Physics

Electrostatics

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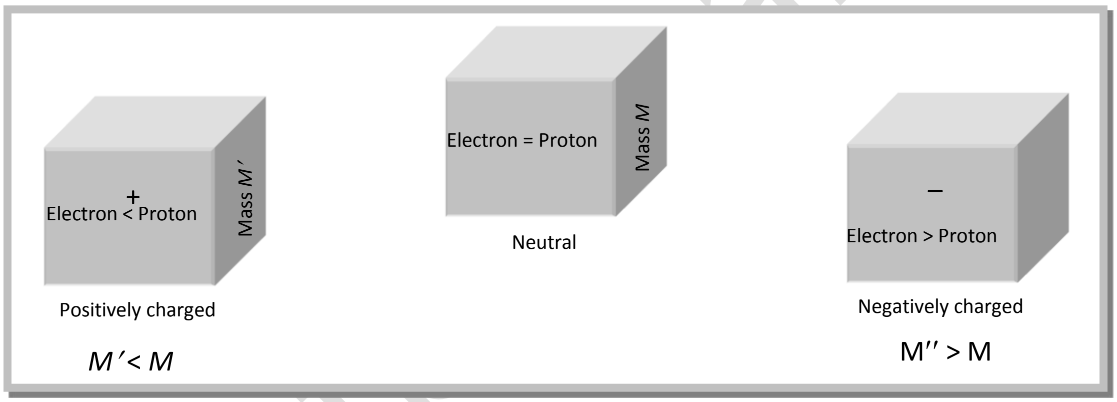
1. Electric Charge

(1) **Definition:** Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

(2) **Origin of electric charge:** It is known that every atom is electrically neutral, containing as many electrons as the number of protons in the nucleus.

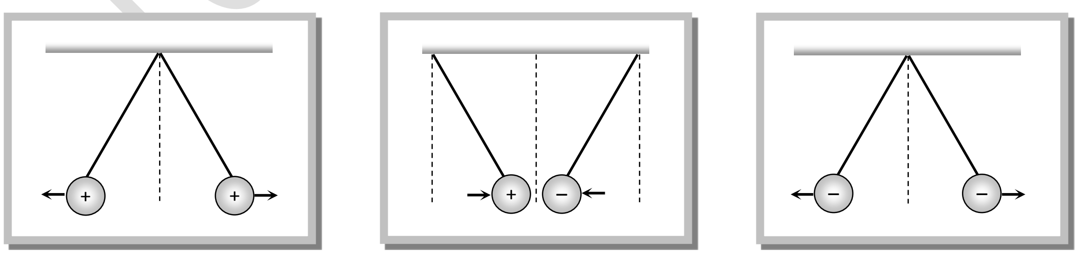
Charged particles can be created by disturbing neutrality of an atom. Loss of electrons gives positive charge (as then $n_p > n_e$) and gain of electrons gives negative charge (as then $n_e > n_p$) to a particle. When an object is negatively charged it gains electrons and therefore its mass increases negligibly. Similarly, on charging a body with positive electricity its mass decreases. Change in mass of object is equal to n × m_e.

Where, n is the number of electrons transferred and m_e is the mass of electron $= 9.1 \times 10^{-31}\ Kg$.



(3) **Type**: There exists two types of charges in nature (I) Positive charge (ii) Negative charge

Charges with the same electrical sign repel each other, and charges with opposite electrical sign attract each other.



(4) **Unit and dimensional formula:** Rate of flow of electric charge is called electric current i.e., $i = \frac{dQ}{dt} \Rightarrow$ $dQ = idt$, hence S.I. unit of charge is – Ampere × sec = coulomb (C), smaller S.I. units are mC, μC, nC

$(1mC = 10^{-3}C, 1\mu C = 10^{-6}C, 1nC = 10^{-9}C)$. C.G.S. unit of charge is – Stat coulomb or e.s.u. Electromagnetic unit of charge is – ab coulomb $1C = 3\times10^{9}\ stat\ coulomb = \frac{1}{10}\ ab\ coulomb$. Dimensional formula $[Q]=[AT]$

Note: Benjamin Franklin was the first to assign positive and negative sign of charge.

- The existence of two type of charges was discovered by Dufog.
- Franklin (i.e., e.s.u. of charge) is the smallest unit of charge while faraday is largest (1 Faraday = 96500 C).
- The e.s.u. of charge is also called stat coulomb or Franklin (Fr) and is related to e.m.u. of charge through the relation $\frac{\text{emu of charge}}{\text{esu of charge}} = 3\times10^{10}$

(5) **Point charge:** A finite size body may behave like a point charge if it produces an inverse square electric field. For example an isolated charged sphere behave like a point charge at very large distance as well as very small distance close to its surface.

(6) **Properties of charge**

(i) **Charge is transferable:** If a charged body is put in contact with an uncharged body, uncharged body becomes charged due to transfer of electrons from one body to the other.

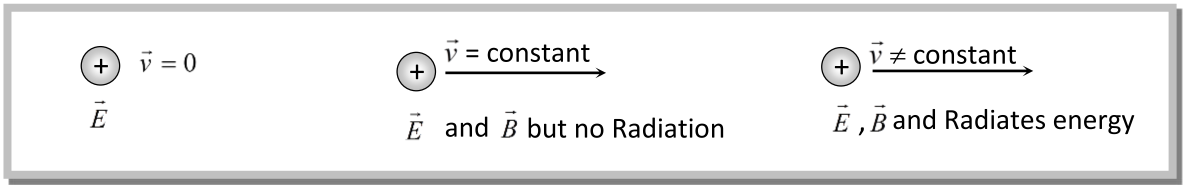
(ii) **Charge is always associated with mass**, i.e., charge cannot exist without mass though mass can exist without charge.

(iii) **Charge is conserved:** Charge can neither be created nor be destroyed. E.g. In radioactive decay the uranium nucleus (charge $= +92e$) is converted into a thorium nucleus (charge $= +90e$) and emits an α-particle (charge $= +2e$)

${}_{92}U^{238} \to {}_{90}Th^{234} + {}_{2}He^{4}$. Thus the total charge is $+92e$ both before and after the decay.

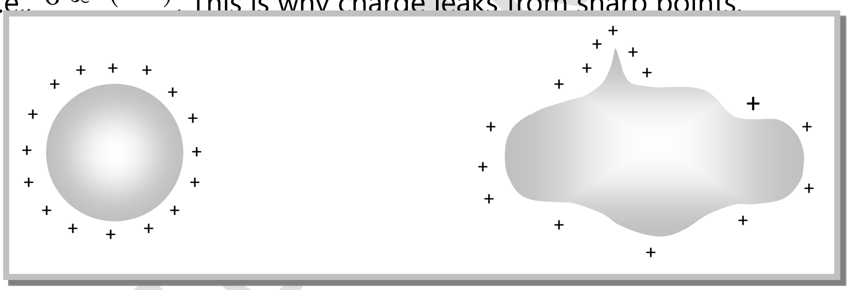
(iv) **Invariance of charge**: The numerical value of an elementary charge is independent of velocity. It is proved by the fact that an atom is neutral. The difference in masses on an electron and a proton suggests that electrons move much faster in an atom than protons. If the charges were dependent on velocity, the neutrality of atoms would be violated.

(v) **Charge produces electric field and magnetic field**: A charged particle at rest produces only electric field in the space surrounding it. However, if the charged particle is in unaccelerated motion it produces both electric and magnetic fields. And if the motion of charged particle is accelerated it not only produces electric and magnetic fields but also radiates energy in the space surrounding the charge in the form of electromagnetic waves.



(vi) **Charge resides on the surface of conductor:** Charge resides on the outer surface of a conductor because like charges repel and try to get as far away as possible from one another and stay at the farthest distance from each other which is outer surface of the conductor. This is why a solid and hollow conducting sphere of same outer radius will hold maximum equal charge and a **soap bubble expands on charging.**

(vii) **Charge leaks from sharp points :** In case of conducting body no doubt charge resides on its outer surface, if surface is uniform the charge distributes uniformly on the surface and for irregular surface the distribution of charge, i.e., charge density is not uniform. It is maximum where the radius of curvature is minimum and vice versa. i.e., $\sigma \propto (1/R)$. This is why charge leaks from sharp points.



(viii) **Quantization of charge:** When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantized. The smallest charge that can exist in nature is the charge of an electron. If the charge of an electron ($= 1.6 \times 10^{-19}\, C$) is taken as elementary unit i.e. quanta of charge the charge on anybody will be some integral multiple of e i.e.

$\boxed{Q = \pm ne}$ With $n = 1, 2, 3 \ldots.$

Charge on a body can never be $\pm \frac{2}{3} e$, $\pm 17.2e$ or $\pm 10^{-5} e$ etc.

Note: Recently it has been discovered that elementary particles such as proton or neutron are composed of quarks having charge $(\pm 1/3)$e and $(\pm 2/3)$e. However, as quarks do not exist in Free State, the quanta of charge is still e.

- Quantization of charge implies that there is a maximum permissible magnitude of charge.

2. Comparison of Charge and Mass.

We are familiar with role of mass in gravitation, and we have just studied some features of electric charge. We can compare the two as shown below

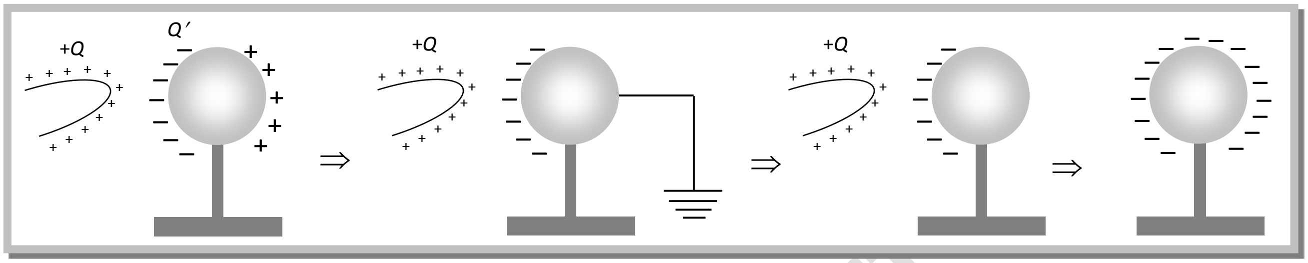
Charge	**Mass**
(1) Electric charge can be positive, negative or zero.	(1) Mass of a body is a positive quantity.
(2) Charge carried by a body does not depend upon velocity of the body.	(2) Mass of a body increases with its velocity as $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ where c is velocity of light in vacuum, m is the mass of the body moving with velocity v and m_0 is rest mass of the body.
(3) Charge is quantized.	(3) The quantization of mass is yet to be established.
(4) Electric charge is always conserved.	(4) Mass is not conserved as it can be changed into energy and vice-versa.
(5) Force between charges can be attractive or repulsive, according as charges are unlike or like charges.	(5) The gravitational force between two masses is always attractive.

3. Methods of Charging.

A body can be charged by following methods:

(1) **By friction:** In friction when two bodies are rubbed together, electrons are transferred from one body to the other. As a result of this one body becomes positively charged while the other negatively charged, e.g., when a glass rod is rubbed with silk, the rod becomes positively charged while the silk negatively. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged. Clouds also become charged by friction. In charging by friction in accordance with conservation of charge, both positive and negative charges in equal amounts appear simultaneously due to transfer of electrons from one body to the other.

(2) **By electrostatic induction:** If a charged body is brought near an uncharged body, the charged body will attract opposite charge and repel similar charge present in the uncharged body. As a result of this one side of neutral body (closer to charged body) becomes oppositely charged while the other is similarly charged. This process is called electrostatic induction.



Note: Inducting body neither gains nor loses charge.

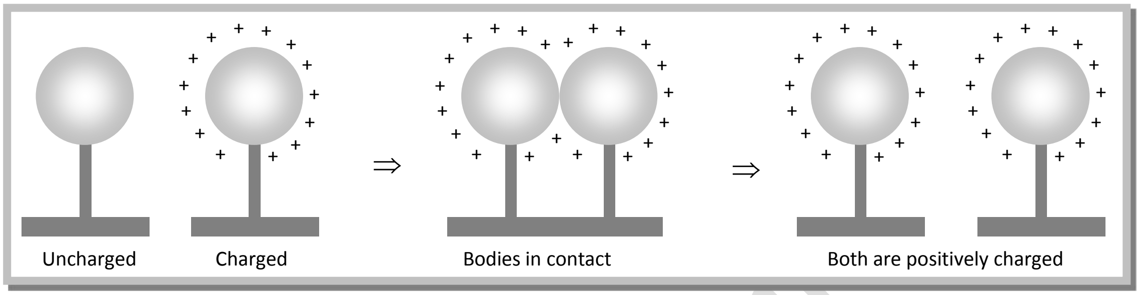
Induced charge can be lesser or equal to inducing charge (but never greater) and its maximum value is given by $Q' = -Q\left[1 - \frac{1}{K}\right]$ where Q is the inducing charge and K is the dielectric constant of the material of the uncharged body. Dielectric constant of different media are shown below

Medium	**K**
Vacuum / air	1
Water	80
Mica	6
Glass	5–10
Metal	∞

Dielectric constant of an insulator cannot be ∞

For metals in electrostatics $K = \infty$ and so $Q' = -Q$; i.e. in metals induced charge is equal and opposite to inducing charge.

