



Knowledge... Everywhere

Physics

Transmission of Heat

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1. Introduction.

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by one of the following modes.

Conduction	Convection	Radiation
Heat flows from hot end to cold end. Particles of the medium simply oscillate but do not leave their place.	Each particle absorbing heat is mobile	Heat flows without any intervening medium in the form of electromagnetic waves.
Medium is necessary for conduction	Medium is necessary for convection	Medium is not necessary for radiation
It is a slow process	It is also a slow process	It is a very fast process
Path of heat flow may be zig-zag	Path may be zig-zag or curved	Path is a straight line
Conduction takes place in solids	Convection takes place in fluids	Radiation takes place in gaseous and transparent media
The temperature of the medium increases through which heat flows	In this process also the temperature of medium increases	There is no change in the temperature of the medium

2. Conduction.

The process of transmission of heat energy in which the heat is transferred from one particle to other particle without dislocation of the particle from their equilibrium position is called conduction.

(i) Conduction is a process which is possible in all states of matter.

(ii) In solids only conduction takes place.

(iii) In non-metallic solids and fluids the conduction takes place only due to vibrations of molecules, therefore they are poor conductors.



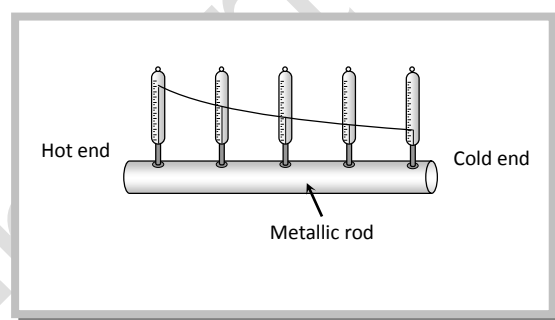
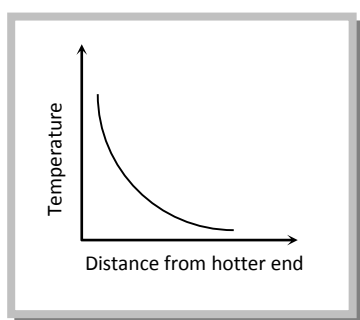
(iv) In metallic solids free electrons carry the heat energy, therefore they are good conductor of heat.

(1) Variable and steady state

When one end of a metallic rod is heated, heat flows by conduction from the hot end to the cold end.

In the process of conduction each cross-section of the rod receives heat from the adjacent cross-section towards the hot end. A part of this heat is absorbed by the cross-section itself whose temperature increases, another part is lost into atmosphere by convection & radiation and the rest is conducted away to the next cross-section.

Because in this state temperature of every cross-section of the rod goes on increasing, hence rod is said to exist in variable state.



After sometime, a state is reached when the temperature of every cross-section of the rod becomes constant. In this state, no heat is absorbed by the rod. The heat that reaches any cross-section is transmitted to the next except that a small part of heat is lost to surrounding from the sides by convection & radiation. This state of the rod in which no part of rod absorbs heat is called steady state.

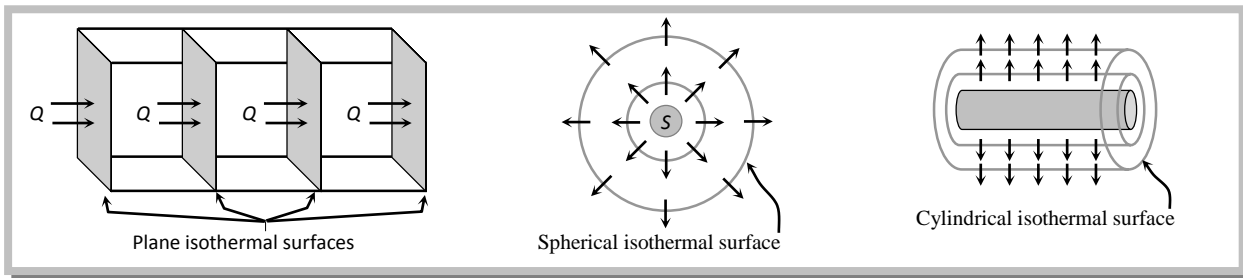
(2) Isothermal surface

Any surface (within a conductor) having its all points at the same temperature, is called isothermal surface. The direction of flow of heat through a conductor at any point is perpendicular to the isothermal surface passing through that point.

- (i) If the material is rectangular or cylindrical rod, the isothermal surface is a plane surface.
- (ii) If a point source of heat is situated at the center of a sphere the isothermal surface will be spherical,



(iii) If steam passes along the axis of the hollow cylinder, heat will flow through the walls of the cylinder so that in this condition the isothermal surface will be cylindrical.



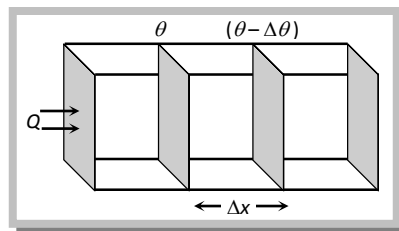
(3) Temperature Gradient

The rate of change of temperature with distance between two isothermal surfaces is called temperature gradient.

If the temperature of two isothermal surfaces be θ and $(\theta - \Delta\theta)$, and the perpendicular distance between them be Δx then Temperature gradient = $\frac{(\theta - \Delta\theta) - \theta}{\Delta x} = \frac{-\Delta\theta}{\Delta x}$

The negative sign show that temperature θ decreases as the distance x increases in the direction of heat flow.

Unit: K/m (S.I.) and Dimensions: $[L^{-1}\theta]$



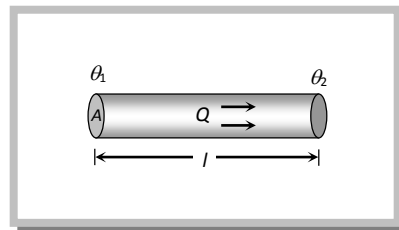
(4) Coefficient of thermal conductivity

If L be the length of the rod, A the area of cross-section and θ_1 and θ_2 are the temperature of its two faces, then the amount of heat flowing from one face to the other face in time t is given by

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

Where K is coefficient of thermal conductivity of material of rod. It is the measure of the ability of a substance to conduct heat through it.

