Physics

Rotational Motion

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1. Introduction

Translation is motion along a straight line but rotation is the motion of wheels, gears, motors, planets, and the hands of a clock, the rotor of jet engines and the blades of helicopters. First figure shows a skater gliding across the ice in a straight line with constant speed. Her motion is called translation but second figure shows her spinning at a constant rate about a vertical axis. Here motion is called rotation.

Up to now we have studied translatory motion of a point mass. In this chapter we will study the rotatory motion of rigid body about a fixed axis.

(1) Rigid body: A rigid body is a body that can rotate with all the parts locked together and without any change in its shape.

(2) System: A collection of any number of particles interacting with one another and are under consideration during analysis of a situation are said to form a system.

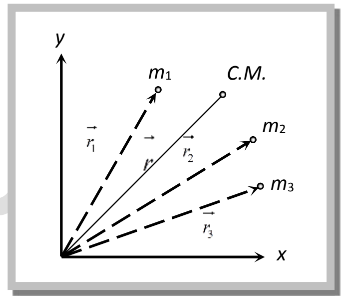
(3) Internal forces: All the forces exerted by various particles of the system on one another are called internal forces. These forces are alone enable the particles to form a well-defined system. Internal forces between two particles are mutual (equal and opposite).

(4) External forces: To move or stop an object of finite size, we have to apply a force on the object from outside. This force exerted on a given system is called an external force.

2. Center of Mass.

Center of mass of a system (body) is a point that moves as though all the mass were concentrated there and all external forces were applied there.

(1) **Position vector of center of mass for n particle system:** If a system consists of n particles of masses $m_1, m_2, m_3 ......m_n$, whose positions vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3 .... \vec{r}_n$ respectively then position vector of center of mass



$$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + ...m_n \vec{r}_n}{m_1 + m_2 + m_3 + ...m_n}$$

Hence the center of mass of n particles is a weighted average of the position vectors of n particles making up the system.

(2) **Position vector of center of mass for two particle system:** $\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$

and the center of mass lies between the particles on the line joining them.

If two masses are equal i.e. $m_1 = m_2$, then position vector of center of mass $\vec{r} = \frac{\vec{r}_1 + \vec{r}_2}{2}$

(3) **Important points about center of mass**

(i) The position of center of mass is independent of the co-ordinate system chosen.

(ii) The position of center of mass depends upon the shape of the body and distribution of mass.

Example: The center of mass of a circular disc is within the material of the body while that of a circular ring is outside the material of the body.

(iii) In symmetrical bodies in which the distribution of mass is homogenous, the center of mass coincides with the geometrical center or center of symmetry of the body.

(iv) Position of center of mass for different bodies

S.No.	Body	Position of center of mass
(a)	Uniform hollow sphere	Center of sphere
(b)	Uniform solid sphere	Center of sphere
(c)	Uniform circular ring	Center of ring
(d)	Uniform circular disc	Center of disc
(e)	Uniform rod	Center of rod
(f)	A plane lamina (Square, Rectangle, Parallelogram)	Point of inter section of diagonals
(g)	Triangular plane lamina	Point of inter section of medians
(h)	Rectangular or cubical block	Points of inter section of diagonals
(i)	Hollow cylinder	Middle point of the axis of cylinder
(j)	Solid cylinder	Middle point of the axis of cylinder
(k)	Cone or pyramid	On the axis of the cone at point distance $\frac{3h}{4}$ from the vertex where h is the height of cone

(v) The center of mass changes its position only under the translatory motion. There is no effect of rotatory motion on center of mass of the body.

(vi) If the origin is at the center of mass, then the sum of the moments of the masses of the system about the center of mass is zero i.e. $\Sigma m_i \vec{r_i} = 0$.

(vii) If a system of particles of masses $m_1, m_2, m_3, \ldots\ldots$ move with velocities $v_1, v_2, v_3, \ldots\ldots$

Then the velocity of center of mass $v_{cm} = \frac{\sum m_i v_i}{\sum m_i}$.

(viii) If a system of particles of masses $m_1, m_2, m_3, \ldots\ldots$ move with accelerations $a_1, a_2, a_3, \ldots\ldots$

Then the acceleration of center of mass $A_{cm} = \frac{\sum m_i a_i}{\sum m_i}$

(ix) If $\vec{r}$ is a position vector of center of mass of a system

Then velocity of center of mass

$$\vec{v}_{cm} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots\ldots}{m_1 + m_2 + m_3 + \ldots\ldots}\right)$$

(x) Acceleration of center of mass

$$\vec{A}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2}{dt^2}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots}{m_1 + m_2 + m_3 + \ldots\ldots}\right)$$

(xi) Force on a rigid body

$$\vec{F} = M\vec{A}_{cm} = M\frac{d^2\vec{r}}{dt^2}$$

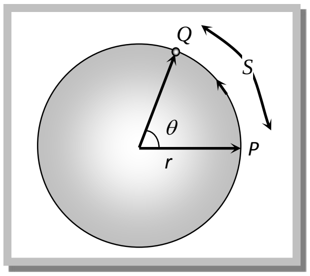
(xii) For an isolated system external force on the body is zero

$$\vec{F} = M\frac{d}{dt}\left(\vec{v}_{cm}\right) = 0 \Rightarrow \vec{v}_{cm} = \text{constant}.$$

i.e., center of mass of an isolated system moves with uniform velocity along a straight-line path.

3. Angular Displacement.

It is the angle described by the position vector $\vec{r}$ about the axis of rotation.



Angular displacement $(\theta) = \dfrac{\text{Linear displacement }(s)}{\text{Radius }(r)}$

(1) Unit: radian

(2) Dimension: $[M^0 L^0 T^0]$

(3) Vector form $\vec{S} = \vec{\theta} \times \vec{r}$

i.e., angular displacement is a vector quantity whose direction is given by right hand rule. It is also known as axial vector. For anti-clockwise sense of rotation direction of θ is perpendicular to the plane, outward and along the axis of rotation and vice-versa.

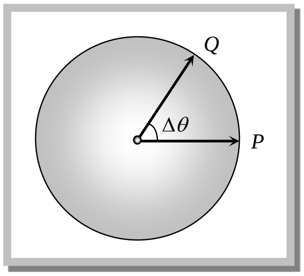
(4) 2π radian $= 360° = 1$ revolution.

(5) If a body rotates about a fixed axis then all the particles will have same angular displacement (although linear displacement will differ from particle to particle in accordance with the distance of particles from the axis of rotation).

4. Angular Velocity.

The angular displacement per unit time is defined as angular velocity.

If a particle moves from P to Q in time Δt, $\omega = \dfrac{\Delta\theta}{\Delta t}$ where $\Delta\theta$ is the angular displacement.



(1) Instantaneous angular velocity $\omega = \lim_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$

(2) Average angular velocity $\omega_{av} = \dfrac{\text{total angular displacement}}{\text{total time}} = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$

(3) Unit: Radian/sec

(4) Dimension: $[M^0 L^0 T^{-1}]$ which is same as that of frequency.

(5) Vector form $\vec{v} = \vec{\omega} \times \vec{r}$ [where $\vec{v}$ = linear velocity, $\vec{r}$ = radius vector]

$\vec{\omega}$ Is an axial vector, whose direction is normal to the rotational plane and its direction is given by right hand screw rule.

(6) $\omega = \frac{2\pi}{T} = 2\pi n$ [where T = time period, n = frequency]

(7) The magnitude of an angular velocity is called the angular speed which is also represented by ω.