(3) **Charging by conduction:** Take two conductors, one charged and other uncharged. Bring the conductors in contact with each other. The charge (whether $-ve$ or $+ve$) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. This is called as charging by conduction (through contact).

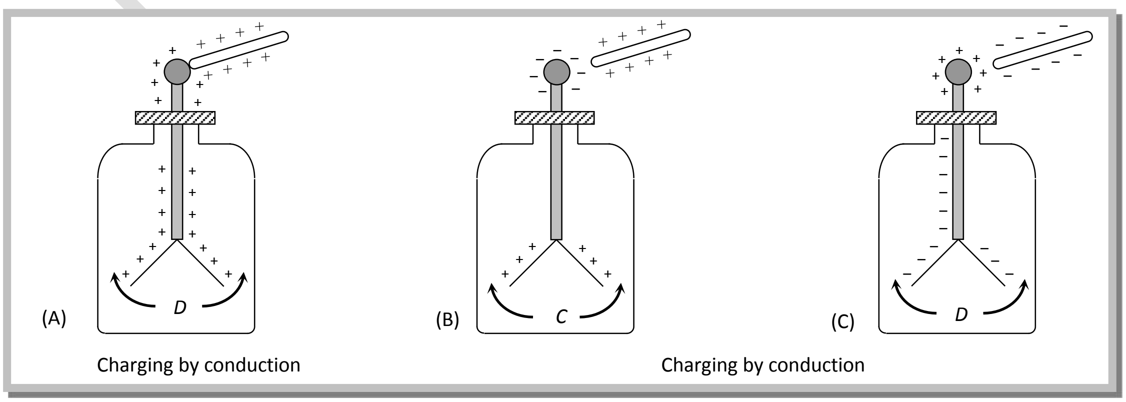


Note: A truck carrying explosives has a metal chain touching the ground, to conduct away the charge produced by friction.

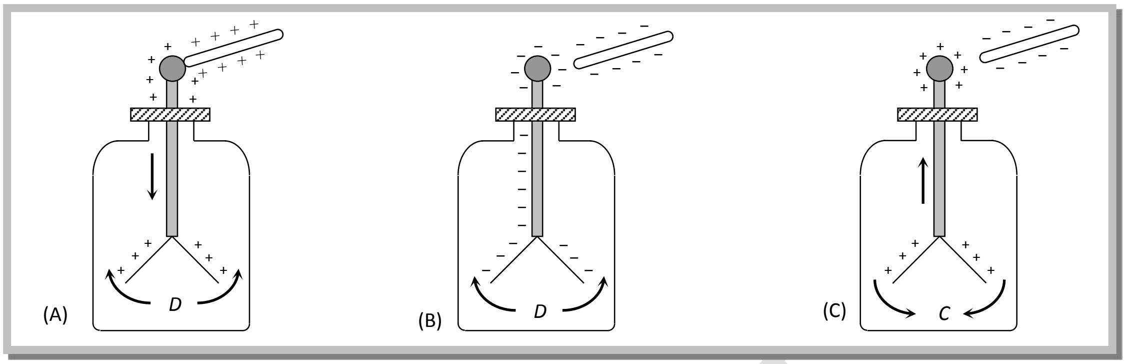
4. Electroscope.

It is a simple apparatus with which the presence of electric charge on a body is detected (see figure). When metal knob is touched with a charged body, some charge is transferred to the gold leaves, which then diverges due to repulsion. The separation gives a rough idea of the amount of charge on the body. If a charged body brought near a charged electroscope the leaves will further diverge. If the charge on body is similar to that on electroscope and will usually converge if opposite. If the induction effect is strong enough leaves after converging may again diverge.

(1) **Uncharged electroscope**



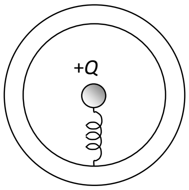
(2) **Charged electroscope**



Concepts

- After earthing a positively charged conductor electrons flow from earth to conductor and if a negatively charged conductor is earthed then electrons flows from conductor to earth.

- When a charged spherical conductor placed inside a hollow insulated conductor and connected if through a fine conducting wire the charge will be completely transferred from the inner conductor to the outer conductor.

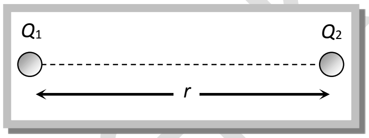


- Lightening-rods arrestors are made up of conductors with one of their ends earthed while the other sharp, and protects a building from lightening either by neutralizing or conducting the charge of the cloud to the ground.
- With rise in temperature dielectric constant of liquid decreases.
- Induction takes place only in bodies (either conducting or non-conducting) and not in particles.

- If X-rays are incident on a charged electroscope, due to ionisation of air by X-rays the electroscope will get discharged and hence its leaves will collapse. However, if the electroscope is evacuated. X-rays will cause photoelectric effect with gold and so the leaves will further diverge if it is positively charged (or uncharged) and will converge if it is negatively charged.
- If only one charge is available than by repeating the induction process, it can be used to obtain a charge many times greater than its equilibrium. (High voltage generator)

5. Coulomb's Law.

If two stationary and point charges Q_1 and Q_2 are kept at a distance r, then it is found that force of attraction



or repulsion between them is $F \propto \frac{Q_1 Q_2}{r^2}$ i.e., $F = \frac{kQ_1 Q_2}{r^2}$; (k = Proportionality constant)

(1) **Dependence of k:** Constant k depends upon system of units and medium between the two charges.

(i) **Effect of units**

(a) In C.G.S. for air $k = 1,$ $F = \frac{Q_1 Q_2}{r^2}$ Dyne

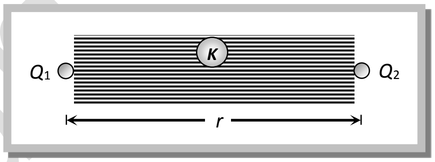
(b) In S.I. for air $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{N-m^2}{C^2}$, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2}$ Newton (1 Newton = 10^5 Dyne)

Note: $\varepsilon_0 =$ Absolute permittivity of air or free space = $8.85 \times 10^{-12} \frac{C^2}{N-m^2} \left(= \frac{Farad}{m}\right)$. Its Dimension is $[ML^{-3}T^4A^2]$

ε_0 Relates with absolute magnetic permeability (μ_0) and velocity of light (c) according to the following relation $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

(ii) **Effect of medium**

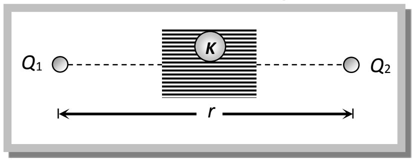
(a) When a dielectric medium is completely filled in between charges rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of K known as **dielectric constant** or specific inductive capacity **(SIC)** of the medium, K is also called relative permittivity ε_r of the medium (relative means with respect to free space).



Hence in the presence of medium $F_m = \frac{F_{air}}{K} = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{Q_1 Q_2}{r^2}$

Here $\varepsilon_0 K = \varepsilon_0 \, \varepsilon_r = \varepsilon$ (permittivity of medium)

(b) If a dielectric medium (dielectric constant K, thickness t) is partially filled between the charges then effective air separation between the charges becomes $(r - t + t\sqrt{K})$



Hence force $F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{(r - t + t\sqrt{K})^2}$

(2) **Vector form of coulomb's law:** Vector form of Coulomb's law is $\vec{F}_{12} = K.\frac{q_1 q_2}{r^3} \vec{r}_{12} = K.\frac{q_1 q_2}{r^2} \hat{r}_{12}$, where $\hat{r}_{12}$ is the unit vector from first charge to second charge along the line joining the two charges.

(3) **A comparative study of fundamental forces of nature**

S.No.	Force	Nature and formula	Range	Relative strength
(i)	Force of gravitation between two masses	Attractive $F = Gm_1m_2/r^2$, obey's Newton's third law of motion, it's a conservative force	Long range (between planets and between electron and proton)	1

(ii)	Electromagnetic force (for stationary and moving charges)	Attractive as well as repulsive, obey's Newton's third law of motion, it's a conservative force	Long (up to few kelometers)	10^{37}
(iii)	Nuclear force (between nucleons)	Exact expression is not known till date. However in some cases empirical formula $U_0 e^{r/r_0}$ can be utilized for nuclear potential energy U_0 and r_0 are constant.	Short (of the order of nuclear size 10^{-15} m)	10^{39} (strongest)
(iv)	Weak force (for processes like β decay)	Formula not known	Short (upto 10^{-15}m)	10^{24}

Note: Coulombs law is not valid for moving charges because moving charges produces magnetic field also.

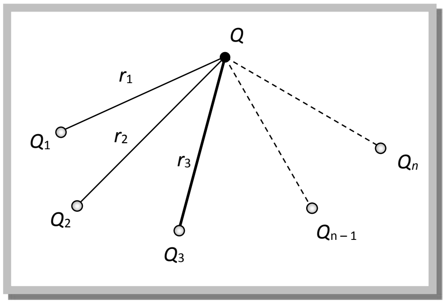
Coulombs law is valid at a distance greater than $10^{-15}\,m$.

A charge Q_1 exert some force on a second charge Q_2. If third charge Q_3 is brought near, the force of Q_1 exerted on Q_2 remains unchanged.

Ratio of gravitational force and electrostatic force between (i) Two electrons is **10^{-43}/1.** (ii) Two protons is **10^{-36}/1** (iii) One proton and one electron **10^{-39}/1**.

Decreasing order to fundamental forces $F_{Nuclear} > F_{Electromagnetic} > F_{Weak} > F_{Gravitational}$

(4) **Principle of superposition:** According to the principle of super position, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges.



Consider number of charge Q_1, Q_2, Q_3 ...are applying force on a charge Q

Net force on Q will be

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots\ldots + \vec{F}_{n-1} + \vec{F}_n$$

Concepts

☞ Two point charges separated by a distance r in vacuum and a force F acting between them. After filling a dielectric medium having dielectric constant K completely between the charges, force between them decreases. To maintain the force as before separation between them changes to $r\sqrt{K}$. This distance known as effective air separation.

6. Electrical Field.

A positive charge or a negative charge is said to create its field around itself. If a charge Q_1 exerts a force on charge Q_2 placed near it, it may be stated that since Q_2 is in the field of Q_1, it experiences some force, or it may also be said that since charge Q_1 is inside the field of Q_2, it experience some force. Thus space around a charge in which another charged particle experiences a force is said to have electrical field in it.

(1) **Electric field intensity** $(\vec{E})$**:** The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point. $\vec{E} = \frac{\vec{F}}{q_0}$

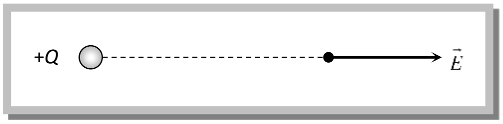
Where $q_0 \to 0$ so that presence of this charge may not affect the source charge Q and its electric field is not changed, therefore expression for electric field intensity can be better written as $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

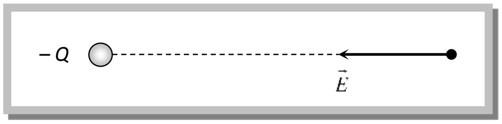
(2) **Unit and Dimensional formula:** its S.I. unit – $\frac{Newton}{coulomb} = \frac{volt}{meter} = \frac{Joule}{coulomb \times meter}$ and

C.G.S. unit – Dyne/stat coulomb.