If $A = 1\text{m}^2$, $(\theta_1 - \theta_2) = 1\text{oC}$, $t = 1$ sec and $l = 1\text{m}$, then $Q = K$.



Thus, thermal conductivity of a material is the amount of heat flowing per second during steady state through its rod of length 1 m and cross-section 1 m² with a unit temperature difference between the opposite faces.

(i) Units: Cal/cm-sec oC (in C.G.S.), kcal/m-sec-K (in M.K.S.) and W/m- K (in S.I.)



(ii) Dimension: $[MLT^{-3}\theta^{-1}]$

(iii) The magnitude of K depends only on nature of the material.

(iv) For perfect conductors, $K = \infty$ and for perfect insulators, $K = 0$

(v) Substances in which heat flows quickly and easily are known as good conductor of heat. They possess large thermal conductivity due to large number of free electrons. Example: Silver, brass etc.

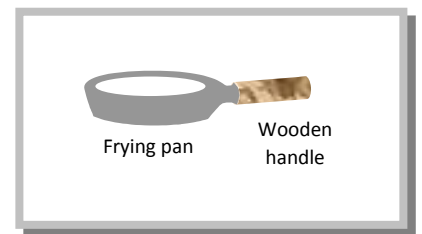
(vi) Substances which do not permit easy flow of heat are called bad conductors. They possess low thermal conductivity due to very few free electrons. Example: Glass, wood etc.

(vii) The thermal conductivity of pure metals decreases with rise in temperature but for alloys thermal conductivity increases with increase of temperature.

(viii) Human body is a bad conductor of heat (but it is a good conductor of electricity).

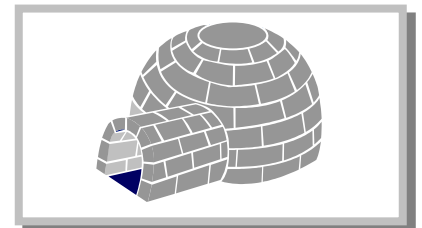
(5) Applications of conductivity in daily life

(i) Cooking utensils are provided with wooden handles, because wood is a poor conductor of heat. The hot utensils can be easily handled from the wooden handles and our hands are saved from burning.



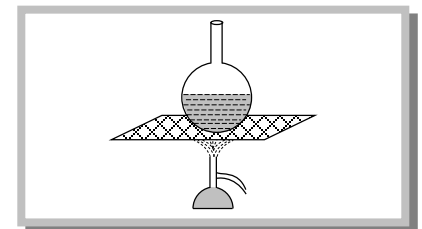
(ii) We feel warmer in a fur coat. The air enclosed in the fur coat being bad conductor heat does not allow the body heat to flow outside. Hence we feel warmer in a fur coat.

(iii) Eskimos make double walled houses of the blocks of ice. Air enclosed in between the double walls prevents transmission of heat from the house to the cold surroundings.



For exactly the same reason, two thin blankets are warmer than one blanket of their combined thickness. The layer of air enclosed in between the two blankets makes the difference.

(iv) Wire gauze is placed over the flame of Bunsen burner while heating the flask or a beaker so that the flame does not go beyond the gauze and hence there is no direct contact between the flame and the flask. The wire gauze being a good conductor of heat, absorb the heat of the flame and transmit it to the flask.



Davy's safety lamp has been designed on this principle. The gases in the mines burn inside the gauze placed around the flame of the lamp. The temperature outside the gauze is not high, so the gases outside the gauze do not catch fire.



(v) Birds often swell their feathers in winter. By doing so, they enclose more air between their bodies and the feathers. The air, being bad conductor of heat prevents the out flow of their body heat. Thus, birds feel warmer in winter by swelling their feathers.

(6) Relation between temperature gradient and thermal conductivity

$$\text{In steady state, rate of flow of heat } \frac{dQ}{dt} = -KA \frac{d\theta}{dx} = -KA \text{ (Temperature gradient)}$$

$$\text{If } \frac{dQ}{dt} \text{ is constant then temperature gradient } \propto \frac{1}{K}$$

Temperature difference between the hot end and the cold end in steady state is inversely proportional to K, i.e. in case of good conductors temperature of the cold end will be very near to hot end.

In ideal conductor where $K = \infty$, temperature difference in steady state will be zero.

(7) Wiedmann-Franz law

At a given temperature T, the ratio of thermal conductivity to electrical conductivity is constant i.e., $(K / \sigma T) = \text{constant}$, i.e., a substance which is a good conductor of heat (e.g., silver) is also a good conductor of electricity. Mica is an exception to above law.

(8) Thermometric conductivity or diffusivity

It is a measure of rate of change of temperature (with time) when the body is not in steady state (i.e., in variable state)

The thermometric conductivity or diffusivity is defined as the ratio of the coefficient of thermal conductivity to the thermal capacity per unit volume of the material.

$$\text{Thermal capacity per unit volume} = \frac{mc}{V} = \rho c \quad (\text{As } \rho \text{ is density of substance})$$

$$\therefore \text{Diffusivity (D)} = \frac{K}{\rho c}$$

Unit: m²/sec and Dimension: $[L^2 T^{-1}]$

(9) Thermal resistance

The thermal resistance of a body is a measure of its opposition to the flow of heat through it.

It is defined as the ratio of temperature difference to the heat current (= Rate of flow of heat)

$$\text{Now, temperature difference} = (\theta_1 - \theta_2) \text{ and heat current, } H = \frac{Q}{t}$$



$$\therefore \text{Thermal resistance, } R = \frac{\theta_1 - \theta_2}{H} = \frac{\theta_1 - \theta_2}{Q/t} = \frac{\theta_1 - \theta_2}{KA(\theta_1 - \theta_2)/l} = \frac{l}{KA}$$

Unit: $^{\circ}C \times sec / cal$ or $K \times sec / kcal$ and Dimension: $[M^{-1}L^{-2}T^3\theta]$

3. Electrical Analogy for Thermal Conduction.

It is an important fact to appreciate that there exists an exact similarity between thermal and electrical conductivities of a conductor.

