|\, dt$

Total displacement $= A_1 + A_2 + A_3$

= Addition of different area considering their sign. i.e. $r = \int \upsilon\, dt$

Here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium .

Acceleration: Let AB is a velocity-time graph for any moving particle



As Acceleration = $\dfrac{\text{Change in velocity}}{\text{Time taken}} = \dfrac{v_2 - v_1}{t_2 - t_1}$ ...(i)

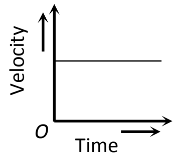
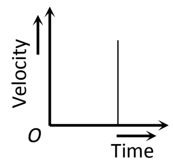
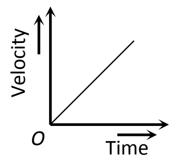
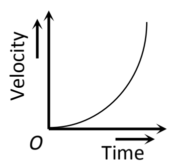
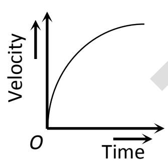
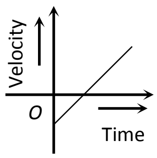
From triangle ABC, $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{v_2 - v_1}{t_2 - t_1}$ ....(ii)

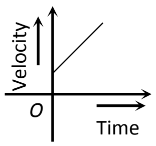
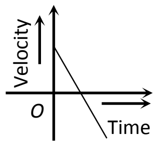
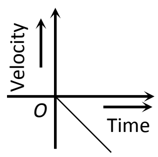
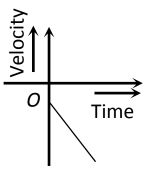
By comparing (i) and (ii)

Acceleration (a) = $\tan\theta$

It is clear that slope of velocity-time graph represents the acceleration of the particle.

Various velocity – time graphs and their interpretation

	θ = 0, a = 0, v = constant i.e., line parallel to time axis represents that the particle is moving with constant velocity.
	$\theta = 90^o$, a = ∞, v = increasing i.e., line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.
	θ =constant, so a = constant and v is increasing uniformly with time i.e., line with constant slope represents uniform acceleration of the particle.
	θ increasing so acceleration increasing i.e., line bending towards velocity axis represent the increasing acceleration in the body.
	θ decreasing so acceleration decreasing i.e. line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because θ is constant and < 90^o but initial velocity of the particle is negative.

	Positive constant acceleration because θ is constant and < 90° but initial velocity of particle is positive.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and > 90° but initial velocity of the particle is negative.

10. Equations of Kinematics.

These are the various relations between u, v, a, t and s for the moving particle where the notations are used as:

u = Initial velocity of the particle at time t = 0 sec

v = Final velocity at time t sec

a = Acceleration of the particle

s = Distance travelled in time t sec

s_n = Distance travelled by the body in n^{th} sec

(1) **When particle moves with zero acceleration**

(i) It is a unidirectional motion with constant speed.

(ii) Magnitude of displacement is always equal to the distance travelled.

(iii) v = u,   s = u t   [As a = 0]

(2) **When particle moves with constant acceleration**

(i) Acceleration is said to be constant when both the magnitude and direction of acceleration remain constant.

(ii) There will be one dimensional motion if initial velocity and acceleration are parallel or anti-parallel to each other.

(iii) Equations of motion in scalar from	Equation of motion in vector from
$\upsilon = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
$\upsilon^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}.\vec{s}$
$s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
$s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

(3) **Important points for uniformly accelerated motion**

(i) If a body starts from rest and moves with uniform acceleration then distance covered by the body in t sec is proportional to t^2 (i.e. $s \propto t^2$).

So we can say that the ratio of distance covered in 1 sec, 2 sec and 3 sec is $1^2 : 2^2 : 3^2$ or 1 : 4 : 9.

(ii) If a body starts from rest and moves with uniform acceleration then distance covered by the body in nth sec is proportional to $(2n-1)$ (i.e. $s_n \propto (2n-1)$)

So we can say that the ratio of distance covered in I sec, II sec and III sec is 1: 3: 5.

(iii) A body moving with a velocity u is stopped by application of brakes after covering a distance s. If the same body moves with velocity nu and same braking force is applied on it then it will come to rest after covering a distance of n^2s.

As $\upsilon^2 = u^2 - 2as \Rightarrow 0 = u^2 - 2as \Rightarrow s = \frac{u^2}{2a}, \quad s \propto u^2$ [since a is constant]

So we can say that if u becomes n times then s becomes n^2 times that of previous value.