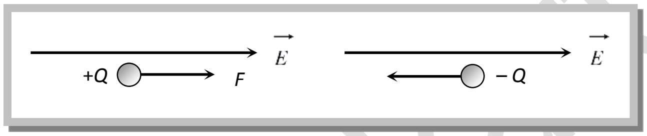
Dimension: [E] = [$MLT^{-3}A^{-1}$]

(3) **Direction of electric field:** Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge





(4) **Relation between electric force and electric field:** In an electric field $\vec{E}$ a charge (Q) experiences a force $F = QE$. If charge is positive then force is directed in the direction of field while if charge is negative force acts on it in the opposite direction of field



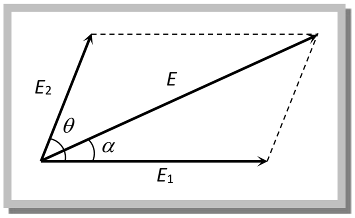
(5) **Super position of electric field** (electric field at a point due to various charges): The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

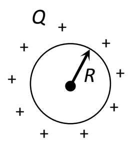
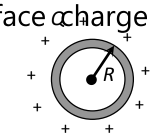
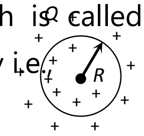
The magnitude of the resultant of two electric fields is given by

$$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\theta}$$ and the direction is given by

$$\tan\alpha = \frac{E_2\sin\theta}{E_1 + E_2\cos\theta}$$



(6) **Electric field due to continuous distribution of charge:** A system of closely spaced electric charges forms a continuous charge distribution

Continuous charge distribution		
Linear charge distribution	**Surface charge distribution**	**Volume charge distribution**
In this distribution charge distributed on a line. For example: charge on a wire, charge on a ring etc. Relevant parameter is λ which is called linear charge density i.e., $\lambda = \frac{\text{charge}}{\text{length}}$ $\lambda = \frac{Q}{2\pi R}$  Circular charged ring	In this distribution charge distributed on the surface. For example: Charge on a conducting sphere, charge on a sheet etc. Relevant parameter is σ which is called surface charge density i.e., $\sigma = \frac{\text{charge}}{\text{area}}$ $\sigma = \frac{Q}{4\pi R^2}$  Spherical shall	In this distribution charge distributed in the whole volume of the body. For example: Non conducting charged sphere. Relevant parameter is ρ which is called volume charge density i.e., $\rho = \frac{\text{charge}}{\text{volume}}$ $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$  Non conducting sphere

To find the field of a continuous charge distribution, we divide the charge into infinitesimal charge elements. Each infinitesimal charge element is then considered, as a point charge and electric field $\vec{dE}$ is determined due to this charge at given point. The Net field at the given point is the summation of fields of all the elements. i.e., $\vec{E} = \int \vec{dE}$

7. Electric Potential.

(1) **Definition:** Potential at a point in a field is defined as the amount of work done in bringing a unit positive test charge, from infinity to that point along any arbitrary path (infinity is point of zero potential).

Electric potential is a scalar quantity, it is denoted by V; $V = \frac{W}{q_0}$

(2) **Unit and dimensional formula:** S. I. unit – $\frac{Joule}{Coulomb} = volt$ C.G.S. unit – Stat volt (e.s.u.); **1 volt** $= \frac{1}{300}$ **Stat volt** Dimension – $[V] = [ML^2T^{-3}A^{-1}]$

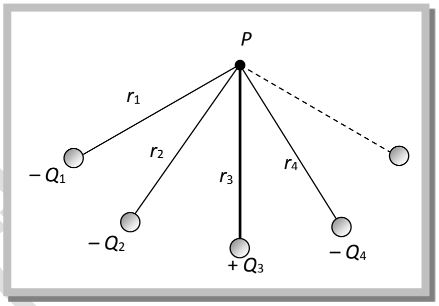
(3) **Types of electric potential:** According to the nature of charge potential is of two types

(i) Positive potential: Due to positive charge.

(ii) Negative potential: Due to negative charge.

(4) **Potential of a system of point charges:** Consider P is a point at which net electric potential is to be determined due to several charges. So net potential at P

$$V = k\frac{Q_1}{r_1} + k\frac{Q_2}{r_2} + k\frac{Q_3}{r_3} + k\frac{(-Q_4)}{r_4} + \ldots$$



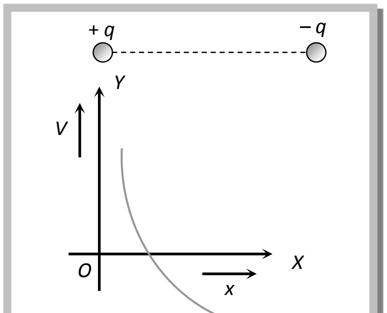
In general $V = \sum_{i=1}^{X} \frac{kQ_i}{r_i}$

Note: At the center of two equal and opposite charge V = 0 but $E \neq 0$

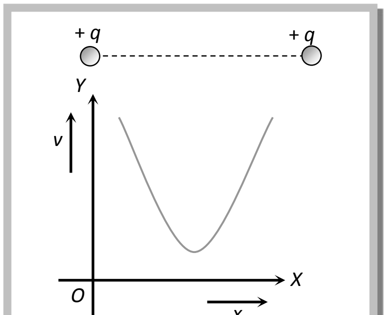
At the center of the line joining two equal and similar charge $V \neq 0, E = 0$

(5) **Electric potential due to a continuous charge distribution :** The potential due to a continuous charge distribution is the sum of potentials of all the infinitesimal charge elements in which the distribution may be divided i.e., $V = \int dV, \; = \int \frac{dQ}{4\pi\varepsilon_0 r}$

(6) **Graphical representation of potential:** When we move from a positive charge towards an equal negative charge along the line joining the two then initially potential decreases in magnitude and at center become zero, but this potential is throughout positive because when we are nearer to positive charge, overall potential must be positive. When we move from center towards the negative charge then though potential remain always negative but increases in magnitude fig. (A). As one move from one charge to other when both charges are like, the potential first decreases, at center become minimum and then increases Fig. (B).



(A)



(B)

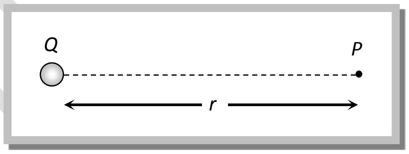
(7) **Potential difference :** In an electric field potential difference between two points A and B is defined as equal to the amount of work done (by external agent) in moving a unit positive charge from point A to point B.

i.e., $V_B - V_A = \dfrac{W}{q_0}$ in general $W = Q.\Delta V$; $\Delta V =$ Potential difference through which charge Q moves.

(8) **Electric Field and Potential Due to Various Charge Distribution.**

(1) **Point charge:** Electric field and potential at point P due to a point charge Q is

$$\boldsymbol{E} = \boldsymbol{k}\frac{\boldsymbol{Q}}{\boldsymbol{r}^2} \text{ or } \vec{E} = k\frac{Q}{r^2}\hat{r} \quad \left(k = \frac{1}{4\pi\varepsilon_0}\right), \quad V = k\frac{Q}{r}$$

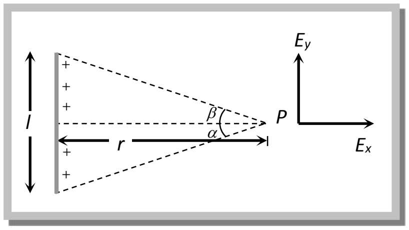


Note: Electric field intensity and electric potential due to a point charge q, at a distance $t_1 + t_2$ where t_1 is thickness of medium of dielectric constant K_1 and t_2 is thickness of medium of dielectric constant K_2 are:

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{Q}}{(\boldsymbol{t}_1\sqrt{\boldsymbol{K}_1} + \boldsymbol{t}_2\sqrt{\boldsymbol{K}_2})^2}; \quad \boldsymbol{V} = \frac{1}{4\pi\varepsilon_0}\frac{\boldsymbol{Q}}{(\boldsymbol{t}_1\sqrt{\boldsymbol{K}_1} + \boldsymbol{t}_2\sqrt{\boldsymbol{K}_2})}$$

(2) **Line charge**

(i) **Straight conductor:** Electric field and potential due to a charged straight conducting wire of length l and charge density λ



(a) **Electric field:** $E_x = \dfrac{k\lambda}{r}(\sin\alpha + \sin\beta)$ and $E_y = \dfrac{k\lambda}{r}(\cos\beta - \cos\alpha)$

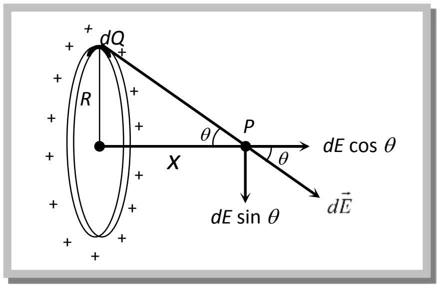
If $\alpha = \beta$; $E_x = \dfrac{2k\lambda}{r}\sin\alpha$ and $E_y = 0$

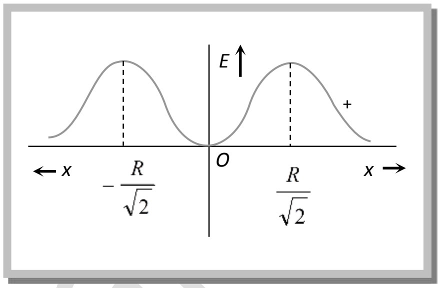
If $l \to \infty$ i.e. $\alpha = \beta = \dfrac{\pi}{2}$; $E_x = \dfrac{2k\lambda}{r}$ and $E_y = 0$ so $E_{net} = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

If $\alpha = 0$, $\beta = \dfrac{\pi}{2}$; $|E_x| = |E_y| = \dfrac{k\lambda}{r}$ so $E_{net} = \sqrt{E_x^2 + E_y^2} = \dfrac{\sqrt{2}\,k\lambda}{r}$

(b) **Potential:** $V = \frac{\lambda}{2\pi\varepsilon_0}\log_e\left[\frac{\sqrt{r^2+l^2}-1}{\sqrt{r^2+l^2}+1}\right]$ for infinitely long conductor $V = \frac{-\lambda}{2\pi\varepsilon_0}\log_e r + c$

(ii) **Charged circular ring:** Suppose we have a charged circular ring of radius R and charge Q. On its axis electric field and potential is to be determined, at a point 'x' away from the center of the ring.





(a) **Electric field:** Consider an element carrying charge dQ. Its electric field $dE = \frac{KdQ}{(R^2+x^2)}$ directed as shown. Its component along the axis is $dE\cos\theta$ and perpendicular to the axis is $dE\sin\theta$. By symmetry $\int dE\sin\theta = 0$, hence $E = \int dE\cos\theta = \int \frac{kdQ}{(R^2+x^2)}.\frac{x}{(R^2+x^2)^{1/2}}$

$$\boldsymbol{E} = \frac{\boldsymbol{kQx}}{(\boldsymbol{R}^2+\boldsymbol{x}^2)^{3/2}}$$ Directed away from the center if Q is positive

(b) **Potential:** $V = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{\sqrt{x^2+R^2}}$

Note: At center x = 0 so E_{centre} = 0 and $V_{centre} = \frac{kQ}{R}$

At a point on the axis such that x >> R $E = \frac{kQ}{x^2}$ and $V = \frac{kQ}{x}$

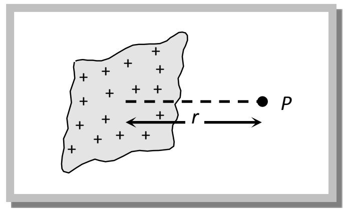
At a point on the axis if $x = \pm\frac{R}{\sqrt{2}}$, $E_{\max} = \frac{Q}{6\sqrt{3}\pi\varepsilon_0 a^2}$

(3) **Surface charge:**

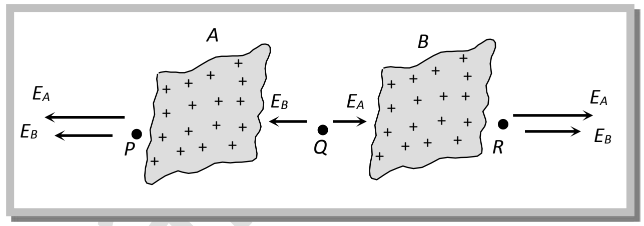
(i) **Infinite sheet of charge:** Electric field and potential at a point P as shown

$$E = \frac{\sigma}{2\varepsilon_0} \quad (E \propto r^o)$$

and
$$V = -\frac{\sigma r}{2\varepsilon_0} + C$$



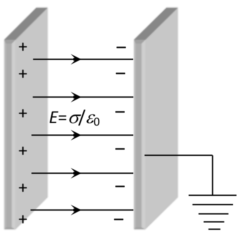
(ii) **Electric field due to two parallel plane sheet of charge:** Consider two large, uniformly charged parallel. Plates A and B, having surface charge densities are σ_A and σ_B respectively. Suppose net electric field at points P, Q and R is to be calculated.



At P, $E_P = (E_A + E_B) = \frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

At Q, $E_Q = (E_A - E_B) = \frac{1}{2\varepsilon_0}(\sigma_A - \sigma_B)$; At R, $E_R = -(E_A + E_B) = -\frac{1}{2\varepsilon_0}(\sigma_A + \sigma_B)$

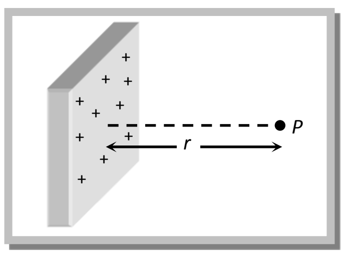
Note: If $\sigma_A = +\sigma$ and $\sigma_B = -\sigma$ then $E_p = 0, E_Q = \frac{\sigma}{\varepsilon_0}, E_R = 0$. Thus in case of two infinite plane sheets of charges having equal and opposite surface charge densities, the field is non-zero only in the space between the two sheets and is independent of the distance between them i.e., field is uniform in this region. It should be noted that this result will hold good for finite plane sheet also, if they are held at a distance much smaller than the dimensions of sheets i.e., parallel plate capacitor.