5. Angular Acceleration.

The rate of change of angular velocity is defined as angular acceleration.

If particle has angular velocity ω_1 at time t_1 and angular velocity ω_2 at time t_2 then,

Angular acceleration $\vec{\alpha} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$

(1) Instantaneous angular acceleration $\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2}$.

(2) Unit: rad/sec^2

(3) Dimension: $[M^0L^0T^{-2}]$.

(4) If $\alpha = 0$, circular or rotational motion is said to be uniform.

(5) Average angular acceleration $\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$.

(6) Relation between angular acceleration and linear acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$.

(7) It is an axial vector whose direction is along the change in direction of angular velocity i.e. normal to the rotational plane, outward or inward along the axis of rotation (depends upon the sense of rotation).

6. Equations of Linear Motion and Rotational Motion.

Linear Motion	Rotational Motion
If linear acceleration is 0, u = constant and s = u t.	If angular acceleration is 0, ω = constant and $\theta = \omega t$
If linear acceleration a = constant, (i) $s = \frac{(u+v)}{2}t$ (ii) $a = \frac{v-u}{t}$ (iii) $v = u + at$ (iv) $s = ut + \frac{1}{2}at^2$ (v) $v^2 = u^2 + 2as$ (vi) $s_{nth} = u + \frac{1}{2}a(2n-1)$	If angular acceleration α = constant then (i) $\theta = \frac{(\omega_1 + \omega_2)}{2}t$ (ii) $\alpha = \frac{\omega_2 - \omega_1}{t}$ (iii) $\omega_2 = \omega_1 + \alpha t$ (iv) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ (v) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (vi) $\theta_{nth} = \omega_1 + (2n-1)\frac{\alpha}{2}$
If acceleration is not constant, the above equation will not be applicable. In this case (i) $v = \frac{dx}{dt}$ (ii) $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ (iii) $vdv = a\,ds$	If acceleration is not constant, the above equation will not be applicable. In this case (i) $\omega = \frac{d\theta}{dt}$ (ii) $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ (iii) $\omega d\omega = \alpha\, d\theta$

7. Moment of Inertia.

Moment of inertia plays the same role in rotational motion as mass plays in linear motion. It is the property of a body due to which it opposes any change in its state of rest or of uniform rotation.

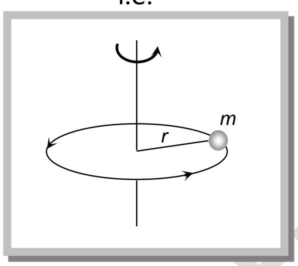
(1) Moment of inertia of a particle $I = mr^2$; where r is the perpendicular distance of particle from rotational axis.

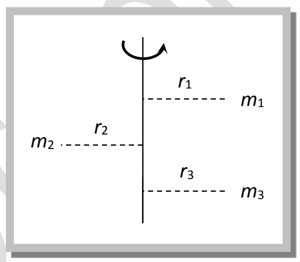
(2) Moment of inertia of a body made up of number of particles (discrete distribution)

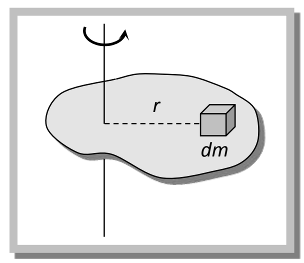
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ...$$

(3) Moment of inertia of a continuous distribution of mass, treating the element of mass dm at position r as particle

$dI = dm\, r^2$ i.e. $I = \int r^2 dm$







(4) Dimension: $[ML^2T^0]$

(5) S.I. unit: kgm^2.

(6) Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation.

(7) Moment of inertia does not depend on angular velocity, angular acceleration, torque, angular momentum and rotational kinetic energy.

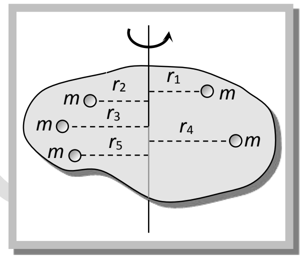
(8) It is not a vector as direction (clockwise or anti-clockwise) is not to be specified and also not a scalar as it has different values in different directions. Actually it is a tensor quantity.

(9) In case of a hollow and solid body of same mass, radius and shape for a given axis, moment of inertia of hollow body is greater than that for the solid body because it depends upon the mass distribution.

8. Radius of Gyration.

Radius of gyration of a body about a given axis is the perpendicular distance of a point from the axis, where if whole mass of the body were concentrated, the body shall have the same moment of inertia as it has with the actual distribution of mass.

When square of radius of gyration is multiplied with the mass of the body gives the moment of inertia of the body about the given axis.



$$I = Mk^2 \text{ Or } k = \sqrt{\frac{I}{M}}.$$

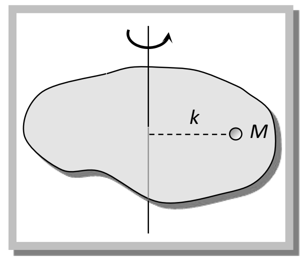
Here k is called radius of gyration.

From the formula of discrete distribution

$$I = mr_1^2 + mr_2^2 + mr_3^2 + ... + mr_n^2$$

If $m_1 = m_2 = m_3 = ... = m$ then

$$I = m(r_1^2 + r_2^2 + r_3^2 + ...r_n^2) \quad ....(i)$$



From the definition of Radius of gyration,

$$I = Mk^2 \quad ....(ii)$$

By equating (i) and (ii)

$$Mk^2 = m(r_1^2 + r_2^2 + r_3^2 + ... + r_n^2)$$

$$nmk^2 = m(r_1^2 + r_2^2 + r_3^2 + ... + r_n^2) \quad [\text{As } M = nm]$$

$$\therefore \quad k = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + ... + r_n^2}{n}}$$

Hence radius of gyration of a body about a given axis is equal to root mean square distance of the constituent particles of the body from the given axis.

(1) Radius of gyration (k) depends on shape and size of the body, position and configuration of the axis of rotation, distribution of mass of the body w.r.t. the axis of rotation.

(2) Radius of gyration (k) does not depends on the mass of body.

(3) Dimension $[M^0 L^1 T^0]$.

(4) S.I. unit: Meter.

(5) Significance of radius of gyration: Through this concept a real body (particularly irregular) is replaced by a point mass for dealing its rotational motion.

Example: In case of a disc rotating about an axis through its center of mass and perpendicular to its plane

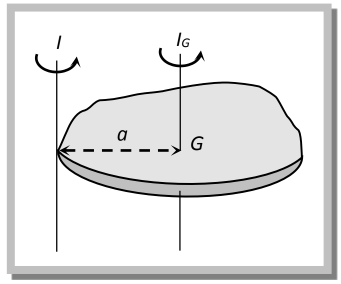
$$k = \sqrt{\frac{I}{M}} = \sqrt{\frac{(1/2)MR^2}{M}} = \frac{R}{\sqrt{2}}$$

So instead of disc we can assume a point mass M at a distance $(R/\sqrt{2})$ from the axis of rotation for dealing the rotational motion of the disc.

Note: For a given body inertia is constant whereas moment of inertia is variable.

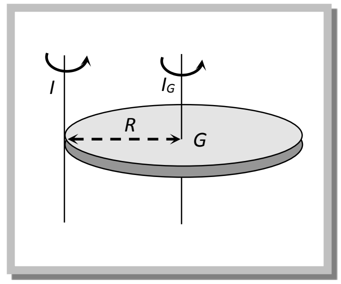
9. Theorem of Parallel Axes.

Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through center of mass of the body I_g and Ma^2 where M is the mass of the body and a is the perpendicular distance between the two axes.



$$I = I_g + Ma^2$$

Example: Moment of inertia of a disc about an axis through its center and perpendicular to the plane is $\frac{1}{2}MR^2$, so moment of inertia about an axis through its tangent and perpendicular to the plane will be



$$I = I_g + Ma^2$$

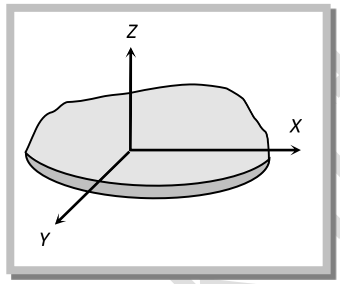
$$I = \frac{1}{2}MR^2 + MR^2$$

$$\therefore \quad I = \frac{3}{2}MR^2$$

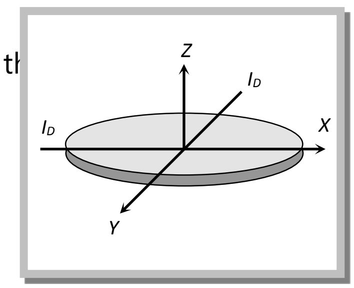
10. Theorem of Perpendicular Axes.

According to this theorem the sum of moment of inertia of a plane lamina about two mutually perpendicular axes lying in its plane is equal to its moment of inertia about an axis perpendicular to the plane of lamina and passing through the point of intersection of first two axes.

$$I_z = I_x + I_y$$



Example: Moment of inertia of a disc about an axis through its center of mass and perpendicular to its plane is $\frac{1}{2}MR^2$, so if the disc is in x–y plane then by th perpendicular axes



i.e. $I_z = I_x + I_y$

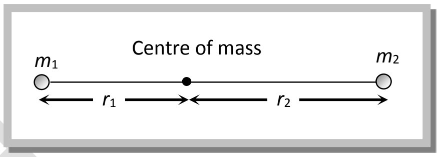
$\Rightarrow \frac{1}{2}MR^2 = 2I_D$ [As ring is symmetrical body so $I_x = I_y = I_D$]

$\Rightarrow I_D = \frac{1}{4}MR^2$

Note: In case of symmetrical two-dimensional bodies as moment of inertia for all axes passing through the center of mass and in the plane of body will be same so the two axes in the plane of body need not be perpendicular to each other.

11. Moment of Inertia of Two Point Masses about Their Center of Mass.

Let m_1 and m_2 be two masses distant r from each-other and r_1 and r_2 be the distances of their center of mass from m_1 and m_2 respectively, then



(1) $r_1 + r_2 = r$

(2) $m_1 r_1 = m_2 r_2$

(3) $r_1 = \frac{m_2}{m_1 + m_2} r$ and $r_2 = \frac{m_1}{m_1 + m_2} r$

(4) $I = m_1 r_1^2 + m_2 r_2^2$

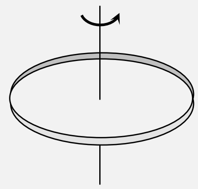
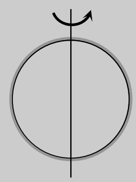
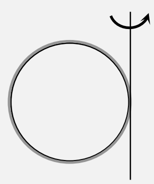
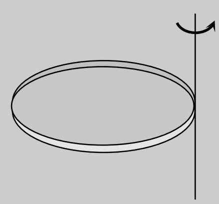
(5) $I = \left[\frac{m_1 m_2}{m_1 + m_2}\right] r^2 = \mu r^2$ [where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is known as reduced mass $\mu < m_1$ and $\mu < m_2$.]

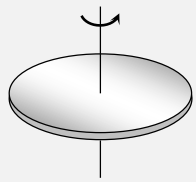
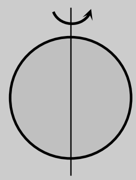
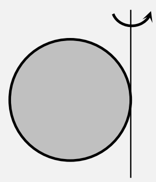
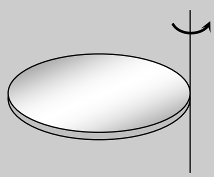
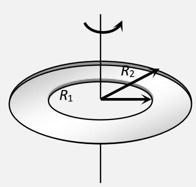
(6) In diatomic molecules like H_2, HCl etc. moment of inertia about their center of mass is derived from above formula.

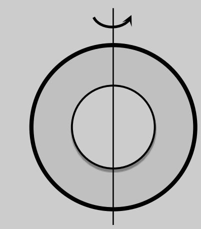
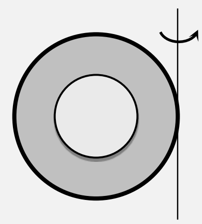
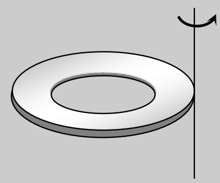
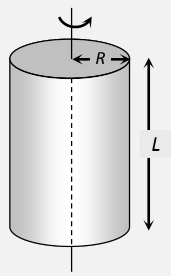
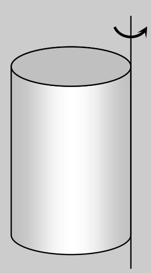
12. Analogy between Translatory Motion and Rotational Motion.

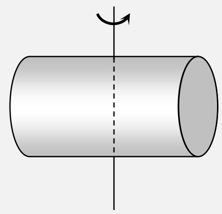
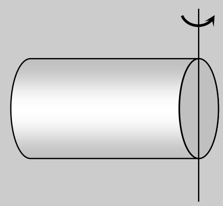
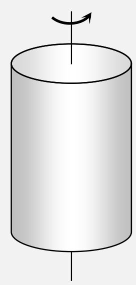
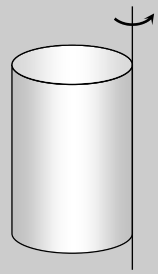
Translatory motion		Rotatory motion	
Mass	(m)	Moment of Inertia	(I)
Linear momentum	$P = mv$ $P = \sqrt{2mE}$	Angular Momentum	$L = I\omega$ $L = \sqrt{2IE}$
Force	$F = ma$	Torque	$\tau = I\alpha$
Kinetic energy	$E = \frac{1}{2}mv^2$ $E = \frac{P^2}{2m}$		$E = \frac{1}{2}I\omega^2$ $E = \frac{L^2}{2I}$

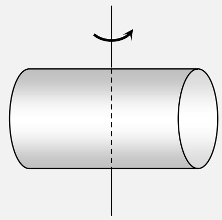
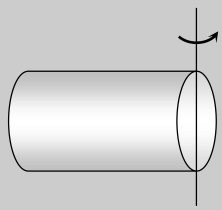
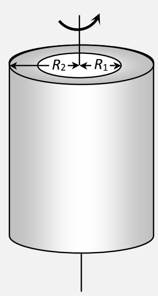
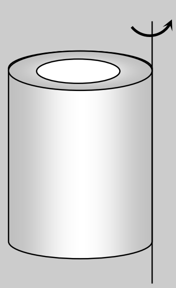
13. Moment of Inertia of Some Standard Bodies about Different Axes.