(iv) A particle moving with uniform acceleration from A to B along a straight line has velocities υ_1 and υ_2 at A and B respectively. If C is the mid-point between A and B then velocity of the particle at C is equal to

$$\upsilon = \sqrt{\frac{\upsilon_1^2 + \upsilon_2^2}{2}}$$

11. Motion of Body under Gravity (Free Fall).

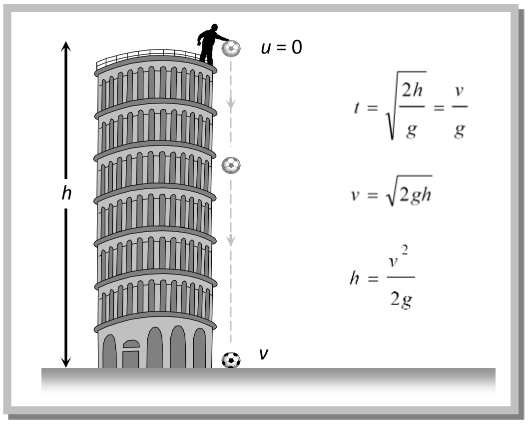
The force of attraction of earth on bodies, is called force of gravity. Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

In the absence of air resistance, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << R) is called free fall.

An ideal one-dimensional motion under gravity in which air resistance and the small changes in acceleration with height are neglected.

(1) **If a body dropped from some height (initial velocity zero)**

(i) Equation of motion: Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have



u = 0 [As body starts from rest]

a = +g [As acceleration is in the direction of motion]

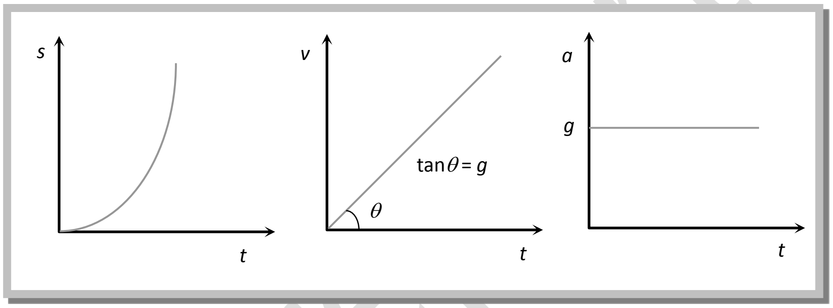
v = g t ...(i)

$h = \frac{1}{2}gt^2$ ...(ii)

$\upsilon^2 = 2gh$ ...(iii)

$h_n = \frac{g}{2}(2n-1)$ ...(iv)

(ii) Graph of distance velocity and acceleration with respect to time:



(iii) As h = (1/2)gt^2, i.e., h $\propto$ t^2, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of integers.

(iv) The distance covered in the nth sec, $h_n = \frac{1}{2}g(2n-1)$

So distance covered in I, II, III sec, etc., will be in the ratio of 1: 3: 5, i.e., odd integers only.

(2) **If a body is projected vertically downward with some initial velocity**

Equation of motion: $\upsilon = u + g\,t$

$$h = ut + \frac{1}{2}g\,t^2$$

$$\upsilon^2 = u^2 + 2gh$$

$$h_n = u + \frac{g}{2}(2n-1)$$

(3) **If a body is projected vertically upward**

(i) Equation of motion: Taking initial position as origin and direction of motion (i.e., vertically up) as positive

$a = -g$ [As acceleration is downwards while motion upwards]

So, if the body is projected with velocity u and after time t it reaches up to height h then

$$\upsilon = u - gt; \quad h = ut - \frac{1}{2}gt^2; \quad \upsilon^2 = u^2 - 2gh; h_n = u - \frac{g}{2}(2n-1)$$