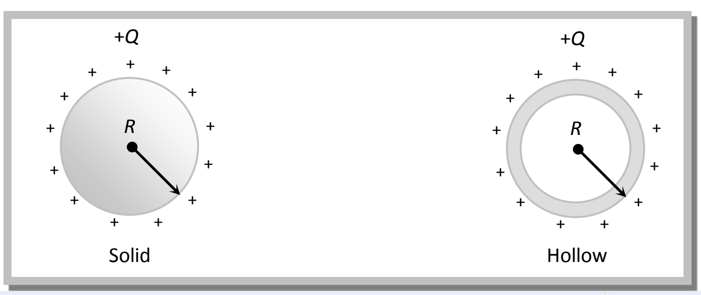
(iii) **Conducting sheet of charge:**

$$E = \frac{\sigma}{\varepsilon_0}$$

$$V = -\frac{\sigma r}{\varepsilon_0} + C$$



(iv) **Charged conducting sphere:** If charge on a conducting sphere of radius R is Q as shown in figure then electric field and potential in different situation are –



(a) **Outside the sphere:** P is a point outside the sphere at a distance r from the center at which electric field and potential is to be determined.

Electric field at P

$$E_{out} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \quad \text{and} \quad V_{out} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r} \quad \begin{cases} Q = \sigma \times A \\ \quad = \sigma \times 4\pi R^2 \end{cases}$$

(b) **At the surface of sphere:** At surface $r = R$

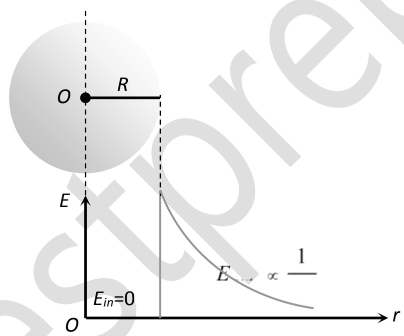
So, $$E_s = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad V_s = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$$

(c) **Inside the sphere:** Inside the conducting charge sphere electric field is zero and potential remains constant everywhere and equals to the potential at the surface.

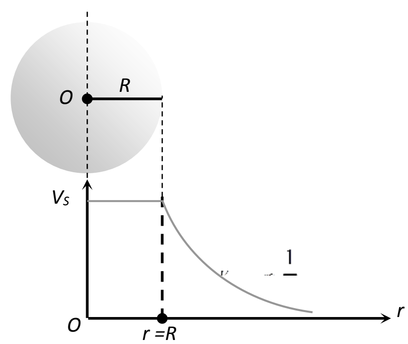
$$E_{in} = 0 \quad \text{and} \quad V_{in} = \text{constant} = V_s$$

Note: Graphical variation of electric field and potential of a charged spherical conductor with distance

E - r graph



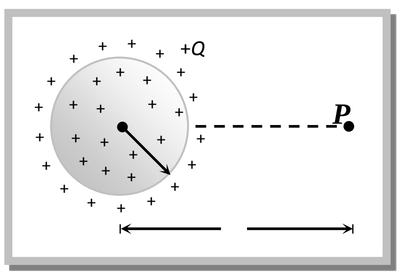
V - r graph



(4) **Volume charge (charged non-conducting sphere):**

Charge given to a non-conducting spheres spreads uniformly throughout its volume.

(i) **Outside the sphere at P**



$$E_{out} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2} \quad \text{and} \quad V_{out} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r} \quad \text{by using} \quad \rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$E_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2} \quad \text{and} \quad V_{out} = \frac{\rho R^3}{3\varepsilon_0 r}$$

(ii) **At the surface of sphere:** At surface $r = R$

$$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\rho R}{3\varepsilon_0} \quad \text{and} \quad V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\rho R^2}{3\varepsilon_0}$$

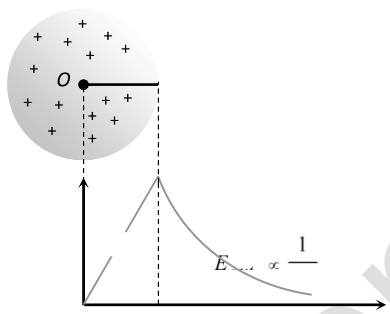
(iii) **Inside the sphere:** At a distance r from the center

$$E_{in} = \frac{1}{4\pi\varepsilon_0}.\frac{Qr}{R^3} = \frac{\rho r}{3\varepsilon_0} \quad \{E_{in} \propto r\} \quad \text{and} \quad V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q[3R^2 - r^2]}{2R^3} = \frac{\rho(3R^2 - r^2)}{6\varepsilon_0}$$

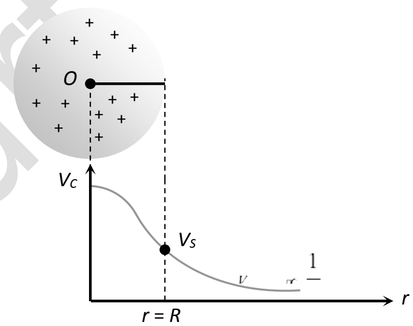
Note: At center $r = 0$ So, $V_{\text{centre}} = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{3}{2}V_s$ i.e., $V_{centre} > V_{surface} > V_{out}$

Graphical variation of electric field and potential with distance

E-r graph



V-r graph



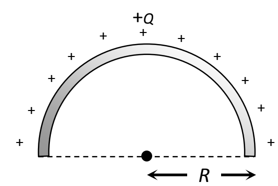
(5) **Electric field and potential in some other cases**

(i) **Uniformly charged semicircular ring :** $\lambda = \frac{\text{charge}}{\text{length}}$

At center :

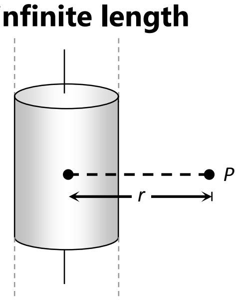
$$E = \frac{2K\lambda}{R} = \frac{Q}{2\pi^2\varepsilon_0 R^2}$$

$$V = \frac{KQ}{R} = \frac{Q}{4\pi\varepsilon_0 R}$$

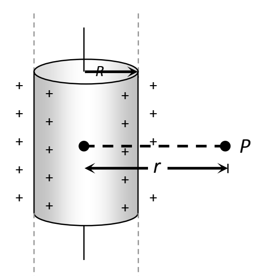


(iii) **Charged cylinder of infinite length**

(a) Conducting



(b) Non-conducting

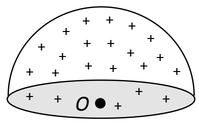


For both type of cylindrical charge distribution $E_{out} = \frac{\lambda}{2\pi\varepsilon_0 r}$, and $E_{suface} = \frac{\lambda}{2\pi\varepsilon_0 R}$ but for conducting $E_{in} = 0$ and for non-conducting $E_{in} = \frac{\lambda r}{2\pi\varepsilon_0 R^2}$. (We can also write formulae in form of ρ i.e., $E_{out} = \frac{\rho R^2}{2\varepsilon_0 r}$ etc.)

(ii) **Hemispherical charged body :**

At center O, $E = \frac{\sigma}{4\varepsilon_0}$

$$V = \frac{\sigma R}{2\varepsilon_0}$$

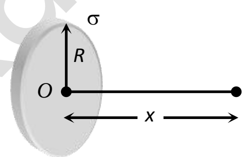


(iv) **Uniformly charged disc**

At a distance x from center O on its axis

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

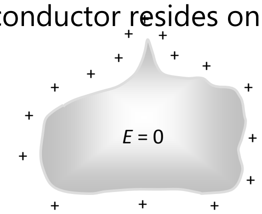


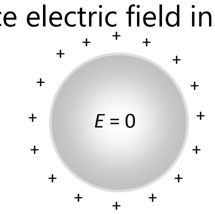
Note: Total charge on disc Q = $\sigma\pi R^2$

If x → 0, $E \simeq \frac{\sigma}{2\varepsilon_0}$ i.e. for points situated near the disc, it behaves as an infinite sheet of charge.

Concepts

- No point charge produces electric field at its own position.
- Since charge given to a conductor resides on its surface hence electric field inside it is zero.





- The electric field on the surface of a conductor is directly proportional to the surface charge density at that point i.e, $E \propto \sigma$

- Two charged spheres having radii r_1 and r_2 charge densities σ_1 and σ_2 respectively, then the ratio of electric field on their surfaces will be $\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{r_2^2}{r_1^2}$ $\left\{\sigma = \frac{Q}{4\pi r^2}\right.$
- In air if intensity of electric field exceeds the value $3 \times 10^6\ N/C$ air ionizes.
- A small ball is suspended in a uniform electric field with the help of an insulated thread. If a high energy x–ray bean falls on the ball, x-rays knock out electrons from the ball so the ball is positively charged and therefore the ball is deflected in the direction of electric field.
- Electric field is always directed from higher potential to lower potential.
- A positive charge if left free in electric field always moves from higher potential to lower potential while a negative charge moves from lower potential to higher potential.
- The practical zero of electric potential is taken as the potential of earth and theoretical zero is taken at infinity.
- An electric potential exists at a point in a region where the electric field is zero and it's vice versa.
- A point charge +Q lying inside a closed conducting shell does not exert force another point charge q placed outside the shell as shown in figure

Actually the point charge +Q is unable to exert force on the charge +q because it cannot produce electric field at the position of +q. All the field lines emerging from the point charge +Q terminate inside as these lines cannot penetrate the conducting medium (properties of lines of force).

The charge q however experiences a force not because of charge +Q but due to charge induced on the outer surface of the shell.

8. Potential Due to Concentric Spheres.

To find potential at a point due to concentric sphere following guideline are to be considered

Guideline 1: Identity the point (P) at which potential is to be determined.

Guideline 2: Start from inner most sphere, you should know where point (P) lies w.r.t. concerning sphere/shell (i.e. outside, at surface or inside)

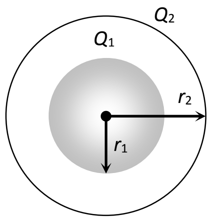
Guideline 3: Then find the potential at the point (P) due to inner most sphere and then due to next and so on.

Guideline 4: Using the principle of superposition find net potential at required shell/sphere.

Standard cases

Case (i): If two concentric conducting shells of radii r_1 and $r_2 (r_2 > r_1)$ carrying uniformly distributed charges Q_1 and Q_2 respectively. What will be the potential of each shell

To find the solution following guidelines are to be taken.



Here after following the above guideline potential at the surface of inner shell is

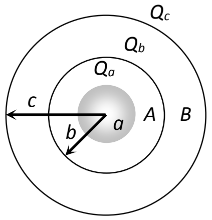
$$V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

and potential at the surface of outer shell

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{r_2}$$

Case (ii) : The figure shows three conducting concentric shell of radii a, b and c (a < b < c) having charges Q_a, Q_b and Q_c respectively what will be the potential of each shell

After following the guidelines discussed above



Potential at A; $V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{a} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$

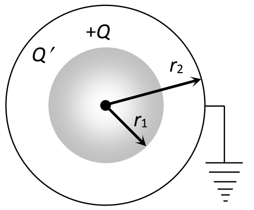
Potential at B; $V_B = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{b} + \frac{Q_b}{b} + \frac{Q_c}{c}\right]$

Potential at C; $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_a}{c} + \frac{Q_b}{c} + \frac{Q_c}{c}\right]$

Case (iii): The figure shows two concentric spheres having radii r_1 and r_2 respectively $(r_2 > r_1)$. If charge on inner sphere is +Q and outer sphere is earthed then determine.

(a) **The charge on the outer sphere**

(b) **Potential of the inner sphere**



(i) Potential at the surface of outer sphere $V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} = 0$

$\Rightarrow Q' = -Q$

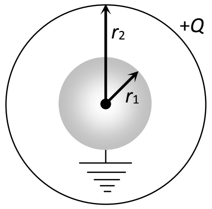
(ii) Potential of the inner sphere $V_1 = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)}{r_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

Case (iv) : In the case III if outer sphere is given a charge +Q and inner sphere is earthed then

(a) **What will be the charge on the inner sphere**

(b) **What will be the potential of the outer sphere**

(i) In this case potential at the surface of inner sphere is zero, so if Q' is the charge induced on inner sphere



then $V_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q'}{r_1} + \frac{Q}{r_2}\right] = 0$ i.e., $Q' = -\frac{r_1}{r_2}Q$

(Charge on inner sphere is less than that of the outer sphere.)