Electrical conduction	Thermal conduction
Electric charge flows from higher potential to lower potential	Heat flows from higher temperature to lower temperature
The rate of flow of charge is called the electric current, i.e. $I = \frac{dq}{dt}$	The rate of flow of heat may be called as heat current i.e. $H = \frac{dQ}{dt}$
The relation between the electric current and the potential difference is given by Ohm's law, that is $I = \frac{V_1 - V_2}{R}$ where R is the electrical resistance of the conductor	Similarly, the heat current may be related with the temperature difference as $H = \frac{\theta_1 - \theta_2}{R}$ where R is the thermal resistance of the conductor
The electrical resistance is defined as $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$ where ρ = Resistivity and σ = Electrical conductivity $\frac{dq}{dt} = I = \frac{V_1 - V_2}{R} = \frac{\sigma A}{l}(V_1 - V_2)$	The thermal resistance may be defined as $R = \frac{l}{KA}$ where K = Thermal conductivity of conductor $\frac{dQ}{dt} = H = \frac{\theta_1 - \theta_2}{R} = \frac{KA}{l}(\theta_1 - \theta_2)$

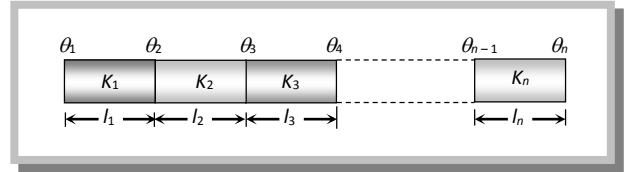


4. Combination of Conductors.

(1) Series combination:

Let n slabs each of cross-sectional area A, lengths $l_1, l_2, l_3, \dots, l_n$ and conductivities $K_1, K_2, K_3, \dots, K_n$ respectively be connected in the series

Heat current is the same in all the conductors.



i.e. $\frac{Q}{t} = H_1 = H_2 = H_3 \dots = H_n$

$$\frac{K_1 A (\theta_1 - \theta_2)}{l_1} = \frac{K_2 A (\theta_2 - \theta_3)}{l_2} = \frac{K_3 A (\theta_3 - \theta_4)}{l_3} = \dots = \frac{K_n A (\theta_{n-1} - \theta_n)}{l_n}$$

(i) Equivalent resistance $R = R_1 + R_2 + R_3 + \dots + R_n$

(ii) If K_s is equivalent conductivity, then from relation $R = \frac{l}{KA}$

$$\frac{l_1 + l_2 + l_3 + \dots + l_n}{K_s} = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A} + \frac{l_3}{K_3 A} + \dots + \frac{l_n}{K_n A}$$

$$K_s = \frac{l_1 + l_2 + l_3 + \dots + l_n}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3} + \dots + \frac{l_n}{K_n}}$$

∴

$$K = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \dots + \frac{1}{K_n}}$$

(iii) Equivalent thermal conductivity for n slabs of equal length

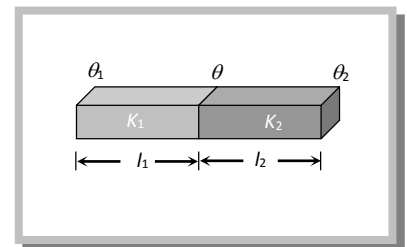
$$K = \frac{2K_1 K_2}{K_1 + K_2}$$

For two slabs of equal length,

(iv) Temperature of interface of composite bar: Let the two bars are arranged in series as shown in the figure.

Then heat current is same in the two conductors.

i.e. $\frac{Q}{t} = \frac{K_1 A (\theta_1 - \theta)}{l_1} = \frac{K_2 A (\theta - \theta_2)}{l_2}$



$$\theta = \frac{\frac{K_1}{l_1} \theta_1 + \frac{K_2}{l_2} \theta_2}{\frac{K_1}{l_1} + \frac{K_2}{l_2}}$$

By solving we get

If ($l_1 = l_2 = l$) then
$$\theta = \frac{K_1 \theta_1 + K_2 \theta_2}{K_1 + K_2}$$

(2) Parallel Combination

Let n slabs each of length l, areas $A_1, A_2, A_3, \dots, A_n$ and thermal conductivities $K_1, K_2, K_3, \dots, K_n$ are connected in parallel then.

(i) Equivalent resistance
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

(ii) Temperature gradient across each slab will be same.

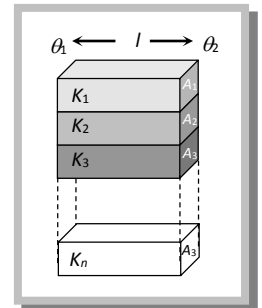
(iii) Heat current in each slab will be different. Net heat current will be the sum of heat currents through individual slabs. i.e. $H = H_1 + H_2 + H_3 + \dots + H_n$

$$\frac{K(A_1 + A_2 + A_3 + \dots + A_n)(\theta_1 - \theta_2)}{l} = \frac{K_1 A_1 (\theta_1 - \theta_2)}{l} + \frac{K_2 A_2 (\theta_1 - \theta_2)}{l} + \frac{K_3 A_3 (\theta_1 - \theta_2)}{l} + \dots + \frac{K_n A_n (\theta_1 - \theta_2)}{l}$$

$$\therefore K = \frac{K_1 A_1 + K_2 A_2 + K_3 A_3 + \dots + K_n A_n}{A_1 + A_2 + A_3 + \dots + A_n}$$

For n slabs of equal area
$$K = \frac{K_1 + K_2 + K_3 + \dots + K_n}{n}$$

Equivalent thermal conductivity for two slabs of equal area
$$K = \frac{K_1 + K_2}{2}$$



5. Ingen-Hauz Experiment.

It is used to compare thermal conductivities of different materials. If l_1 and l_2 are the lengths of wax

melted on rods, then the ratio of thermal conductivities is $\frac{K_1}{K_2} = \frac{l_1^2}{l_2^2}$

i.e., in this experiment, we observe Thermal conductivity $\propto (\text{length})^2$

6. Searle's Experiment.

It is a method of determination of K of a metallic rod. Here we are not much interested in the detailed description of the experimental setup. We will only understand its essence, which is the essence of solving many numerical problems.