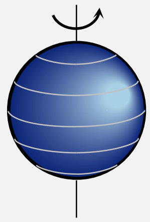
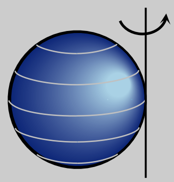
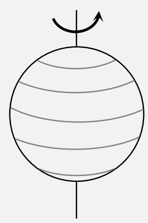
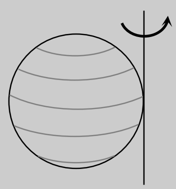
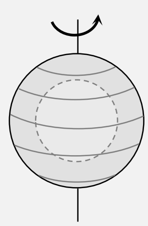
Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Ring	About an axis passing through C.G. and perpendicular to its plane		MR^2	R	1
Ring	About its diameter		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Ring	About a tangential axis in its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Ring	About a tangential axis perpendicular to its own plane		$2MR^2$	$\sqrt{2}R$	2

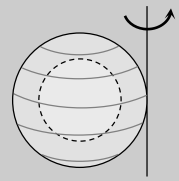
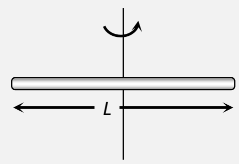
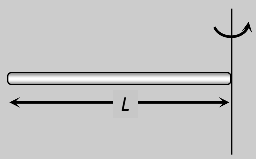
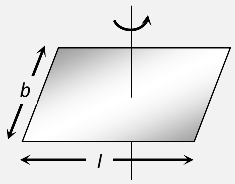
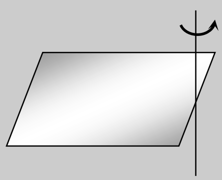
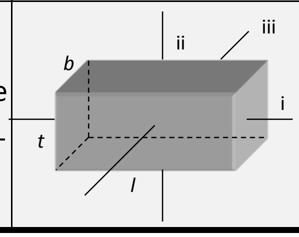
Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Disc	About an axis passing through C.G. and perpendicular to its plane		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Disc	About its Diameter		$\frac{1}{4}MR^2$	$\frac{R}{2}$	$\frac{1}{4}$
Disc	About a tangential axis in its own plane		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$	$\frac{5}{4}$
Disc	About a tangential axis perpendicular to its own plane		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$
Annular disc inner radius = R_1 and outer radius = R_2	Passing through the center and perpendicular to the plane		$\frac{M}{2}[R_1^2 + R_2^2]$	–	–

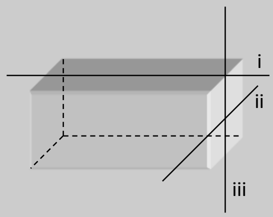
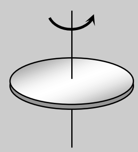
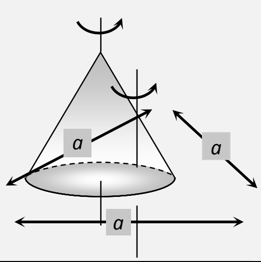
Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Annular disc	Diameter		$\frac{M}{4}[R_1^2+R_2^2$	–	–
Annular disc	Tangential and Parallel to the diameter		$\frac{M}{4}[5R_1^2+R$	–	–
Annular disc	Tangential and perpendicular to the plane		$\frac{M}{2}[3R_1^2+R$	–	–
Solid cylinder	About its own axis		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$	$\frac{1}{2}$
Solid cylinder	Tangential (Generator)		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$	$\frac{3}{2}$

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Solid cylinder	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12}+\frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{12}+}$	
Solid cylinder	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3}+\frac{R^2}{4}\right]$	$\sqrt{\frac{L^2}{3}+}$	
Cylindrical shell	About its own axis		MR^2	R	1
Cylindrical shell	Tangential (Generator)		$2MR^2$	$\sqrt{2}R$	2

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Cylindrical shell	About an axis passing through its C.G. and perpendicular to its own axis		$M\left[\frac{L^2}{12}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{12}+}$	
Cylindrical shell	About the diameter of one of faces of the cylinder		$M\left[\frac{L^2}{3}+\frac{R^2}{2}\right]$	$\sqrt{\frac{L^2}{3}+}$	
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Axis of cylinder		$\frac{M}{2}(R_1^2+R_2^2)$		
Hollow cylinder with inner radius = R_1 and outer radius = R_2	Tangential		$\frac{M}{2}(R_1^2+3R_2^2)$		

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Solid Sphere	About its diametric axis		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$	$\frac{2}{5}$
Solid sphere	About a tangential axis		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$	$\frac{7}{5}$
Spherical shell	About its diametric axis		$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$	$\frac{2}{3}$
Spherical shell	About a tangential axis		$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$	$\frac{5}{3}$
Hollow sphere of inner radius R_1 and outer radius R_2	About its diametric axis		$\frac{2}{5}M\left[\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}\right]$		

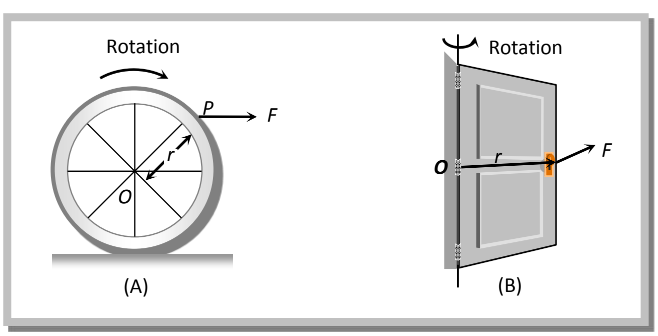
Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Hollow sphere	Tangential		$\frac{2M[R_2^5 - R_1^5]}{5(R_2^3 - R_1^3)} + M$		
Long thin rod	About on axis passing through its center of mass and perpendicular to the rod.		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$	
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$	
Rectangular lamina of length l and breadth b	Passing through the center of mass and perpendicular to the plane		$\frac{M}{12}[l^2 + b^2]$		
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of breadth		$\frac{M}{12}[4l^2 + b^2]$		
Rectangular lamina	Tangential perpendicular to the plane and at the mid-point of length		$\frac{M}{12}[l^2 + 4b^2]$		

Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Rectangular parallelopiped length l, breadth b, thickness t	Passing through center of mass and parallel to (i) Length (x) (ii) breadth (z) (iii) thickness (y)		(i) $\frac{M[b^2+t^2]}{12}$ (ii) $\frac{M[l^2+t^2]}{12}$ (iii) $\frac{M[b^2+l^2]}{12}$		
Rectangular parallelepiped length l, breath b, thickness t	Tangential and parallel to (i) length (x) (ii) breadth (y) (iii) thickness(z)		(i) $\frac{M}{12}[3l^2+b^2+t^2]$ (ii) $\frac{M}{12}[l^2+3b^2+t^2]$ (iii) $\frac{M}{12}[l^2+b^2+3t^2]$		
Elliptical disc of semimajor axis = a and semiminor axis = b	Passing through CM and perpendicular to the plane		$\frac{M}{4}[a^2+b^2]$		
Solid cone of radius R and height h	Axis joining the vertex and center of the base		$\frac{3}{10}MR^2$		

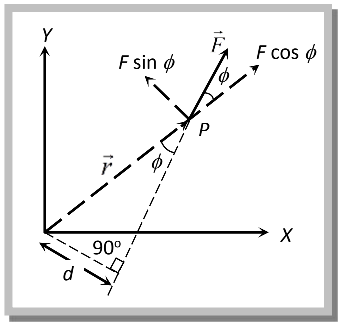
Body	Axis of Rotation	Figure	Moment of inertia	k	k^2/R^2
Equilateral triangular lamina with side a	Passing through CM and perpendicular to the plane		$\frac{Ma^2}{6}$		
Right angled triangular lamina of sides a, b, c	Along the edges		(1) $\frac{Mb^2}{6}$ (2) $\frac{Ma^2}{6}$ (3) $\frac{M}{6}\left[\frac{a^2b^2}{a^2+b^2}\right]$		

14. Torque.

If a pivoted, hinged or suspended body tends to rotate under the action of a force, it is said to be acted upon by a torque. or the turning effect of a force about the axis of rotation is called moment of force or torque due to the force.