(ii) Potential at the surface of outer sphere

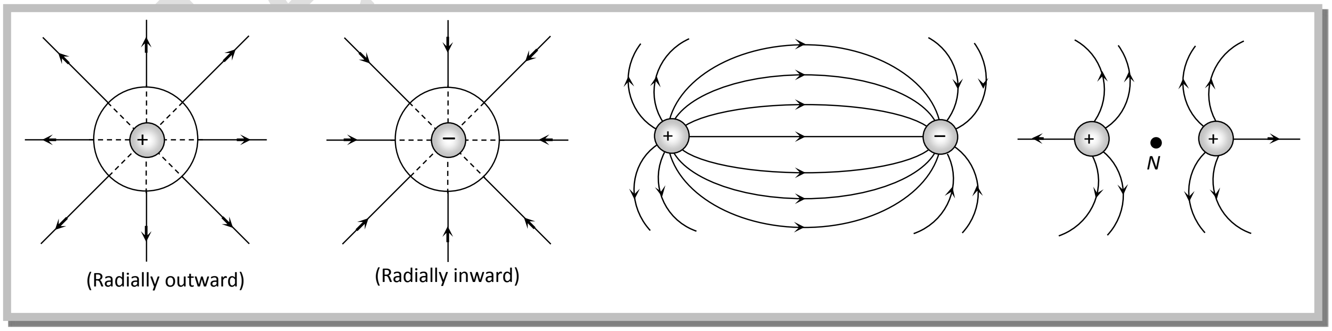
$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{Q'}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r_2}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0 r_2}\left[-Q\frac{r_1}{r_2} + Q\right] = \frac{Q}{4\pi\varepsilon_0 r_2}\left[1 - \frac{r_1}{r_2}\right]$$

9. Electric Lines of Force.

(1) **Definition:** The electric field in a region is represented by continuous lines (also called lines of force). Field line is an imaginary line along which a positive test charge will move if left free.

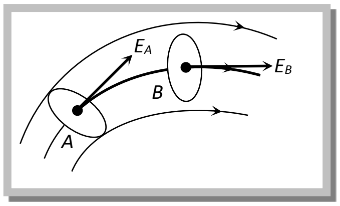
Electric lines of force due to an isolated positive charge, isolated negative charge and due to a pair of charge are shown below



(2) **Properties of electric lines of force**

(i) Electric field lines come out of positive charge and go into the negative charge.

(ii) Tangent to the field line at any point gives the direction of the field at that point.

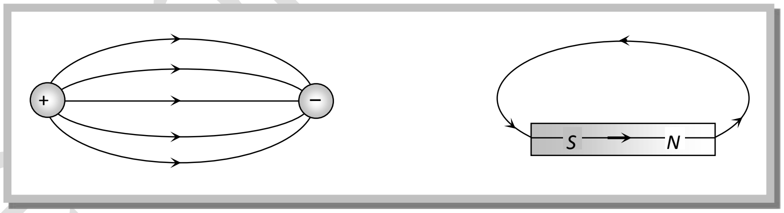


(iii) Field lines never cross each other.

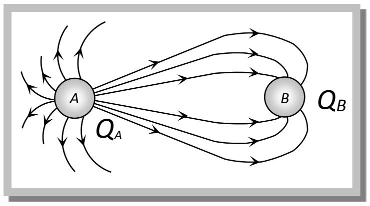
(iv) Field lines are always normal to conducting surface.

(v) Field lines do not exist inside a conductor.

(vi) The electric field lines never form closed loops. (While magnetic lines of forces form closed loop)

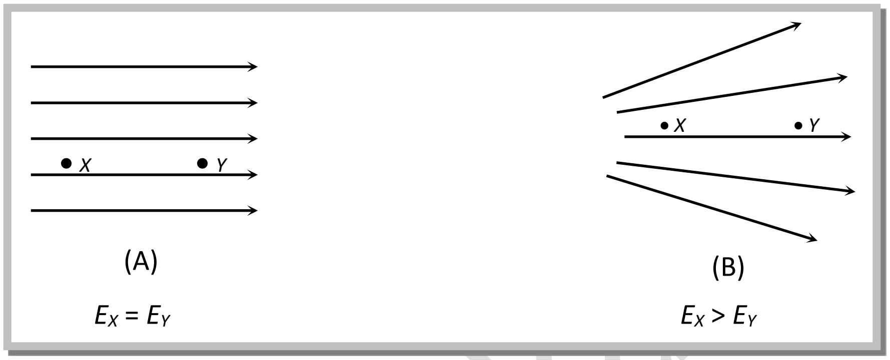


(vii) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In the following figure electric lines of force are originating from A and terminating at B hence Q_A is positive while Q_B is negative, also number of electric lines at force linked with Q_A are more than those linked with Q_B hence $|Q_A| > |Q_B|$

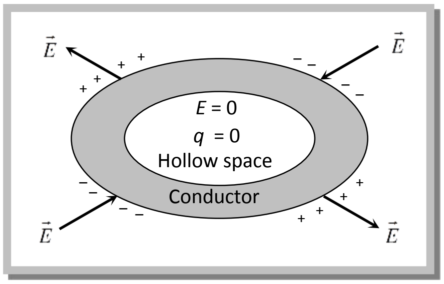


(viii) Number of lines of force per unit area normal to the area at a point represents magnitude of intensity (concept of electric flux i.e. $\phi = EA$)

(ix) If the lines of forces are equidistant and parallel straight lines the field is uniform and if either lines of force are not equidistant or straight line or both the field will be non-uniform, also the density of field lines is proportional to the strength of the electric field. For example see the following figures.



(3) **Electrostatic shielding:** Electrostatic shielding/screening is the phenomenon of protecting a certain region of space from external electric field. Sensitive instruments and appliances are affected seriously with strong external electrostatic fields. Their working suffers and they may start misbehaving under the effect of unwanted fields.

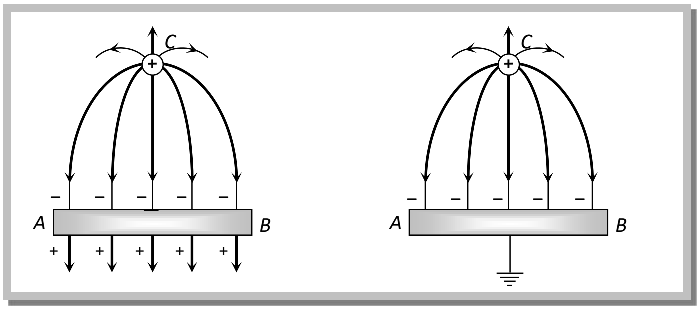


The electrostatic shielding can be achieved by protecting and enclosing the sensitive instruments inside a hollow conductor because inside hollow conductors, electric fields is zero.

(i) It is for this reason that it is safer to sit in a car or a bus during lightening rather than to stand under a tree or on the open ground.

(ii) A high voltage generator is usually enclosed in such a cage which is earthen. This would prevent the electrostatic field of the generator from spreading out of the cage.

(iii) An earthed conductor also acts as a screen against the electric field. When conductor is not earthed field of the charged body C due to electrostatic induction continues beyond AB. If AB is earthed, induced positive charge neutralizes and the field in the region beyond AB disappears.



10. Equipotential Surface or Lines.

If every point of a surface is at same potential, then it is said to be an equipotential surface

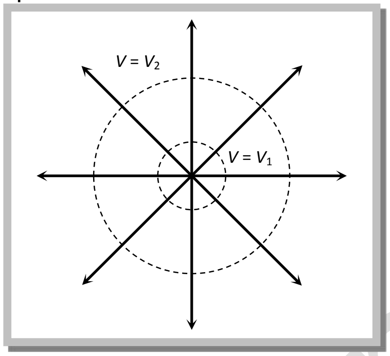
or

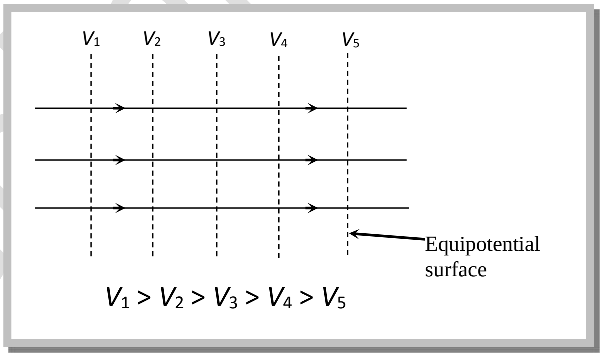
For a given charge distribution, locus of all points having same potential is called "equipotential surface" regarding equipotential surface following points should keep in mind:

(1) The density of the equipotential lines gives an idea about the magnitude of electric field. Higher the density larger the field strength.

(2) The direction of electric field is perpendicular to the equipotential surfaces or lines.

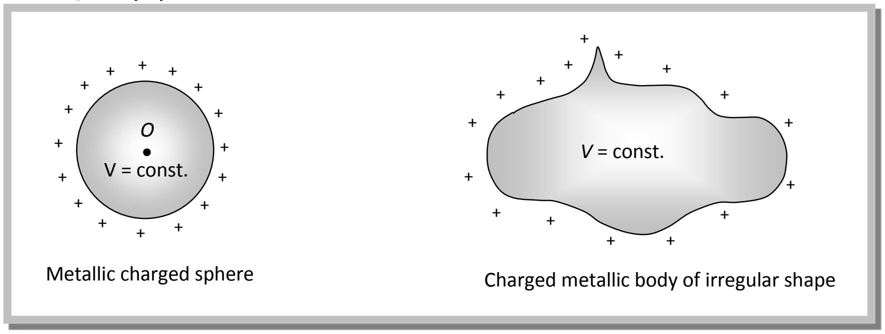
(3) The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.





(4) For a uniform electric field, the equipotential surfaces are a family of plane perpendicular to the field lines.

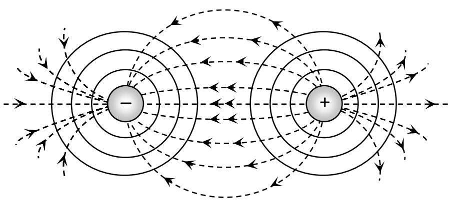
(5) A metallic surface of any shape is an equipotential surface e.g. When a charge is given to a metallic surface, it distributes itself in a manner such that its every point comes at same potential even if the object is of irregular shape and has sharp points on it.



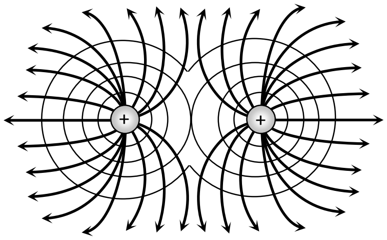
If it is not so, that is say if the sharp points are at higher potential then due to potential difference between these points connected through metallic portion, charge will flow from points of higher potential to points of lower potential until the potential of all points become same.

(6) Equipotential surfaces can never cross each other.

(7) Equipotential surface for pair of charges



Pair of two equal and opposite charges



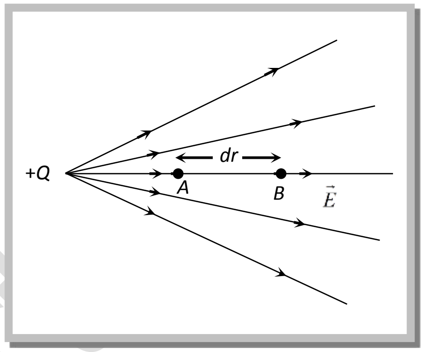
Pair of two equal and similar charges

Concepts

- Unit field i.e. 1N/C is defined arbitrarily as corresponding to unit density of lines of force.
- Number of lines originating from a unit charge is $\frac{1}{\varepsilon_0}$
- It is a common misconception that the path traced by a positive test charge is a field line but actually the path traced by a unit positive test charge represents a field full line only when it moves along a straight line.
- Both the equipotential surfaces and the lines of force can be used to depict electric field in a certain region of space. The advantage of using equipotential surfaces over the lines of force is that they give a visual picture of both the magnitude and direction of the electric field.

11. Relation between Electric Field and Potential.

In an electric field rate of change of potential with distance is known as **potential gradient.** It is a vector quantity and its direction is opposite to that of electric field. Potential gradient relates with electric field according to the following relation $E = -\frac{dV}{dr}$; this relation gives another unit of electric field is $\frac{volt}{meter}$. In the above relation negative sign indicates that in the direction of electric field potential decreases.

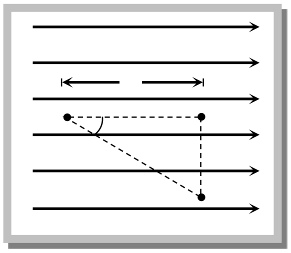


In space around a charge distribution we can also write $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

Where $E_x = -\frac{dV}{dx}$, $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$

With the help of formula $E = -\frac{dV}{dr}$, potential difference between any two points in an electric field can be determined by knowing the boundary conditions $dV = -\int_{r_1}^{r_2} \vec{E}.\vec{dr} = -\int_{r_1}^{r_2} \vec{E}.\vec{dr}\cos\theta$.