In this experiment a temperature difference $(\theta_1 - \theta_2)$ is maintained across a rod of length l and area of cross section A . If the thermal conductivity of the material of the rod is K , then the amount of heat

transmitted by the rod from the hot end to the cold end in time t is given by, $Q = \frac{KA(\theta_1 - \theta_2)t}{l}$

.....(i)

In Searle's experiment, this heat reaching the other end is utilized to raise the temperature of certain amount of water flowing through pipes circulating around the other end of the rod. If temperature of the water at the inlet is θ_3 and at the outlet is θ_4 , then the amount of heat absorbed by water is given by, $Q = mc(\theta_4 - \theta_3)$ (ii)

Where, m is the mass of the water which has absorbed this heat and temperature is raised and c is the specific heat of the water

$$K = \frac{mc(\theta_4 - \theta_3)l}{A(\theta_1 - \theta_2)t}$$

Equating (i) and (ii), K can be determined i.e.

Note: In numerical we may have the situation where the amount of heat travelling to the other end may be required to do some other work e.g., it may be required to melt the given amount of ice. In that case equation (i) will have to be equated to mL .

$$mL = \frac{KA(\theta_1 - \theta_2)t}{l}$$

i.e.



7. Growth of Ice on Lake.

Water in a lake starts freezing if the atmospheric temperature drops below $0^\circ C$. Let y be the thickness of ice layer in the lake at any instant t and atmospheric temperature is $-\theta^\circ C$. The temperature of water in contact with lower surface of ice will be zero. If A is the area of lake, heat escaping through ice in time dt is

$$dQ_1 = \frac{KA[0 - (-\theta)]dt}{y}$$

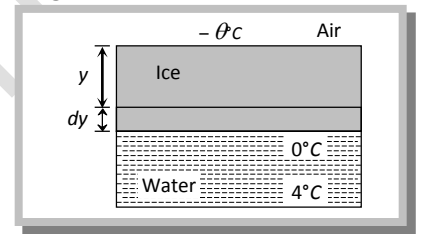
Now, suppose the thickness of ice layer increases by dy in time dt , due to escaping of above heat. Then

$$dQ_2 = mL = \rho(dy A)L$$

As $dQ_1 = dQ_2$, hence, rate of growth of ice will be $(dy/dt) = (K\theta/\rho Ly)$

So, the time taken by ice to grow to a thickness y is $t = \frac{\rho L}{K\theta} \int_0^y y dy = \frac{\rho L}{2K\theta} y^2$

If the thickness is increased from y_1 to y_2 then time taken $t = \frac{\rho L}{K\theta} \int_{y_1}^{y_2} y dy = \frac{\rho L}{2K\theta} (y_2^2 - y_1^2)$



(i) Take care and do not apply a negative sign for putting values of temperature in formula and also do not convert it to absolute scale.

(ii) Ice is a poor conductor of heat, therefore the rate of increase of thickness of ice on ponds decreases with time.

(iii) It follows from the above equation that time taken to double and triple the thickness, will be in the ratio of

$$t_1 : t_2 : t_3 :: 1^2 : 2^2 : 3^2, \text{ i.e., } t_1 : t_2 : t_3 :: 1 : 4 : 9$$

(iv) The time intervals to change the thickness from 0 to y , from y to $2y$ and so on will be in the ratio

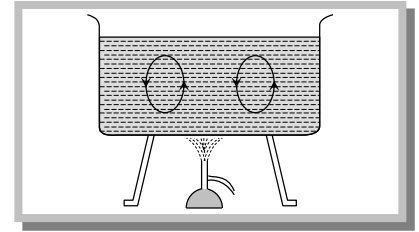
$$\Delta t_1 : \Delta t_2 : \Delta t_3 :: (1^2 - 0^2) : (2^2 - 1^2) : (3^2 - 2^2), \Delta t_1 : \Delta t_2 : \Delta t_3 :: 1 : 3 : 5$$



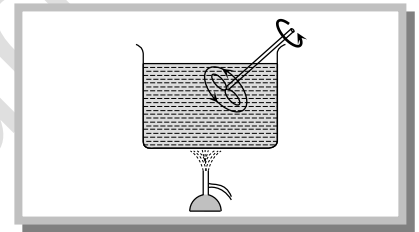
8. Convection.

Mode of transfer of heat by means of migration of material particles of medium is called convection. It is of two types.

(1) Natural convection: This arise due to difference of densities at two places and is a consequence of gravity because on account of gravity the hot light particles rise up and cold heavy particles try setting down. It mostly occurs on heating a liquid/fluid.



(2) Forced convection: If a fluid is forced to move to take up heat from a hot body then the convection process is called forced convection. In this case Newton's law of cooling holds good. According to which rate of loss of heat from a hot body due to moving fluid is directly proportional to the surface area of body and excess temperature of body over its surroundings



i.e.
$$\frac{Q}{t} \propto A(T - T_0)$$

$$\frac{Q}{t} = h A(T - T_0)$$

Where h = Constant of proportionality called convection coefficient,

T = Temperature of body and T₀ = Temperature of surrounding

Convection coefficient (h) depends on properties of fluid such as density, viscosity, specific heat and thermal conductivity.

- (i) Natural convection takes place from bottom to top while forced convection in any direction.
- (ii) In case of natural convection, convection currents move warm air upwards and cool air downwards. That is why heating is done from base, while cooling from the top.
- (iii) Natural convection plays an important role in ventilation, in changing climate and weather and in forming land and sea breezes and trade winds.
- (iv) Natural convection is not possible in a gravity free region such as a free falling lift or an orbiting satellite.
- (v) The force of blood in our body by heart helps in keeping the temperature of body constant.
- (vi) If liquids and gases are heated from the top (so that convection is not possible) they transfer heat (from top to bottom) by conduction.
- (vii) Mercury though a liquid is heated by conduction and not by convection.



9. Radiation.

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

Precisely it is electromagnetic energy transfer in the form of electromagnetic wave through any medium. It is possible even in vacuum.

For example, the heat from the sun reaches the earth through radiation.

Properties of thermal radiation

(1) The wavelength of thermal radiations ranges from $7.8 \times 10^{-7} \text{ m}$ to $4 \times 10^{-4} \text{ m}$. They belong to infra-red region of the electromagnetic spectrum. That is why thermal radiations are also called infra-red radiations.