If the particle rotating in xy plane about the origin under the effect of force $\vec{F}$ and at any instant the position vector of the particle is $\vec{r}$ then,



Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = r F \sin\phi$

[Where ϕ is the angle between the direction of $\vec{r}$ and $\vec{F}$]

(1) Torque is an axial vector. i.e., its direction is always perpendicular to the plane containing vector $\vec{r}$ and $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

(2) Rectangular components of force

$\vec{F}_r = F\cos\phi =$ radial component of force, $\vec{F}_\phi = F\sin\phi =$ transverse component of force

As $\tau = r F \sin\phi$

or $\tau = r F_\phi$ = (position vector) □ (transverse component of force)

Thus the magnitude of torque is given by the product of transverse component of force and its perpendicular distance from the axis of rotation i.e., Torque is due to transverse component of force only.

(3) As $\tau = r F \sin\phi$

or □ $\tau = F(r\sin\phi) = Fd$ [As $d = r\sin\phi$ from the figure]

i.e. Torque = Force □ Perpendicular distance of line of action of force from the axis of rotation.

Torque is also called as moment of force and d is called moment or lever arm.

(4) Maximum and minimum torque: As $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = r F \sin\phi$

$\tau_{maximum} = rF$	When $\lvert\sin\phi\rvert = \max = 1$ i.e., $\phi = 90°$	$\vec{F}$ is perpendicular to $\vec{r}$
$\tau_{minimum} = 0$	When $\lvert\sin\phi\rvert = \min = 0$ $i.e.\ \phi = 0°$ or $180°$	$\vec{F}$ is collinear to $\vec{r}$

(5) For a given force and angle, magnitude of torque depends on r. The more is the value of r, the more will be the torque and easier to rotate the body.

Example: (i) Handles are provided near the free edge of the Planck of the door.

(ii) The handle of screw driver is taken thick.

(iii) In villages handle of flourmill is placed near the circumference.

(iv) The handle of hand-pump is kept long.

(v) The arm of wrench used for opening the tap, is kept long.

(6) Unit: Newton-meter (M.K.S.) and Dyne-cm (C.G.S.)

(7) Dimension: $[ML^2T^{-2}]$.

(8) If a body is acted upon by more than one force, the total torque is the vector sum of each torque.

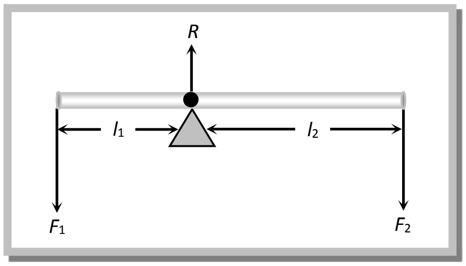
$$\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + ...$$

(9) A body is said to be in rotational equilibrium if resultant torque acting on it is zero i.e. $\Sigma \vec{\tau} = 0$.

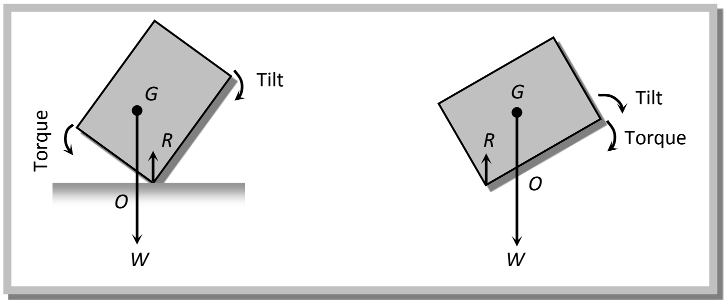
(10) In case of beam balance or see-saw the system will be in rotational equilibrium if,

$\vec{\tau}_1 + \vec{\tau}_2 = 0$ or $F_1 l_1 - F_2 l_2 = 0 \therefore F_1 l_1 = F_2 l_2$

However if, $\vec{\tau}_1 > \vec{\tau}_2$ L.H.S. will move downwards and if $\vec{\tau}_1 < \vec{\tau}_2$. R.H.S. will move downward. and the system will not be in rotational equilibrium.



(11) On tilting, a body will restore its initial position due to torque of weight about the point O till the line of action of weight passes through its base on tilting, a body will topple due to torque of weight about O, if the line of action of weight does not pass through the base.



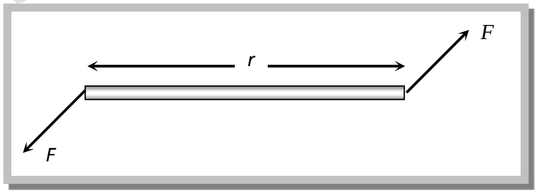
(12) Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translatory motion i.e., torque is rotational analogue of force. This all is evident from the following correspondences between rotatory and translatory motion.

Rotatory Motion	Translatory Motion
$\vec{\tau} = I\vec{\alpha}$	$\vec{F} = m\vec{a}$
$W = \int \vec{\tau} \cdot d\vec{\theta}$	$W = \int \vec{F} \cdot d\vec{s}$
$P = \vec{\tau} \cdot \vec{\omega}$	$P = \vec{F} \cdot \vec{v}$
$\vec{\tau} = \frac{d\vec{L}}{dt}$	$\vec{F} = \frac{d\vec{P}}{dt}$

15. Couple.

A special combination of forces even when the entire body is free to move can rotate it. This combination of forces is called a couple.

(1) A couple is defined as combination of two equal but oppositely directed force not acting along the same line. The effect of couple is known by its moment of couple or torque by a couple $\vec{\tau} = \vec{r} \times \vec{F}$.

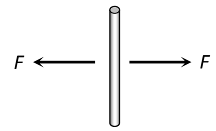
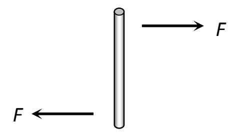
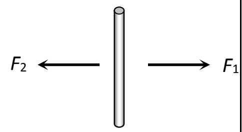
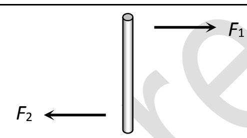


(2) Generally both couple and torque carry equal meaning. The basic difference between torque and couple is the fact that in case of couple both the forces are externally applied while in case of torque one force is externally applied and the other is reactionary.

(3) Work done by torque in twisting the wire $W = \frac{1}{2}C\theta^2$.

Where $\tau = C\theta$; C is known as twisting coefficient or couple per unit twist.

16. Translatory and Rotatory Equilibrium.

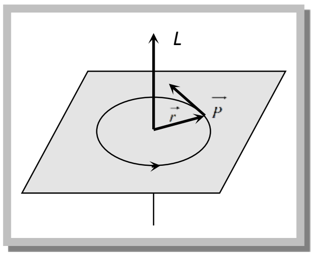
Forces are equal and act along the same line.		$\sum F = 0$ and $\sum \tau = 0$	Body will remain stationary if initially it was at rest.
Forces are equal and does not act along the same line.		$\sum F = 0$ and $\sum \tau \neq 0$	Rotation i.e. spinning.
Forces are unequal and act along the same line.		$\sum F \neq 0$ and $\sum \tau = 0$	Translation i.e. slipping or skidding.
Forces are unequal and does not act along the same line.		$\sum F \neq 0$ and $\sum \tau \neq 0$	Rotation and translation both i.e. rolling.