For example: Suppose A, B and C are three points in a uniform electric field as shown in figure.



(i) Potential difference between point A and B is

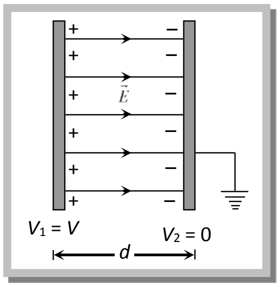
$$V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dr}$$

Since displacement is in the direction of electric field, hence θ = 0°

So, $$V_B - V_A = -\int_A^B E\cdot dr\cos 0 = -\int_A^B E\cdot dr = -Ed$$

In general we can say that in an uniform electric field $E = -\frac{V}{d}$ or $|E| = \frac{V}{d}$

Another example



$$E = \frac{V}{d}$$

(ii) Potential difference between points A and C is:

$$V_C - V_A = -\int_A^C E\,dr\cos\theta = -E(AC)\cos\theta = -E(AB) = -Ed$$

Above relation proves that potential difference between A and B is equal to the potential difference between A and C i.e. points B and C are at same potential.

Concept

☞ Negative of the slope of the V-r graph denotes intensity of electric field i.e. $\tan\theta = \frac{V}{r} = -E$

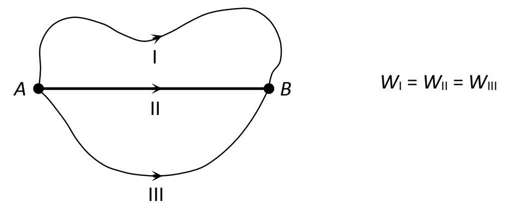
12. Work Done in Displacing a Charge.

(1) **Definition :** If a charge Q displaced from one point to another point in electric field then work done in this process is $W = Q \times \Delta V$ where ΔV = Potential difference between the two position of charge Q. ($\Delta V = \vec{E}.\Delta\vec{r} = E\Delta r\cos\theta$ where θ is the angle between direction of electric field and direction of motion of charge).

(2) **Work done in terms of rectangular component of $\vec{E}$ and $\vec{r}$:** If charge Q is given a displacement $\vec{r} = (r_1\hat{i} + r_2\hat{j} + r_3\hat{k})$ in an electric field $\vec{E} = (E_1\hat{i} + E_2\hat{j} + E_3\hat{k})$. The work done is $W = Q(\vec{E}.\vec{r}) = Q(E_1 r_1 + E_2 r_2 + E_3 r_3)$.

13. Conservation of Electric Field.

As electric field is conservation, work done and hence potential difference between two points is path independent and depends only on the position of points between. Which the charge is moved.



Concept

☞ No work is done in moving a charge on an equipotential surface.

14. Equilibrium of Charge.

(1) **Definition:** A charge is said to be in equilibrium, if net force acting on it is zero. A system of charges is said to be in equilibrium if each charge is separately in equilibrium.

(2) **Type of equilibrium:** Equilibrium can be divided in following type:

(i) **Stable equilibrium:** After displacing a charged particle from its equilibrium position, if it returns back then it is said to be in stable equilibrium. If U is the potential energy then in case of stable equilibrium $\frac{d^2U}{dx^2}$ is positive i.e., U is minimum.

(ii) **Unstable equilibrium :** After displacing a charged particle from its equilibrium position, if it never returns back then it is said to be in unstable equilibrium and in unstable equilibrium $\frac{d^2U}{dx^2}$ is negative i.e., U is maximum.

(iii) **Neutral equilibrium :** After displacing a charged particle from its equilibrium position if it neither comes back, nor moves away but remains in the position in which it was kept it is said to be in neutral equilibrium and in neutral equilibrium $\frac{d^2U}{dx^2}$ is zero i.e., U is constant

(3) **Guidelines to check the equilibrium**

(i) Identify the charge for which equilibrium is to be analyzed.

(ii) Check, how many forces acting on that particular charge.

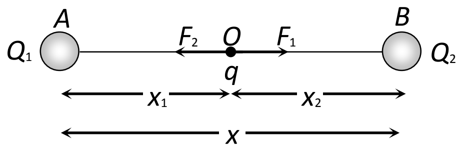
(iii) There should be at least two forces acts oppositely on that charge.

(iv) If magnitude of these forces are equal then charge is said to be in equilibrium then identify the nature of equilibrium.

(v) If all the charges of system are in equilibrium then system is said to be in equilibrium

(4) **Different cases of equilibrium of charge**

<u>Case – 1</u> : Suppose three similar charge Q_1, q and Q_2 are placed along a straight line as shown below



Charge q will be in equilibrium if $|F_1| = |F_2|$

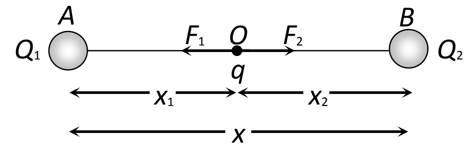
i.e., $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$; This is the condition of equilibrium of charge q. After following the guidelines we can say that charge q is in stable equilibrium and this system is not in equilibrium

Note : $x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}$

and $x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$

e.g. if two charges +4μC and +16 μC are separated by a distance of 30 cm from each other then for equilibrium a third charge should be placed between them at a distance $x_1 = \frac{30}{1+\sqrt{16/4}} = 10\,cm$ or $x_2 = 20\,cm$

<u>Case – 2</u> : Two similar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other and a third dissimilar charge q is placed in between them as shown below



Charge q will be in equilibrium if $|F_1| = |F_2|$

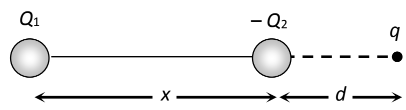
i.e., $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$.

Note : ❑ Same short trick can be used here to find the position of charge q as we discussed in Case–1 i.e.,

$x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}}$ and $x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$

❑ It is very important to know that magnitude of charge q can be determined if one of the extreme charge (either Q_1 or Q_2) is in equilibrium i.e. if Q_2 is in equilibrium then $|q| = Q_1\left(\frac{x_2}{x}\right)^2$ and if Q_1 is in equilibrium then $|q| = Q_2\left(\frac{x_1}{x}\right)^2$ (It should be remember that sign of q is opposite to that of $Q_1 (\text{or } Q_2)$)

Case – 3: Two dissimilar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other, a third charge q should be placed outside the line joining Q_1 and Q_2 for it to experience zero net force.



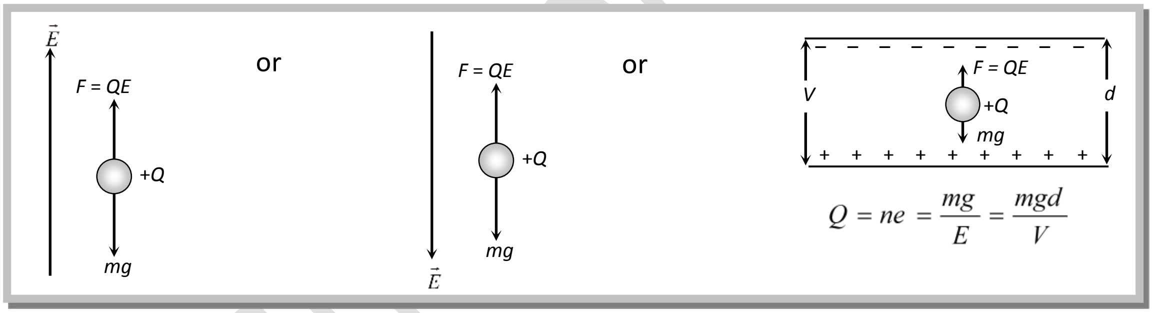
(Let $|Q_2| < |Q_1|$)

Short Trick :

For its equilibrium. Charge q lies on the side of charge which is smallest in magnitude and $d = \dfrac{x}{\sqrt{Q_1/Q_2} - 1}$

(5) **Equilibrium of suspended charge in an electric field**

(i) **Freely suspended charged particle :** To suspend a charged a particle freely in air under the influence of electric field it's downward weight should be balanced by upward electric force for example if a positive charge is suspended freely in an electric field as shown then

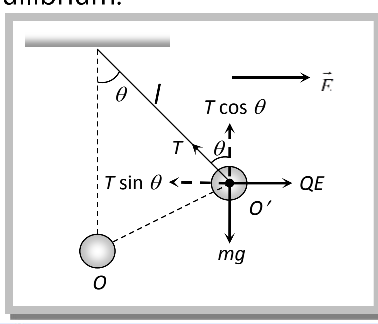


In equilibrium $QE = mg \Rightarrow E = \dfrac{mg}{Q}$

Note: In the above case if direction of electric field is suddenly reversed in any figure then acceleration of charge particle at that instant will be a = 2g.

(ii) **Charged particle suspended by a massless insulated string** (like simple pendulum): Consider a charged particle (like Bob) of mass m, having charge Q is suspended in an electric field as shown under the influence of electric field. It turned through an angle (sayθ) and comes in equilibrium.

So, in the position of equilibrium (O′ position)



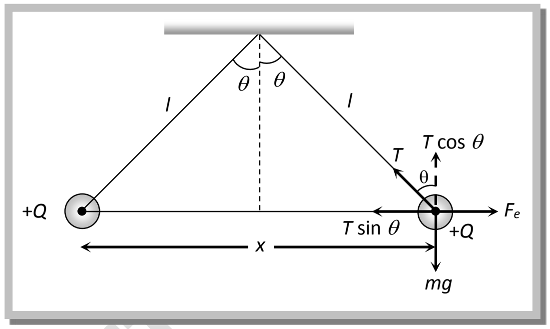
$T\sin\theta = QE$ ....(i)

$T\cos\theta = mg$ ....(ii)

By squaring and adding equation (i) and (ii) $T = \sqrt{(QE)^2 + (mg)^2}$

Dividing equation (i) by (ii) $\tan\theta = \frac{QE}{mg} \Rightarrow \theta = \tan^{-1}\frac{QE}{mg}$

(iii) **Equilibrium of suspended point charge system:** Suppose two small balls having charge +Q on each are suspended by two strings of equal length l. Then for equilibrium position as shown in figure.



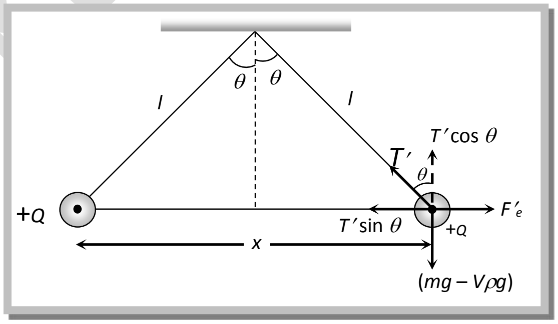
$T\sin\theta = F_e$ ....(I)

$T\cos\theta = mg$ ....(ii)

$T^2 = (F_e)^2 + (mg)^2$

and $\tan\theta = \frac{F_e}{mg}$; here $F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{x^2}$ and $\frac{x}{2} = l\sin\theta$

(iv) **Equilibrium of suspended point charge system in a liquid:** In the previous discussion if point charge system is taken into a liquid of density ρ such that θ remain same then



In equilibrium $Fe' = T'\sin\theta$ and $(mg - V\rho g) = T'\cos\theta$

$\therefore \tan\theta = \frac{Fe'}{(mg - V\rho g)} = \frac{Q^2}{4\pi\varepsilon_0 K(mg - V\rho g)x^2}$

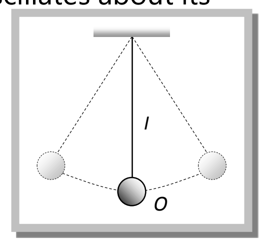
When this system was in air $\tan\theta = \frac{Fe}{mg} = \frac{Q^2}{4\pi\varepsilon_0\, mgx^2}$

$\therefore$ So equating these two gives us $\frac{1}{m} = \frac{1}{k(m - V\rho)} \Rightarrow K = \frac{m}{m - V\rho} = \frac{1}{\left(1 - \frac{V}{m}\rho\right)}$

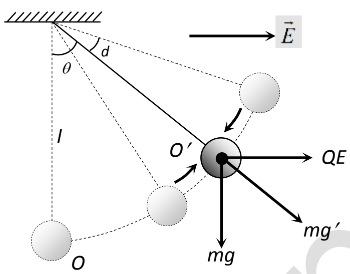
If σ is the density of material of ball then $K = \frac{1}{\left(1 - \frac{\rho}{\sigma}\right)}$

15. Time Period of Oscillation of a Charged Body.

(1) **Simple pendulum based:** If a simple pendulum having length l and mass of bob m oscillates about its mean position than its time period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$



Case – 1 : If some charge say +Q is given to bob and an electric field E is applied in the direction as shown in figure then equilibrium position of charged bob (point charge) changes from O to O′.