Radiation	Frequency	Wavelength
Cosmic rays	$> 10^{21} \text{ Hz}$	$< 10^{-13} \text{ m}$
Gamma rays	$10^{18} - 10^{21} \text{ Hz}$	$10^{-13} - 10^{-10} \text{ m}$
X-rays	$10^{16} - 10^{19} \text{ Hz}$	$10^{-11} - 10^{-8} \text{ m}$ (0.1 Å - 100 Å)
Ultraviolet rays	$7.5 \times 10^{14} - 2 \times 10^{16} \text{ Hz}$	$1.4 \times 10^{-8} - 4 \times 10^{-7} \text{ m}$ (140 Å - 4000 Å)
Visible rays	$4 \times 10^{14} - 7.5 \times 10^{14} \text{ Hz}$	$4 \times 10^{-7} - 7.8 \times 10^{-7} \text{ m}$ (4000 Å - 7800 Å)
Infrared rays (Heat)	$3 \times 10^{11} - 4 \times 10^{14} \text{ Hz}$	$7.8 \times 10^{-7} - 10^{-3}$ (7800 Å - 3×10^5 Å)
Microwaves	$3 \times 10^8 - 3 \times 10^{11} \text{ Hz}$	$10^{-3} \text{ m} - 0.1 \text{ m}$
Radio waves	$10^4 - 3 \times 10^9 \text{ Hz}$	$0.1 \text{ m} - 10^4 \text{ m}$

(2) Medium is not required for the propagation of these radiations.

(3) They produce sensation of warmth in us but we can't see them.

(4) Everybody whose temperature is above zero Kelvin emits thermal radiation.

(5) Their speed is equal to that of light i.e. ($= 3 \times 10^8 \text{ m/s}$).



- (6) Their intensity is inversely proportional to the square of distance of point of observation from the source (i.e. $I \propto 1/d^2$).
- (7) Just as light waves, they follow laws of reflection, refraction, interference, diffraction and polarization.
- (8) When these radiations fall on a surface then exert pressure on that surface which is known as radiation pressure.
- (9) While travelling these radiations travel just like photons of other electromagnetic waves. They manifest themselves as heat only when they are absorbed by a substance.
- (10) Spectrum of these radiations cannot be obtained with the help of glass prism because it absorbs heat radiations. It is obtained by quartz or rock salt prism because these materials do not have free electrons and interatomic vibrational frequency is greater than the radiation frequency, hence they do not absorb heat radiations.

10. Some Definition about Radiations.

- (1) Diathermanous Medium: A medium which allows heat radiations to pass through it without absorbing them is called diathermanous medium. Thus the temperature of a diathermanous medium does not increase irrespective of the amount of the thermal radiations passing through it e.g., dry air, SO_2 , rock salt (NaCl).
- (i) Dry air does not get heated in summers by absorbing heat radiations from sun. It gets heated through convection by receiving heat from the surface of earth.
- (ii) In winters heat from sun is directly absorbed by human flesh while the surrounding air being diathermanous is still cool. This is the reason that sun's warmth in winter season appears very satisfying to us.
- (2) Athermanous medium: A medium which partly absorbs heat rays is called a thermous medium as a result temperature of an athermanous medium increases when heat radiations pass through it e.g., wood, metal, moist air, simple glass, human flesh etc.
- Glass and water vapors transmit shorter wavelengths through them but reflects longer wavelengths. This concept is utilized in Greenhouse effect. Glass transmits those waves which are emitted by a source at a temperature greater than 100°C . So, heat rays emitted from sun are able to enter through glass enclosure but heat emitted by small plants growing in the nursery gets trapped inside the enclosure.
- (3) Reflectance, Absorptance and transmittance



When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted.

(i) Reflectance or reflecting power (r): It is defined as the ratio of the amount of thermal radiations reflected (Q_r) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

(ii) Absorptance or absorbing power (a): It is defined as the ratio of the amount of thermal radiations absorbed (Q_a) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

(iii) Transmittance or transmitting power (t): It is defined as the ratio of the amount of thermal radiations transmitted (Q_t) by the body in a given time to the total amount of thermal radiations incident on the body in that time.

From the above definitions $r = \frac{Q_r}{Q}$, $a = \frac{Q_a}{Q}$ and $t = \frac{Q_t}{Q}$

By adding we get $r + a + t = \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = \frac{(Q_r + Q_a + Q_t)}{Q} = 1$

$\therefore r + a + t = 1$

(a) r, a and t all are the pure ratios so they have no unit and dimension.

(b) For perfect reflector : $r = 1$, $a = 0$ and $t = 0$

(c) For perfect absorber : $a = 1$, $r = 0$ and $t = 0$ (Perfectly black body)

(d) For perfect transmitter: $t = 1$, $a = 0$ and $r = 0$

(e) If body does not transmit any heat radiation, $t = 0 \therefore r + a = 1$ or $a = 1 - r$

So if r is more, a is less and vice-versa. It means good reflectors are bad absorbers.

(4) Monochromatic Emittance or Spectral emissive power

For a given surface it is defined as the radiant energy emitted per sec per unit area of the surface with in

a unit wavelength around λ i.e. lying between $\left(\lambda - \frac{1}{2}\right)$ to $\left(\lambda + \frac{1}{2}\right)$.

Spectral emissive power $(E_\lambda) = \frac{\text{Energy}}{\text{Area} \times \text{time} \times \text{wavelength}}$

Unit: $\frac{\text{Joule}}{m^2 \times \text{sec} \times \text{\AA}}$ and Dimension: $[ML^{-1}T^{-3}]$



(5) Total emittance or total emissive power

It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all

possible wavelengths.
$$E = \int_0^{\infty} E_{\lambda} d\lambda$$

Unit: $\frac{\text{Joule}}{m^2 \times \text{sec}}$ or $\frac{\text{Watt}}{m^2}$ and Dimension: $[MT^{-3}]$

(6) Monochromatic absorptance or spectral absorptive power

It is defined as the ratio of the amount of the energy absorbed in a certain time to the total heat energy incident upon it in the same time, both in the unit wavelength interval. It is dimensionless and unit less quantity. It is represented by a_{λ} .

(7) Total absorptance or total absorption power: It is defined as the total amount of thermal energy absorbed per unit time, per unit area of the body for all possible wavelengths.

$$a = \int_0^{\infty} a_{\lambda} d\lambda$$

It is also unit less and dimensionless quantity.

(8) Emissivity (e): Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body ($E_{\text{practical}}$) to the total emissive power of a perfect black body (E_{black}) at that temperature.

$$e = \frac{E_{\text{practical}}}{E_{\text{black}}}$$

i.e.