17. Angular Momentum.

The turning momentum of particle about the axis of rotation is called the angular momentum of the particle.

Or

The moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If $\vec{P}$ is the linear momentum of particle and $\vec{r}$ its position vector from the point of rotation then angular momentum.



$$\vec{L} = \vec{r} \times \vec{P}$$

$$\vec{L} = r\,P\sin\phi\,\hat{n}$$

Angular momentum is an axial vector i.e. always directed perpendicular to the plane of rotation and along the axis of rotation.

(1) S.I. Unit: kg-m^2-s^{-1} or J-sec.

(2) Dimension: $[ML^2T^{-1}]$ and it is similar to Planck's constant (h).

(3) In Cartesian co-ordinates if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

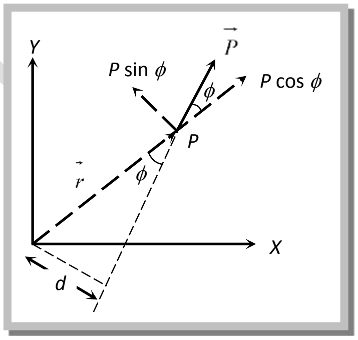
Then $\vec{L} = \vec{r} \times \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ P_x & P_y & P_z \end{vmatrix} = (yP_z - zP_y)\hat{i} - (xP_z - zP_x)\hat{j} + (xP_y - yP_x)\hat{k}$

(4) As it is clear from the figure radial component of momentum $\vec{P}_r = P\cos\phi$

Transverse component of momentum $\vec{P}_\phi = P\sin\phi$

So magnitude of angular momentum $L = rP\sin\phi$

$$L = rP_\phi$$



 Angular momentum = Position vector × Transverse component of angular momentum

i.e., The radial component of linear momentum has no role to play in angular momentum.

(5) Magnitude of angular momentum $L = P\ (r\sin\phi) = L = Pd$ [As $d = r\sin\phi$ from the figure.]

∴ Angular momentum = (Linear momentum) × (Perpendicular distance of line of action of force from the axis of rotation)

(6) Maximum and minimum angular momentum: We know $\vec{L} = \vec{r} \times \vec{P}$

∴ $\vec{L} = m[\vec{r} \times \vec{v}] = mvr\sin\phi = Pr\sin\phi$ [As $\vec{P} = m\vec{v}$]

$L_{maximum} = mvr$	When $\lvert\sin\phi\rvert = \max = 1$ i.e., $\phi = 90°$	$\vec{v}$ is perpendicular to $\vec{r}$
$L_{minimum} = 0$	When $\lvert\sin\phi\rvert = \min = 0$ $i.e.\ \phi = 0°$ or $180°$	$\vec{v}$ is parallel or anti-parallel to $\vec{r}$

(7) A particle in translatory motion always have an angular momentum unless it is a point on the line of motion because $L = mvr\sin\phi$ and $L > 1$ if $\phi \neq 0^o$ or 180^o

(8) In case of circular motion, $\vec{L} = \vec{r} \times \vec{P} = m(\vec{r} \times \vec{v})$ = $mvr\sin\phi$

$\therefore$ $L = mvr = mr^2\omega$ [As $\vec{r} \perp \vec{v}$ and $v = r\omega$]

or $L = I\omega$ [As $mr^2 = I$]

In vector form $\vec{L} = I\vec{\omega}$

(9) From $\vec{L} = I\vec{\omega}$ $\therefore$ $\frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt}$ = $I\vec{\alpha} = \vec{\tau}$ [As $\frac{d\vec{\omega}}{dt} = \vec{\alpha}$ and $\vec{\tau} = I\vec{\alpha}$]

I.e. the rate of change of angular momentum is equal to the net torque acting on the particle. [Rotational analogue of Newton's second law]

(10) If a large torque acts on a particle for a small time then 'angular impulse' of torque is given by

$$\vec{J} = \int \vec{\tau}\, dt = \vec{\tau}_{av} \int_{t_1}^{t_2} dt$$

or Angular impulse $\vec{J} = \vec{\tau}_{av}\, \Delta t = \Delta\vec{L}$

$\therefore$ Angular impulse = Change in angular momentum

(11) The angular momentum of a system of particles is equal to the vector sum of angular momentum of each particle i.e., $\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ... + \vec{L}_n$.

(12) According to Bohr Theory angular momentum of an electron in n^{th} orbit of atom can be taken as,

$L = n\frac{h}{2\pi}$ [Where n is an integer used for number of orbit]

18. Law of Conservation of Angular Momentum.

Newton's second law for rotational motion $\vec{\tau} = \frac{d\vec{L}}{dt}$

So if the net external torque on a particle (or system) is zero then $\frac{d\vec{L}}{dt} = 0$

i.e. $\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + ...$ = constant.

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As $L = I\omega$ so if $\vec{\tau} = 0$ then $I\omega = \text{constant}$ $\therefore\ I \propto \frac{1}{\omega}$

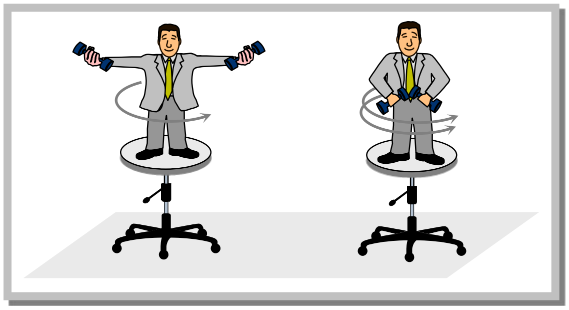
Since angular momentum $I\omega$ remains constant so when I decreases, angular velocity ω increases and vice-versa.

Examples of law of conservation of angular momentum:

(1) The angular velocity of revolution of a planet around the sun in an elliptical orbit increases when the planet come closer to the sun and vice-versa because when planet comes closer to the sun, its moment of inertia I decreases therefore ω increases.

(2) A circus acrobat performs feats involving spin by bringing his arms and legs closer to his body or vice-versa. On bringing the arms and legs closer to body, his moment of inertia I decreases. Hence ω increases.

(3) A person-carrying heavy weight in his hands and standing on a rotating platform can change the speed of platform. When the person suddenly folds his arms. Its moment of inertia decreases and in accordance the angular speed increases.



(4) A diver performs somersaults by Jumping from a high diving board keeping his legs and arms out stretched first and then curling his body.

(5) Effect of change in radius of earth on its time period

Angular momentum of the earth $L = I\omega = \text{constant}$

$$L = \frac{2}{5}MR^2 \times \frac{2\pi}{T} = \text{constant}$$

$$\therefore \quad T \propto R^2$$

[If M remains constant]

If R becomes half then time period will become one-fourth i.e. $\frac{24}{4} = 6 hrs.$

19. Work, Energy and Power for Rotating Body.

(1) **Work:** If the body is initially at rest and angular displacement is $d\theta$ due to torque then work done on the body.

$W = \int \tau \, d\theta$ [Analogue to work in translatory motion $W = \int F \, dx$]

(2) **Kinetic energy:** The energy, which a body has by virtue of its rotational motion is called rotational kinetic energy. A body rotating about a fixed axis possesses kinetic energy because its constituent particles are in motion, even though the body as a whole remains in place.