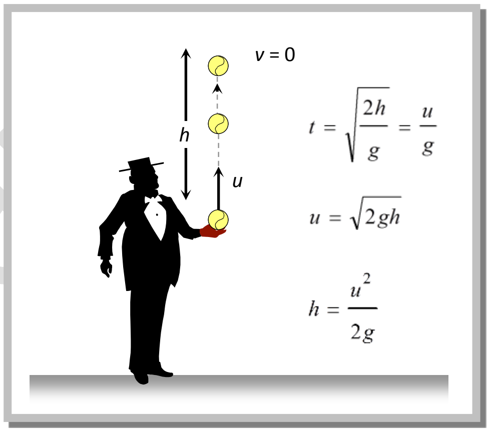
(ii) For maximum height v = 0

So from above equation

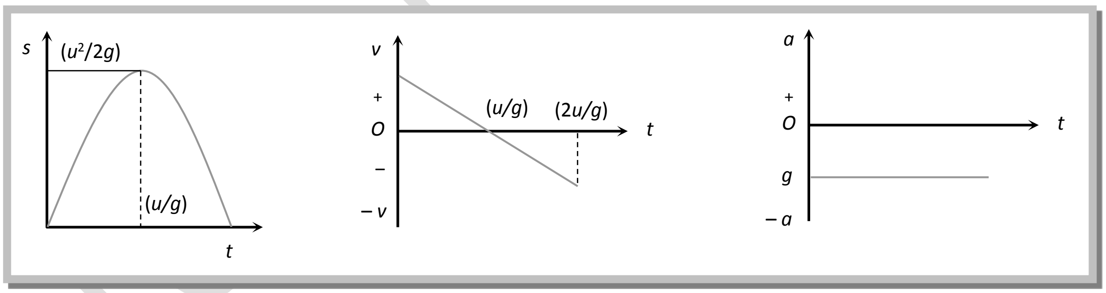
$u = gt$,

$$h = \frac{1}{2}gt^2$$

and $u^2 = 2gh$



(iii) Graph of distance, velocity and acceleration with respect to time (for maximum height):



It is clear that both quantities do not depend upon the mass of the body or we can say that in absence of air resistance, all bodies fall on the surface of the earth with the same rate.

(4) In case of motion under gravity for a given body, mass, acceleration, and mechanical energy remain constant while speed, velocity, momentum, kinetic energy and potential energy change.

(5) The motion is independent of the mass of the body, as in any equation of motion, mass is not involved. That is why a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.

(6) In case of motion under gravity time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_1) = time of ascent (t_2) = u/g

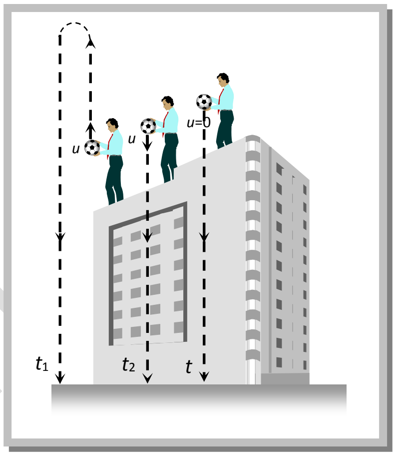
$\therefore$ Total time of flight T = $t_1 + t_2 = \frac{2u}{g}$

(7) In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.

As well as the magnitude of velocity at any point on the path is same whether the body is moving in upwards or downward direction.

(8) A ball is dropped from a building of height h and it reaches after t seconds on earth. From the same building if two ball are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively then

$$t = \sqrt{t_1 t_2}$$



(9) A body is thrown vertically upwards. If air resistance is to be taken into account, then the time of ascent is less than the time of descent. $t_2 > t_1$

Let u is the initial velocity of body then time of ascent $t_1 = \frac{u}{g+a}$ and $h = \frac{u^2}{2(g+a)}$

Where g is acceleration due to gravity and a is retardation by air resistance and for upward motion both will work vertically downward.

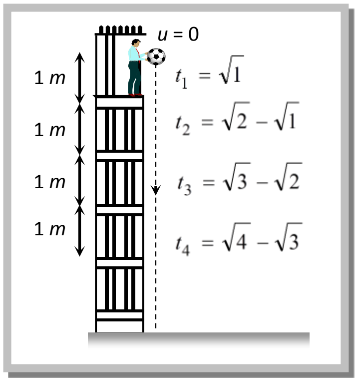
For downward motion a and g will work in opposite direction because a always work in direction opposite to motion and g always work vertically downward.

So $$h = \frac{1}{2}(g-a)t_2^2 \Rightarrow \frac{u^2}{2(g+a)} = \frac{1}{2}(g-a)t_2^2 \Rightarrow t_2 = \frac{u}{\sqrt{(g+a)(g-a)}}$$

Comparing t_1 and t_2 we can say that $t_2 > t_1$ since $(g + a) > (g - a)$

(10) A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1m each will then be in the ratio of the difference in the square roots of the integer's i.e.

$$\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}).......(\sqrt{4}-\sqrt{3}),........$$



12. Motion with Variable Acceleration.

(i) If acceleration is a function of time

$a = f(t)$ Then $v = u + \int_0^t f(t)dt$ and $s = ut + \int \left(\int f(t)dt \right) dt$

(ii) If acceleration is a function of distance

$a = f(x)$ Then $v^2 = u^2 + 2\int_{x_0}^{x} f(x)dx$

(iii) If acceleration is a function of velocity

a = f (v) Then $t = \int_u^v \frac{dv}{f(v)}$ and $x = x_0 + \int_u^v \frac{vdv}{f(v)}$