$e = 1$ for perfectly black body but for practical bodies emissivity (e) lies between zero and one ($0 < e < 1$).

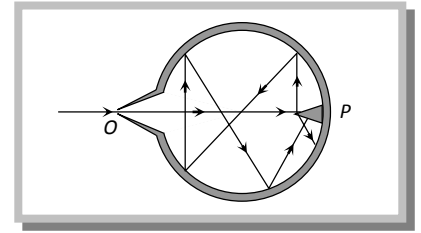
(9) Perfectly black body: A perfectly black body is that which absorbs completely the radiations of all wavelengths incident on it. As a perfectly black body neither reflects nor transmits any radiation, therefore the absorptance of a perfectly black body is unity i.e. $t = 0$ and $r = 0 \therefore a = 1$.

We know that the color of an opaque body is the color (wavelength) of radiation reflected by it. As a black body reflects no wavelength so, it appears black, whatever be the color of radiations incident on it.

When perfectly black body is heated to a suitable high temperature, it emits radiation of all possible wavelengths. For example, temperature of the sun is very high (6000 K approx.) it emits all possible radiation so it is an example of black body.



(10) Ferry's black body: A perfectly black body can't be realized in practice. The nearest example of an ideal black body is the Ferry's black body. It is a doubled walled evacuated spherical cavity whose inner wall is blackened. There is a fine hole in it. All the radiations incident upon this hole are absorbed by this black body. If this black body is heated to high temperature then it emits radiations of all wavelengths.



11. Prevots Theory of Heat Exchange.

- (1) Everybody emits heat radiations at all finite temperature (Except 0 K) as well as it absorbs radiations from the surroundings.
- (2) Exchange of energy along various bodies takes place via radiation.
- (3) The process of heat exchange among various bodies is a continuous phenomenon.
- (4) If the amount of radiation absorbed by a body is greater than that emitted by it then the temperature of body increases and it appears hotter.
- (5) If the amount of radiation absorbed by a body is less than that emitted by it, then the temperature of the body decreases and consequently the body appears colder.
- (6) If the amount of radiation absorbed by a body is equal to that emitted by the body, then the body will be in thermal equilibrium and the temperature of the body remains constant.
- (7) At absolute zero temperature (0 K or -273°C) this law is not applicable because at this temperature the heat exchange among various bodies ceases.



12. Kirchoff's Law.

The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Thus if $a_{\text{practical}}$ and $E_{\text{practical}}$ represent the absorptive and emissive power of a given surface, while

$$\frac{E_{\text{practical}}}{a_{\text{practical}}} = \frac{E_{\text{black}}}{a_{\text{black}}}$$

a_{black} and E_{black} for a perfectly black body, then according to law

$$\frac{E_{\text{practical}}}{a_{\text{practical}}} = E_{\text{black}}$$

But for a perfectly black body $a_{\text{black}} = 1$ so

$$\left(\frac{E_{\lambda}}{a_{\lambda}} \right)_{\text{practical}} = (E_{\lambda})_{\text{black}}$$

If emissive and absorptive powers are considered for a particular wavelength λ ,

Now since $(E_{\lambda})_{\text{black}}$ is constant at a given temperature, according to this law if a surface is a good absorber of a particular wavelength it is also a good emitter of that wavelength.

This in turn implies that a good absorber is a good emitter (or radiator)

Applications of Kirchoff's law

(1) Sand is rough black, so it is a good absorber and hence in deserts, days (when radiation from the sun is incident on sand) will be very hot. Now in accordance with Kirchoff's law, good absorber is a good emitter so nights (when sand emits radiation) will be cold. This is why days are hot and nights are cold in desert.

(2) Sodium vapors, on heating, emit two bright yellow lines. These are called D1, D2 lines of sodium. When continuous white light from an arc lamp is made to pass through sodium vapors at low temperature, the continuous spectrum is intercepted by two dark lines exactly in the same places as D1 and D2 lines. Hence sodium vapors when cold, absorb the same wavelength, as they emit while hot. This is in accordance with Kirchoff's law.

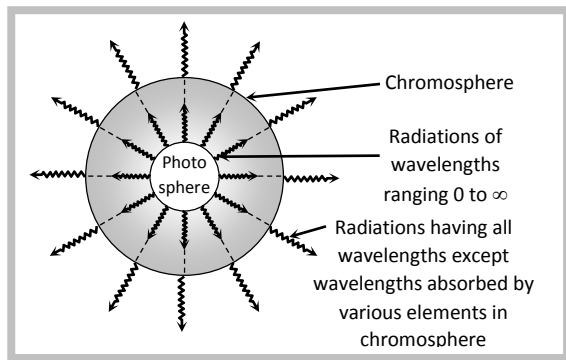
(3) When a shining metal ball having some black spots on its surface is heated to a high temperature and is seen in dark, the black spots shine brightly and the shining ball becomes dull or invisible. The reason is that the black spots on heating absorb radiation and so emit these in dark while the polished shining part reflects radiations and absorb nothing and so does not emit radiations and becomes invisible in the dark.

(4) When a green glass is heated in furnace and taken out, it is found to glow with red light. This is because red and green are complimentary colors. At ordinary temperatures, a green glass appears green, because it transmits green color and absorb red color strongly. According to Kirchoff's law, this green



glass, on heating must emit the red color, which is absorbed strongly. Similarly when a red glass is heated to a high temperature it will glow with green light.

(5) Kirchoff' law also explains the existence of Fraunhofer lines. These are some dark lines observed in the otherwise spectrum of the sun. According to Fraunhofer, the central portion of the sun, called photosphere, is at a very high temperature and emits continuous light of all wavelengths. Before reaching us, the light passes through outer portion of the sun, called chromosphere. The chromosphere has some terrestrial elements in vapor form at lower temperature than that of photosphere. These elements absorb those wavelength which they would emit while hot. These absorbed wavelengths, which are missing appear as dark lines in the spectrum of the sun.



But during total solar eclipse these lines appear bright because the gases and vapor present in the chromosphere start emitting those radiation which they had absorbed.

(6) A person with black skin experiences more heat and colder as compared to a person of white skin because when the outside temperature is greater, the person with black skin absorbs more heat and when the outside temperature is less the person with black skin radiates more energy.