On displacing the bob from its equilibrium position 0′. It will oscillate under the effective acceleration g′, where

$$mg' = \sqrt{(mg)^2 + (QE)^2}$$

$$\Rightarrow g' = \sqrt{g^2 + (QE/m)^2}$$

Hence the new time period is $T_1 = 2\pi\sqrt{\frac{l}{g'}}$

$$T_1 = 2\pi\sqrt{\frac{l}{\left(g^2 + (QE/m)^2\right)^{\frac{1}{2}}}}$$

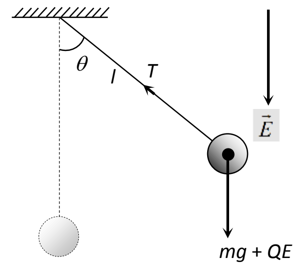
Since g'>g, hence **T_1 < T**

i.e. time period of pendulum will decrease.

Case – 2 : If electric field is applied in the downward direction then.

Effective acceleration

$g' = g + QE/m$



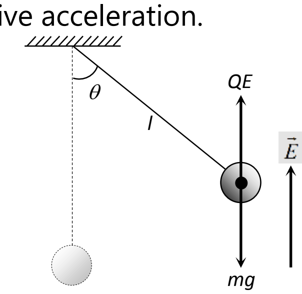
So new time period

$$T_2 = 2\pi\sqrt{\frac{l}{g + (QE/m)}}$$

T_2 < T

Case – 3 : In case 2 if electric field is applied in upward direction then, effective acceleration.

$g' = g - QE/m$



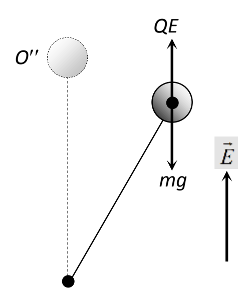
So new time period

$$T_3 = 2\pi\sqrt{\frac{l}{g - (QE/m)}}$$

T_3 > T

Case – 4 : In the case 3,

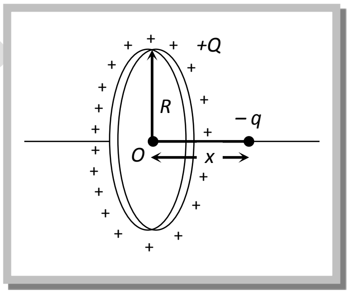
if $T_3 = \frac{T}{2}$ i.e., $2\pi\sqrt{\frac{l}{(g - QE/m)}}$ $= \frac{1}{2}2\pi\sqrt{\frac{l}{g}}$ $\Rightarrow$ QE = 3 mg



i.e., effective vertical force (gravity + electric) on the bob = mg – 3 mg = – 2 mg, hence the equilibrium

	position O′′ of the bob will be above the point of suspension and bob will oscillate under on effective acceleration 2g directed upward. Hence new time period $T_4 = 2\pi\sqrt{\frac{l}{2g}}$, **$T_4 < T$**

(2) **Charged circular ring:** A thin stationary ring of radius R has a positive charge +Q unit. If a negative charge – q (mass m) is placed at a small distance x from the center. Then motion of the particle will be simple harmonic motion.



Electric field at the location of – q charge $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qx}{(x^2 + R^2)^{\frac{3}{2}}}$

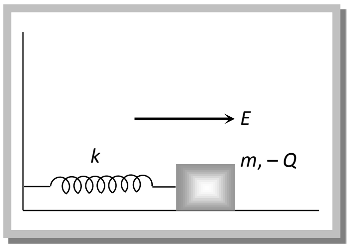
Since x<< R, So x^2 neglected hence $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qx}{R^3}$

Force experienced by charge – q is $F = -q\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qx}{R^3}$

$\Rightarrow F \propto -x$ Hence motion is simple harmonic

Having time period $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}}$

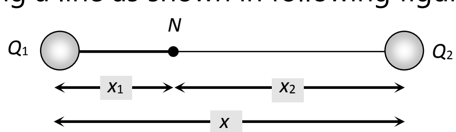
(3) **Spring mass system:** A block of mass m containing a negative charge – Q is placed on a frictionless horizontal table and is connected to a wall through an upstretched spring of spring constant k as shown. If electric field E applied as shown in figure the block experiences an electric force, hence spring compress and block comes in new position. This is called the equilibrium position of block under the influence of electric field. If block compressed further or stretched, it execute oscillation having time period $T = 2\pi\sqrt{\frac{m}{k}}$. Maximum compression in the spring due to electric field $= \frac{QE}{k}$



16. Neutral Point.

A neutral point is a point where resultant electrical field is zero. It is obtained where two electrical field are equal and opposite. Thus neutral points can be obtained only at those points where the resultant field is subtractive. Thus it can be obtained.

(1) **At an internal point along the line joining two like charges (Due to a system of two like point charge):** Suppose two like charges. Q_1 and Q_2 are separated by a distance x from each other along a line as shown in following figure.



If N is the neutral point at a distance x_1 from Q_1 and at a distance $x_2(= x - x_1)$ from Q_2 then –

At N $|\ \textbf{E.F. due to } Q_1\ | = |\ \textbf{E.F. due to } Q_2\ |$

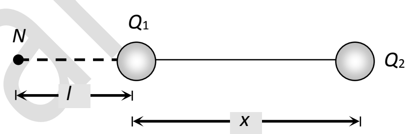
i.e., $\frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{x_1^2} = \frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{x_2^2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$

Short trick : $x_1 = \frac{x}{1 + \sqrt{Q_2/Q_1}}$ and

$x_2 = \frac{x}{1 + \sqrt{Q_1/Q_2}}$

Note: In the above formula if $Q_1 = Q_2$, neutral point lies at the center so remember that resultant field at the midpoint of two equal and like charges is zero.

(2) **At an external point along the line joining two like charges (Due to a system of two unlike point charge):** Suppose two unlike charge Q_1 and Q_2 separated by a distance x from each other.



Here neutral point lies outside the line joining two unlike charges and also it lies nearer to charge which is smaller in magnitude.

If $|Q_1| < |Q_2|$ then neutral point will be obtained on the side of Q_1, suppose it is at a distance l from Q_1

Hence at neutral point ; $\frac{kQ_1}{l^2} = \frac{kQ_2}{(x+l)^2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{l}{x+l}\right)^2$

Short trick : $l = \frac{x}{\left(\sqrt{Q_2/Q_1} - 1\right)}$

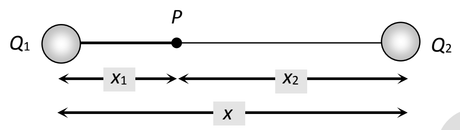
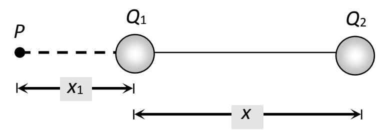
Note: In the above discussion if $|\ Q_1\ | = |\ Q_2\ |$ neutral point will be at infinity.

17. Zero Potential Due to a System of Two Point Charge.

If both charges are like then resultant potential is not zero at any finite point because potentials due to like charges will have same sign and can therefore never add up to zero. Such a point can be therefore obtained only at infinity.

If the charges are unequal and unlike then all such points where resultant potential is zero lies on a closed curve, but we are interested only in those points where potential is zero along the line joining the two charges.

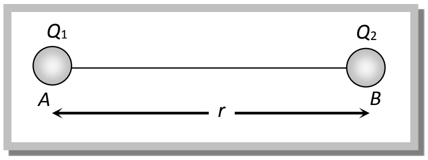
Two such points exist, one lies inside and one lies outside the charges on the line joining the charges. Both the above points lie nearer the smaller charge, as potential created by the charge larger in magnitude will become equal to the potential created by smaller charge at the desired point at larger distance from it.

I. For internal point :	II. For External point :
 (It is assumed that $\lvert Q_1 \rvert < \lvert Q_2 \rvert$). $\frac{Q_1}{x_1} = \frac{Q_2}{(x - x_1)} \Rightarrow x_1 = \frac{x}{(Q_2/Q_1 + 1)}$	 $\frac{Q_1}{x_1} = \frac{Q_2}{(x + x_1)} \Rightarrow x_1 = \frac{x}{(Q_2/Q_1 - 1)}$

18. Electric Potential Energy.

(1) **Potential energy of a charge:** Work done in bringing the given charge from infinity to a point in the electric field is known as potential energy of the charge. Potential can also be written as potential energy per unit charge. i.e. $V = \frac{W}{Q} = \frac{U}{Q}$.

(2) **Potential energy of a system of two charges:** Since work done in bringing charge Q_2 from ∞ to point B is $W = Q_2 V_B$, where V_B is potential of point B due to charge Q_1 i.e. $V_B = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r}$



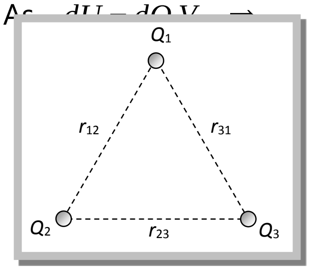
So, $W = U_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r}$

This is the potential energy of charge Q_2, similarly potential energy of charge Q_1 will be $U_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r}$

Hence potential energy of Q_1 = Potential energy of Q_2 = potential energy of system $U = k\frac{Q_1 Q_2}{r}$ (in C.G.S. $U = \frac{Q_1 Q_2}{r}$)

Note: Electric potential energy is a scalar quantity so in the above formula take sign of Q_1 and Q_2.

(3) **Potential energy of a system of n charges:** In a system of n charges electric potential energy is calculated for each pair and then all energies so obtained are added algebraically. i.e. $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_2 Q_3}{r_{23}} + ...\right]$ and in case of continuous distribution of charge. As $dU = dQ\,V \rightarrow$ $U = \int V\,dQ$



E.g. Electric potential energy for a system of three charges

Potential energy $= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_2 Q_3}{r_{23}} + \frac{Q_3 Q_1}{r_{31}}\right]$

While potential energy of any of the charge say Q_1 is $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1 Q_2}{r_{12}} + \frac{Q_3 Q_1}{r_{31}}\right]$

Note: For the expression of total potential energy of a system of n charges consider $\frac{n(n-1)}{2}$ number of pair of charges.

(4) **Electron volt (eV):** It is the smallest practical unit of energy used in atomic and nuclear physics. As electron volt is defined as "the energy acquired by a particle having one quantum of charge 1e when accelerated by 1volt" i.e. $1eV = 1.6 \times 10^{-19} C \times \frac{1J}{C} = 1.6 \times 10^{-19} J$ = 1.6×10^{-12} erg

Energy acquired by a charged particle in eV when it is accelerated by V volt is E = (charge in quanta) × (p.d. in volt)

Commonly asked examples:

S.No.	Charge	Accelerated by p.d.	Gain in K.E.
(i)	Proton	5×10^4 V	K = e × 5 × 10^4 V = 5 × 10^4 eV = 8 × 10^{-15} J **[JIPMER 1999]**
(ii)	Electron	100 V	K = e × 100 V = 100 eV = 1.6 × 10^{-17} J **[MP PMT 2000; AFMC 1999]**
(iii)	Proton	1 V	K = e × 1 V = 1 eV = 1.6 × 10^{-19} J **[CBSE 1999]**
(iv)	0.5 C	2000 V	K = 0.5 × 2000 = 1000 J **[JIPMER 2002]**
(v)	α-particle	10^6 V	K = (2e) × 10^6 V = 2 MeV **[MP PET/PMT 1998]**

(5) **Electric potential energy of a uniformly charged sphere:** Consider a uniformly charged sphere of radius R having a total charge Q. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere.

$$U = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

(6) **Electric potential energy of a uniformly charged thin spherical shell:**

$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

(7) **Energy density:** The energy stored per unit volume around a point in an electric field is given by

$U_e = \frac{U}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$. If in place of vacuum some medium is present then $U_e = \frac{1}{2}\varepsilon_0 \varepsilon_r E^2$

Concepts

- Electric potential energy is not localized but is distributed all over the field
- If a charge moves from one position to another position in an electric field so it's potential energy change and work done in this changing is $W = U_f - U_i$
- If two similar charge comes closer potential energy of system increases while if two dissimilar charge comes closer potential energy of system decreases.

19. Motion of Charged Particle in an Electric Field.

(1) **When charged particle initially at rest is placed in the uniform field:**

Let a charge particle of mass m and charge Q be initially at rest in an electric field of strength E

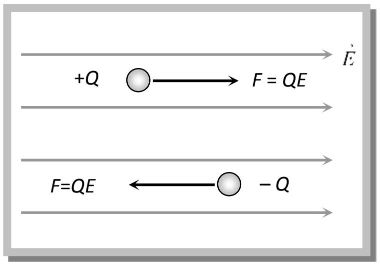


Fig. (A)

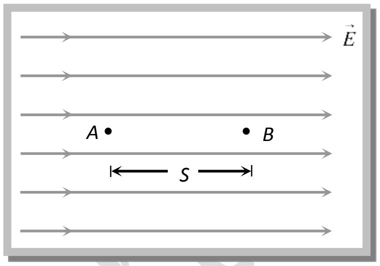


Fig. (B)

(i) **Force and acceleration:** The force experienced by the charged particle is $F = QE$. Positive charge experiences force in the direction of electric field while negative charge experiences force in the direction opposite to the field. [Fig. (A)]

Acceleration produced by this force is $a = \frac{F}{m} = \frac{QE}{m}$

Since the field E in constant the acceleration is constant, thus motion of the particle is uniformly accelerated.