Rotational kinetic energy	Analogue to translatory kinetic energy
$K_R = \frac{1}{2}I\omega^2$	$K_T = \frac{1}{2}mv^2$
$K_R = \frac{1}{2}L\omega$	$K_T = \frac{1}{2}Pv$
$K_R = \frac{L^2}{2I}$	$K_T = \frac{P^2}{2m}$

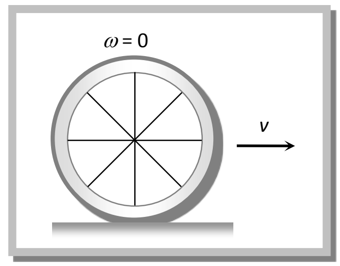
(3) **Power:** Rate of change of kinetic energy is defined as power

$$P = \frac{d}{dt}(K_R) = \frac{d}{dt}\left[\frac{1}{2}I\omega^2\right] = I\omega\frac{d\omega}{dt} = I\omega\alpha = I\alpha\omega = \tau\omega$$

In vector form Power $= \vec{\tau}\cdot\vec{\omega}$ [Analogue to power in translatory motion $P = \vec{F}\cdot\vec{v}$]

20. Slipping, Spinning and Rolling.

(1) **Slipping:** When the body slides on a surface without rotation then its motion is called slipping motion.

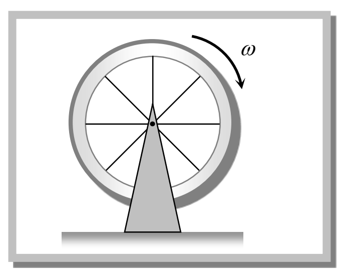


In this condition friction between the body and surface $F = 0$.

Body possess only translatory kinetic energy $K_T = \frac{1}{2}mv^2$.

Example: Motion of a ball on a frictionless surface.

(2) **Spinning:** When the body rotates in such a manner that its axis of rotation does not move then its motion is called spinning motion.



In this condition axis of rotation of a body is fixed.

Example: Motion of blades of a fan.

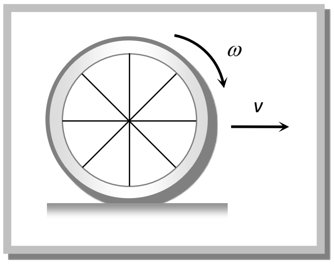
In spinning, body possess only rotatory kinetic energy $K_R = \frac{1}{2}I\omega^2$.

or $$K_R = \frac{1}{2}mK^2\frac{v^2}{R^2} = \frac{1}{2}mv^2\left(\frac{K^2}{R^2}\right)$$

i.e., Rotatory kinetic energy = $\left(\frac{K^2}{R^2}\right)$ times translatory kinetic energy.

Here $\frac{K^2}{R^2}$ is a constant for different bodies. Value of $\frac{K^2}{R^2} = 1$ (ring), $\frac{K^2}{R^2} = \frac{1}{2}$ (disc) and $\frac{K^2}{R^2} = \frac{1}{2}$ (solid sphere)

(3) **Rolling:** If in case of rotational motion of a body about a fixed axis, the axis of rotation also moves, the motion is called combined translatory and rotatory.



Example: (i) Motion of a wheel of cycle on a road.

(ii) Motion of football rolling on a surface.

In this condition friction between the body and surface $F \neq 0$.

Body possesses both translational and rotational kinetic energy.

Net kinetic energy = (Translatory + Rotatory) kinetic energy.

$$K_N = K_T + K_R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2\frac{K^2}{R^2}$$

$$\therefore \qquad K_N = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

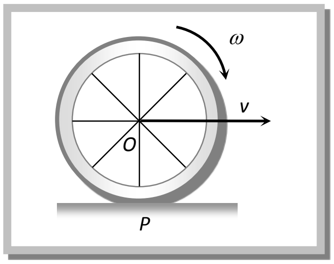
21. Rolling Without Slipping.

In case of combined translatory and rotatory motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping.

Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact.

Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity ω.

By the law of conservation of energy



$$K_N = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad [\because \text{As } v = R\omega]$$

$$= \frac{1}{2}mR^2\omega^2 + \frac{1}{2}I\omega^2$$

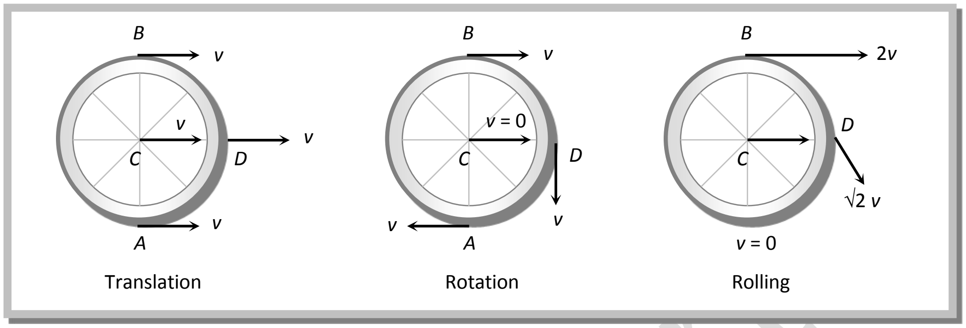
$$= \frac{1}{2}\omega^2[mR^2 + I]$$

$$= \frac{1}{2}\omega^2[I + mR^2] = \frac{1}{2}I_P\omega^2 \quad [\text{As } I_P = I + mR^2]$$

By theorem of parallel axis, where I = moment of inertia of rolling body about its center 'O' and I_P = moment of inertia of rolling body about point of contact 'P'.

(1) **Linear velocity of different points in rolling:** In case of rolling, all points of a rigid body have same angular speed but different linear speed.

Let A, B, C and D are four points then their velocities are shown in the following figure.

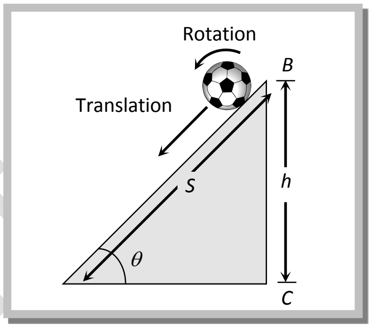


(2) **Energy distribution table for different rolling bodies:**

Body	$\frac{K^2}{R^2}$	**Translatory (K_T)** $\frac{1}{2}mv^2$	**Rotatory (K_R)** $\frac{1}{2}mv^2\frac{K^2}{R^2}$	**Total (K_N)** $\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$	$\frac{K_T}{K_N}$ **(%)**	$\frac{K_R}{K_N}$ **(%)**
Ring Cylindrical shell	1	$\frac{1}{2}mv^2$	$\frac{1}{2}mv^2$	mv^2	$\frac{1}{2}$ (50%)	$\frac{1}{2}$ (50%)
Disc solid cylinder	$\frac{1}{2}$	$\frac{1}{2}mv^2$	$\frac{1}{4}mv^2$	$\frac{3}{4}mv^2$	$\frac{2}{3}$ (66.6%)	$\frac{1}{3}$ (33.3%)
Solid sphere	$\frac{2}{5}$	$\frac{1}{2}mv^2$	$\frac{1}{5}mv^2$	$\frac{7}{10}mv^2$	$\frac{5}{7}$ (71.5%)	$\frac{2}{7}$ (28.5%)
Hollow sphere	$\frac{2}{3}$	$\frac{1}{2}mv^2$	$\frac{1}{3}mv^2$	$\frac{5}{6}mv^2$	$\frac{3}{5}$ (60%)	$\frac{2}{5}$ (40%)

22. Rolling on an Inclined Plane.

When a body of mass m and radius R rolls down on inclined plane of height 'h' and angle of inclination θ, it loses potential energy. However it acquires both linear and angular speeds and hence, gain kinetic energy of translation and that of rotation.