13. Distribution of Energy in the Spectrum of Black Body.

Langley and later on Lummer and Pringsheim investigated the distribution of energy amongst the different wavelengths in the thermal spectrum of a black body radiation. The results obtained are shown in figure. From these curves it is clear that

(1) At a given temperature energy is not uniformly distributed among different wavelengths.



(2) At a given temperature intensity of heat radiation increases with wavelength, reaches a maximum at a particular wavelength and with further increase in wavelength it decreases.

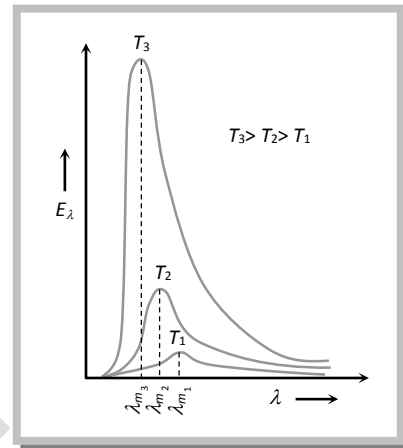
(3) With increase in temperature wavelength λ_m corresponding to most intense radiation decreases in such a way that $\lambda_m \times T = \text{constant}$. [Wien's law]

(4) For all wavelengths an increase in temperature causes an increase in intensity.

(5) The area under the curve $= \int E_\lambda d\lambda$ will represent the total intensity of radiation at a particular temperature. This area increases with rise in temperature of the body. It is found to be directly proportional to the fourth power of absolute temperature of the body, i.e.

$$E = \int E_\lambda d\lambda \propto T^4 \quad \text{[Stefan's law]}$$

(6) The energy (E_{\max}) emitted corresponding to the wavelength of maximum emission (λ_m) increases with fifth power of the absolute temperature of the black body i.e. $E_{\max} \propto T^5$



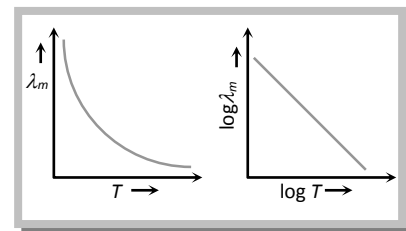
14. Wien's Displacement Law.

When a body is heated it emits radiations of all wavelength. However the intensity of radiations of different wavelength is different.

According to Wien's law the product of wavelength corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, i.e. $\lambda_m T = b = \text{constant}$

Where b is Wien's constant and has value $2.89 \times 10^{-3} \text{ m} \cdot \text{K}$.

This law is of great importance in 'Astrophysics' as through the analysis of radiations coming from a distant star, by finding λ_m the temperature of the star $T(= b / \lambda_m)$ is determined.



15. Law of Distribution of Energy.

The theoretical explanation of black body radiation was done by Planck.

If the walls of hollow enclosure are maintained at a constant temperature, then the inside of enclosure are filled with the electromagnetic radiation.

The radiation coming out from a small hole in the enclosure are called black body radiation. According to Max Planck, the radiation inside the enclosure may be assumed to be produced by a number of harmonic oscillators.

A harmonic oscillator oscillating with frequency ν can possess energies, which are integral multiples of $h\nu$. Where h is a constant, called Planck's constant. Thus the harmonic oscillator can possess energies given by $E = nh\nu$ where n is an integer.

$$E_{\lambda}d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{[e^{hc/\lambda KT} - 1]} d\lambda$$

According to Planck's law

This law is valid for radiations of all wavelengths ranging from zero to infinite.

For radiations of short wavelength $\left(\lambda \ll \frac{hc}{KT}\right)$

Planck's law reduces to Wien's energy distribution law $E_{\lambda}d\lambda = \frac{A}{\lambda^5} e^{-B/\lambda T} d\lambda$

For radiations of long wavelength $\left(\lambda \gg \frac{hc}{KT}\right)$

Planck's law reduces to Rayleigh-Jeans energy distribution law $E_{\lambda}d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$



16. Stefan's Law.

According to it the radiant energy emitted by a perfectly black body per unit area per sec (i.e. emissive power of black body) is directly proportional to the fourth power of its absolute temperature,

i.e. $E \propto T^4$ or $E = \sigma T^4$

Where σ a constant is called Stefan's constant having dimension $[MT^{-3}\theta^{-4}]$ and value $5.67 \times 10^{-8} W/m^2 K^4$.

(i) If e is the emissivity of the body then $E = e\sigma T^4$

(ii) If Q is the total energy radiated by the body then $E = \frac{Q}{A \times t} = e\sigma T^4 \Rightarrow Q = Ate\sigma T^4$

(iii) If a body at temperature T is surrounded by a body at temperature T_0 , then Stefan's law may be put as

$$E = e\sigma(T^4 - T_0^4)$$

(iv) Cooling by radiation: If a body at temperature T is in an environment of temperature $T_0 (< T)$, the body is losing as well as receiving so net rate of loss of energy

$$\frac{dQ}{dt} = eA\sigma(T^4 - T_0^4) \quad \dots(i)$$

Now if m is the mass of body and c its specific heat, the rate of loss of heat at temperature T must be

$$\frac{dQ}{dt} = mc \frac{dT}{dt} \quad \dots(ii)$$

From equation (i) and (ii) $mc \frac{dT}{dt} = eA\sigma(T^4 - T_0^4)$

\therefore Rate of fall of temperature or rate of cooling, $\frac{dT}{dt} = \frac{eA\sigma}{mc}(T^4 - T_0^4) \quad \dots(iii)$

i.e. when a body cools by radiation the rate of cooling depends on

- (a) Nature of radiating surface i.e. greater the emissivity, faster will be the cooling.
- (b) Area of radiating surface, i.e. greater the area of radiating surface, faster will be the cooling.
- (c) Mass of radiating body i.e. greater the mass of radiating body slower will be the cooling.



- (d) Specific heat of radiating body i.e. greater the specific heat of radiating body slower will be cooling.
- (e) Temperature of radiating body i.e. greater the temperature of body faster will be cooling.
- (f) Temperature of surrounding i.e. greater the temperature of surrounding slower will be cooling.

17. Newton's Law of Cooling.

If in case of cooling by radiation the temperature T of body is not very different from that of surrounding

i.e. $T = T_0 + \Delta T$

$$T^4 - T_0^4 = [(T_0 + \Delta T)^4 - T_0^4] = T_0^4 \left[\left(1 + \frac{\Delta T}{T_0} \right)^4 - 1 \right] = T_0^4 \left(1 + \frac{4\Delta T}{T_0} - 1 \right)$$

[Using Binomial

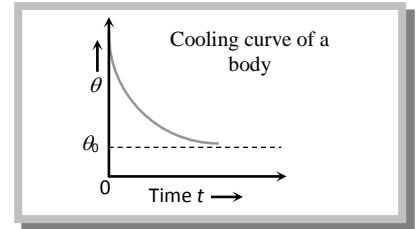
theorem]

$$= 4T_0^3 \Delta T \quad \dots(i)$$

By Stefan's law, $\frac{dT}{dt} = \frac{eA\sigma}{mc} [T^4 - T_0^4]$

From equation (i), $\frac{dT}{dt} = \frac{eA\sigma}{mc} 4T_0^3 \Delta T$

So $\frac{dT}{dt} \propto \Delta T$ or $\frac{d\theta}{dt} \propto \theta - \theta_0$



i.e., if the temperature of body is not very different from surrounding, rate of cooling is proportional to temperature difference between the body and its surrounding. This law is called Newton's law of cooling.

(1) Practical examples

- (i) Hot water loses heat in smaller duration as compared to moderate warm water.
- (ii) Adding milk in hot tea reduces the rate of cooling.

(2) Greater the temperature difference between body and its surrounding greater will be the rate of cooling.



(3) If $\theta = \theta_0$, $\frac{d\theta}{dt} = 0$ i.e. a body can never be cooled to a temperature lesser than its surrounding by radiation.

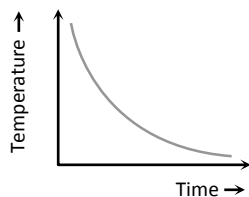
(4) If a body cools by radiation from $\theta_1^\circ C$ to $\theta_2^\circ C$ in time t , then $\frac{d\theta}{dt} = \frac{\theta_1 - \theta_2}{t}$ and $\theta = \theta_{av} = \frac{\theta_1 + \theta_2}{2}$

The Newton's law of cooling becomes $\left[\frac{\theta_1 - \theta_2}{t} \right] = K \left[\frac{\theta_1 + \theta_2}{2} - \theta_0 \right]$

This form of law helps in solving numerical.

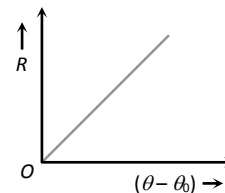
(5) Cooling curves:

Curve between temperature of body θ and time.



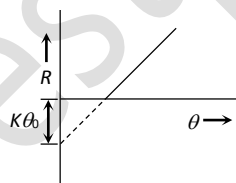
$\theta - \theta_0 = Ae^{-kt}$, which indicates temperature decreases exponentially with increasing time.

Curve between rate of cooling (R) and temperature difference between body (θ) and surrounding (θ_0)



$R \propto (\theta - \theta_0)$. This is a straight line passing through origin.

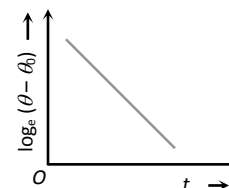
Curve between the rate of cooling (R) and body temperature (θ).



$$R = K(\theta - \theta_0) = K\theta - K\theta_0$$

This is a straight line intercept R-axis at $-K\theta_0$

Curve between $\log_e(\theta - \theta_0)$ and time



As $\frac{d\theta}{dt} \propto -(\theta - \theta_0) \Rightarrow \frac{d\theta}{(\theta - \theta_0)} = -Kdt$

Integrating $\log_e(\theta - \theta_0) = -Kt + C$

$$\log_e(\theta - \theta_0) = -Kt + \log_e A$$

This is a straight line with negative slope



(6) Determination of specific heat of a liquid : If volume, radiating surface area, nature of surface, initial temperature and surrounding of water and given liquid are equal and they are allowed to cool down (by radiation) then rate of loss of heat and fall in temperature of both will be same.

i.e.
$$\left(\frac{dQ}{dt}\right)_{\text{water}} = \left(\frac{dQ}{dt}\right)_{\text{liquid}}$$

$$(ms + W)\frac{(\theta_1 - \theta_2)}{t_1} = (m_1s_1 + W)\frac{(\theta_1 - \theta_2)}{t_2}$$

or
$$\left[\frac{ms + W}{t_1}\right] = \left[\frac{m_1s_1 + W}{t_2}\right]$$
 [where W = water equivalent of calorimeter]

If density of water and liquid is ρ and ρ' respectively then $m = V\rho$ and $m' = V\rho'$

18. Temperature of the Sun and Solar Constant.

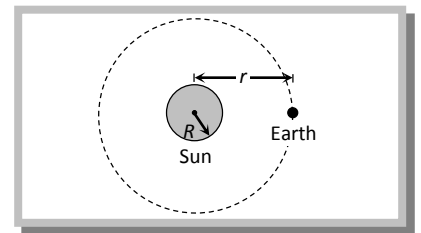
If R is the radius of the sun and T its temperature, then the energy emitted by the sun per sec through radiation in accordance with Stefan's law will be given by

$$P = eA \sigma T^4 = 4\pi R^2 \sigma T^4$$

In reaching earth this energy will spread over a sphere of radius r (= average distance between sun and earth); so the intensity of solar radiation at the surface of earth (called solar constant S) will be given by

$$S = \frac{P}{4\pi r^2} = \frac{4\pi R^2 \sigma T^4}{4\pi r^2}$$

i.e.
$$T = \left[\left(\frac{r}{R}\right)^2 \frac{S}{\sigma}\right]^{1/4} = \left[\left(\frac{1.5 \times 10^8}{7 \times 10^5}\right)^2 \times \frac{1.4 \times 10^3}{5.67 \times 10^{-8}}\right]^{1/4} \approx 5800 \text{ K}$$



As $r = 1.5 \times 10^8 \text{ km}$, $R = 7 \times 10^5 \text{ km}$, $S = 2 \frac{\text{cal}}{\text{cm}^2 \text{min}} = 1.4 \frac{\text{kW}}{\text{m}^2}$ and $\sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{K}^4}$

This result is in good agreement with the experimental value of temperature of sun, i.e., 6000 K.

The difference in the two values is attributed to the fact that sun is not a perfectly black body.