By conservation of mechanical energy $mgh = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

(1) **Velocity at the lowest point:** $v = \sqrt{\dfrac{2gh}{1+\frac{k^2}{R^2}}}$

(2) **Acceleration in motion:** From equation $v^2 = u^2 + 2aS$

By substituting $u = 0,\ S = \dfrac{h}{\sin\theta}$ and $v = \sqrt{\dfrac{2gh}{1+\frac{k^2}{R^2}}}$ we get

$$a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

(3) **Time of descent:** From equation $v = u + at$

By substituting u = 0 and value of v and a from above expressions

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left[1+\frac{k^2}{R^2}\right]}$$

From the above expressions it is clear that, $v \propto \dfrac{1}{\sqrt{1+\frac{k^2}{R^2}}}$; $a \propto \dfrac{1}{1+\frac{k^2}{R^2}}$; $t \propto \sqrt{1+\frac{k^2}{R^2}}$

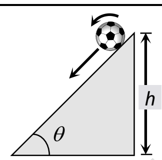
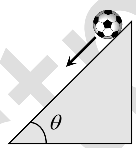
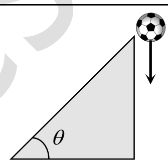
Note: Here factor $\left(\frac{k^2}{R^2}\right)$ is a measure of moment of inertia of a body and its value is constant for given shape of the body and it does not depend on the mass and radius of a body.

- Velocity, acceleration and time of descent (for a given inclined plane) all depends on $\frac{k^2}{R^2}$. Lesser the moment of inertia of the rolling body lesser will be the value of $\frac{k^2}{R^2}$. So greater will be its velocity and acceleration and lesser will be the time of descent.
- If a solid and hollow body of same shape are allowed to roll down on inclined plane then as $\left(\frac{k^2}{R^2}\right)_S < \left(\frac{k^2}{R^2}\right)_H$, solid body will reach the bottom first with greater velocity.

- If a ring, cylinder, disc and sphere runs a race by rolling on an inclined plane then as $\left(\frac{k^2}{R^2}\right)_{\text{sphere}} = \text{minimum}$ while $\left(\frac{k^2}{R^2}\right)_{\text{Ring}} = \text{maximum}$, the sphere will reach the bottom first with greatest velocity while ring at last with least velocity.
- Angle of inclination has no effect on velocity, but time of descent and acceleration depends on it.

  Velocity $\propto \theta^{\circ}$, time of decent $\propto \theta^{-1}$ and acceleration $\propto \theta$.

23. Rolling Sliding and Falling of a Body.

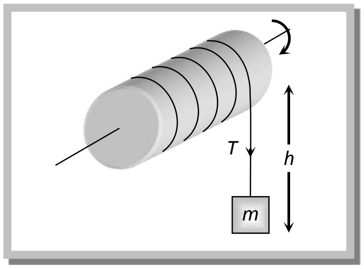
		Figure	Velocity	Acceleration	Time
Rolling	$\frac{k^2}{R^2} \neq 0$		$\sqrt{\frac{2gh}{1+k^2/R^2}}$	$\frac{g \sin\theta}{1+K^2/R^2}$	$\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$
Sliding	$\frac{k^2}{R^2} = 0$		$\sqrt{2gh}$	$g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$
Falling	$\frac{k^2}{R^2} = 0$ θ = 90°		$\sqrt{2gh}$	g	$\sqrt{\frac{2h}{g}}$

24. Velocity, Acceleration and Time for Different Bodies.

Body	$\frac{k^2}{R^2}$	Velocity $v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$	Acceleration $a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$	Time of descent $t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$
Ring or Hollow cylinder	1	$\sqrt{gh}$	$\frac{1}{2}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{4h}{g}}$
Disc or solid cylinder	$\frac{1}{2}$ or 0.5	$\sqrt{\frac{4gh}{3}}$	$\frac{2}{3}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{3h}{g}}$
Solid sphere	$\frac{2}{5}$ or 0.4	$\sqrt{\frac{10}{7}gh}$	$\frac{5}{7}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{14}{5}\frac{h}{g}}$
Hollow sphere	$\frac{2}{3}$ or 0.66	$\sqrt{\frac{6}{5}gh}$	$\frac{3}{5}g\sin\theta$	$\frac{1}{\sin\theta}\sqrt{\frac{10}{3}\frac{h}{g}}$

25. Motion of Connected Mass.

A point mass is tied to one end of a string which is wound round the solid body [cylinder, pulley, disc]. When the mass is released, it falls vertically downwards and the solid body rotates unwinding the string



m = mass of point-mass, M = mass of a rigid body

R = radius of a rigid body, I = moment of inertia of rotating body

(1) **Downwards acceleration of point mass** $a = \frac{g}{1+\frac{I}{mR^2}}$

(2) **Tension in string** $T = mg\left[\frac{I}{I+mR^2}\right]$

(3) **Velocity of point mass** $v = \sqrt{\frac{2gh}{1+\frac{I}{mR^2}}}$

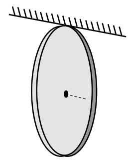
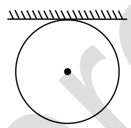
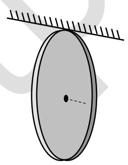
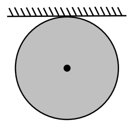
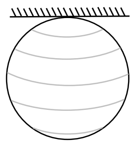
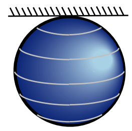
(4) **Angular velocity of rigid body** $\omega = \sqrt{\frac{2mgh}{I+mR^2}}$

26. Time Period of Compound Pendulum

Time period of compound pendulum is given by, $T = 2\pi\sqrt{\frac{L}{g}}$ where $L = \frac{l^2 + k^2}{l}$

Here l = distance of center of mass from point of suspension

k = radius of gyration about the parallel axis passing through center of mass.

Body	Axis of rotation	Figure	l	k^2	$L = \frac{l^2 + k^2}{l}$	$T = 2\pi\sqrt{\frac{L}{g}}$
Ring	Tangent passing through the rim and perpendicular to the plane		R	R^2	$2R$	$T = 2\pi\sqrt{\frac{2R}{g}}$
	Tangent parallel to the plane		R	$\frac{R^2}{2}$	$\frac{3}{2}R$	$T = 2\pi\sqrt{\frac{3R}{2g}}$
Disc	Tangent, Perpendicular to plane		R	$\frac{R^2}{2}$	$\frac{3}{2}R$	$T = 2\pi\sqrt{\frac{3R}{2g}}$
	Tangent parallel to the plane		R	$\frac{R^2}{4}$	$\frac{5}{4}R$	$T = 2\pi\sqrt{\frac{5R}{4g}}$
Spherical shell	Tangent		R	$\frac{2}{3}R^2$	$\frac{5}{3}R$	$T = 2\pi\sqrt{\frac{5R}{3g}}$
Solid sphere	Tangent		R	$\frac{2}{5}R^2$	$\frac{7}{5}R$	$T = 2\pi\sqrt{\frac{7R}{5g}}$