(ii) **Velocity:** Suppose at point A particle is at rest and in time t, it reaches the point B [Fig. (B)]

V = Potential difference between A and B; S = Separation between A and B

(a) By using $v = u + at$, $v = 0 + Q\frac{E}{m}t$, $\Rightarrow v = \frac{QEt}{m}$

(b) By using $v^2 = u^2 + 2as$, $v^2 = 0 + 2 \times \frac{QE}{m} \times s$ $v^2 = \frac{2QV}{m}$ $\left\{\because E = \frac{V}{s}\right\}$ $\Rightarrow v = \sqrt{\frac{2QV}{m}}$

(iii) **Momentum:** Momentum p = mv, $p = m \times \frac{QEt}{m} = QEt$ or $p = m \times \sqrt{\frac{2QV}{m}} = \sqrt{2mQV}$

(iv) **Kinetic energy:** Kinetic energy gained by the particle in time t is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{(QEt)^2}{m} = \frac{Q^2E^2t^2}{2m}$$

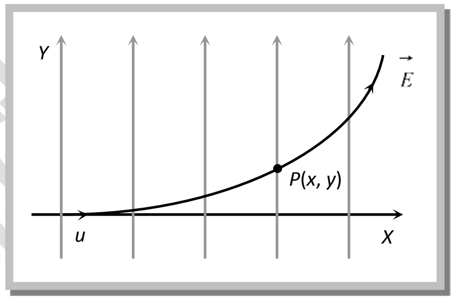
or $$K = \frac{1}{2} m \times \frac{2QV}{m} = QV$$

(2) **When a charged particle enters with an initial velocity at right angle to the uniform field:**

When charged particle enters perpendicularly in an electric field, it describe a parabolic path as shown

(i) Equation of trajectory: Throughout the motion particle has uniform velocity along x-axis and horizontal displacement (x) is given by the equation x = ut

Since the motion of the particle is accelerated along y–axis, we will use equation of motion for uniform acceleration to determine displacement y. From $S = ut + \frac{1}{2}at^2$



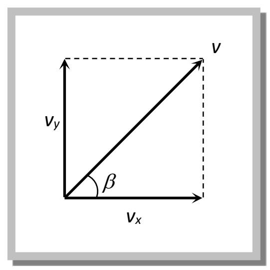
We have $u = 0$ (along y-axis) so $y = \frac{1}{2}at^2$

i.e., displacement along y-axis will increase rapidly with time (since $y \propto t^2$

From displacement along x-axis $t = \frac{x}{u}$

So $y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{u}\right)^2$; this is the equation of parabola which shows $y \propto x^2$

(ii) Velocity at any instant: At any instant t, $v_x = u$ and $v_y = \frac{QEt}{m}$



So $$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{Q^2E^2t^2}{m^2}}$$

If β is the angle made by v with x-axis than $\tan\beta = \frac{v_y}{v_x} = \frac{QEt}{mu}$.

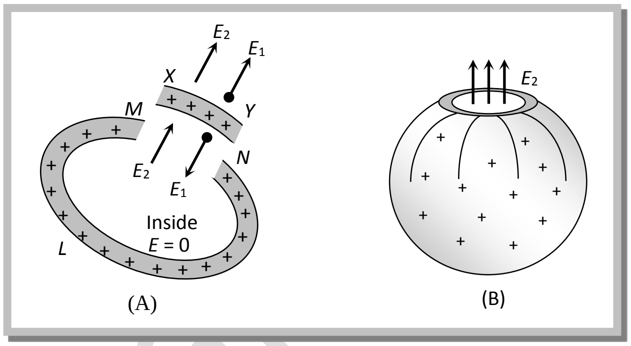
Concepts

☞ An electric field is completely characterized by two physical quantities Potential and Intensity. Force characteristic of the field is intensity and work characteristic of the field is potential.

☞ If a charge particle (say positive) is left free in an electric field, it experiences a force $(F = QE)$ in the direction of electric field and moves in the direction of electric field (which is desired by electric field), so its kinetic energy increases, potential energy decreases, then work is done by the electric field and it is negative.

20. Force on a Charged Conductor.

To find force on a charged conductor (due to repulsion of like charges) imagine a small part XY to be cut and just separated from the rest of the conductor MLN. The field in the cavity due to the rest of the conductor is E_2, while field due to small part is E_1. Then



Inside the conductor $E = E_1 - E_2 = 0$ or $E_1 = E_2$

Outside the conductor $E = E_1 + E_2 = \frac{\sigma}{\varepsilon_0}$

Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

To find force, imagine charged part XY (having charge $\sigma\, dA$ placed in the cavity MN having field E_2). Thus force $dF = (\sigma\, dA)E_2$ or $dF = \frac{\sigma^2}{2\varepsilon_0} dA$. The force per unit area or electric pressure is $\frac{dF}{dA} = \frac{\sigma^2}{2\varepsilon_0}$

The force is always outwards as $(\pm\sigma)^2$ is positive i.e., whether charged positively or negatively, this force will try to expand the charged body.

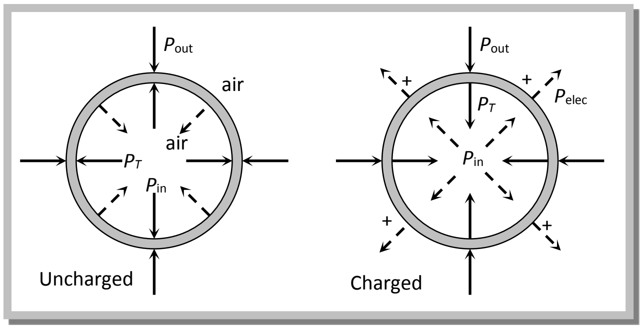
A soap bubble or rubber balloon expands on given charge to it (charge of any kind + or –).

21. Equilibrium of Charged Soap Bubble.

For a charged soap bubble of radius R and surface tension T and charge density σ. the pressure due to surface tension $4\frac{T}{R}$ and atmospheric pressure P_{out} act radially inwards and the electrical pressure (P_{el}) acts radially outward.

The total pressure inside the soap bubble $P_{\text{in}} = P_{\text{out}} + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$

Excess pressure inside the charged soap bubble $P_{\text{in}} - P_{\text{out}} = P_{\text{excess}} = \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}$. If air pressure inside and outside are assumed equal then $P_{\text{in}} = P_{\text{out}}$ i.e., $P_{\text{excess}} = 0$. So, $\frac{4T}{R} = \frac{\sigma^2}{2\varepsilon_0}$



This result give us the following formulae

(1) Radius of bubble $R = \dfrac{8\,\varepsilon_0\,T}{\sigma^2}$

(2) Surface tension $T = \dfrac{\sigma^2 R}{8\varepsilon_0}$

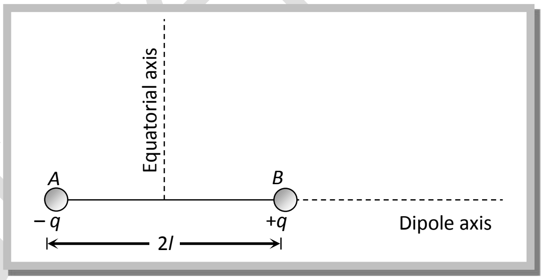
(3) Total charge on the bubble $Q = 8\pi R\sqrt{2\varepsilon_0 TR}$

(4) Electric field intensity at the surface of the bubble $E = \sqrt{\dfrac{8T}{\varepsilon_0 R}} = \sqrt{\dfrac{32\pi\,kT}{R}}$

(5) Electric potential at the surface $V = \sqrt{3\pi RTk} = \sqrt{\dfrac{8RT}{\varepsilon_0}}$

22. Electric Dipole.

(1) **General information:** System of two equal and opposite charges separated by a small fixed distance is called a dipole.



(i) **Dipole axis:** Line joining negative charge to positive charge of a dipole is called its axis. It may also be termed as its longitudinal axis.

(ii) **Equatorial axis:** Perpendicular bisector of the dipole is called its equatorial or transverse axis as it is perpendicular to length.

(iii) **Dipole length:** The distance between two charges is known as dipole length (L = 2l)

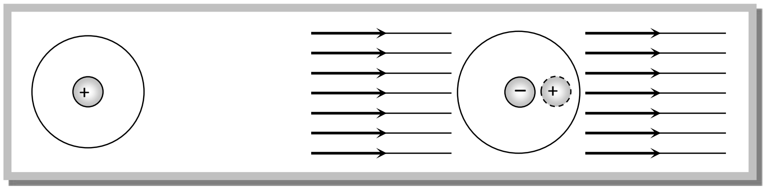
(iv) **Dipole moment:** It is a quantity which gives information about the strength of dipole. It is a vector quantity and is directed from negative charge to positive charge along the axis. It is denoted as $\vec{p}$ and is defined as the product of the magnitude of either of the charge and the dipole length.

i.e. $\vec{p} = q(2\vec{l})$

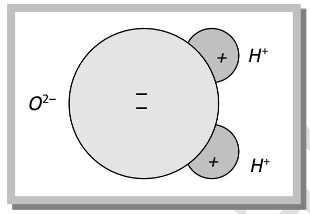
Its S.I. unit is **coulomb-meter** or **Debye** (1 Debye = 3.3 × 10^{-30} C × m) and its dimensions are $M^0L^1T^1A^1$.

Note: A region surrounding a stationary electric dipole has electric field only.

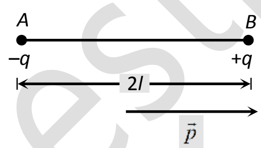
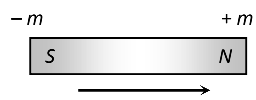
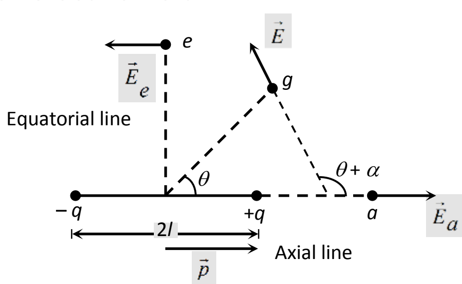
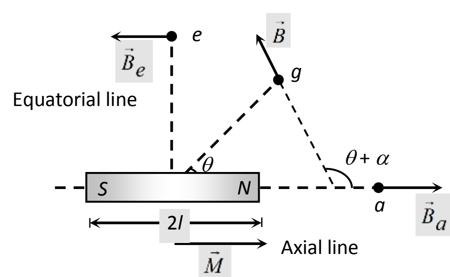
When a dielectric is placed in an electric field, its atoms or molecules are considered as tiny dipoles.



Water (H_2O), Chloroform ($CHCl_3$), Ammonia (NH_3), HCl, CO molecules are some example of permanent electric dipole.



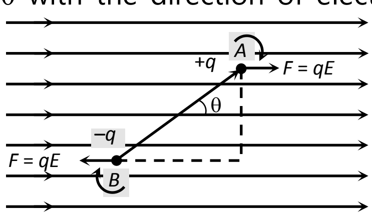
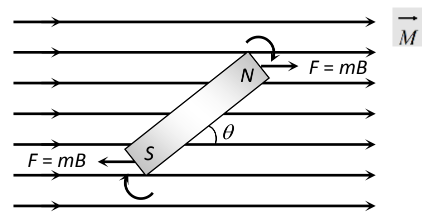
(2) **Electric field and potential due to an electric dipole:** It is better to understand electric dipole with magnetic dipole.

S.No.	Electric dipole	Magnetic dipole
(i)	System of two equal and opposite charges separated by a small fixed distance. 	System of two equal and opposite magnetic poles (Bar magnet) separated by a small fixed distance. 
(ii)	**Electric dipole moment:** $\vec{p} = q(2\vec{l})$, directed from $-q$ to +q. Its S.I. unit is coulomb × meter or Debye.	**Magnetic dipole moment:** $\vec{M} = m(2\vec{l})$, directed from S to N. Its S.I. unit is ampere × meter^2.
(iii)	**Intensity of electric field** 	**Intensity of magnetic field** 

If a, e and g are three points on axial, equatorial and general position at a distance r from the center of dipole on axial point $E_a = \frac{1}{4\pi\varepsilon_0}.\frac{2p}{r^3}$ (directed from – q to +q) on equatorial point $E_e = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^3}$ (directed from +q to –q) on general point $E_a = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^3}\sqrt{(3\cos^2\theta+1)}$ Angle between – $\vec{E}_a$ and $\vec{p}$ is 0°, $\vec{E}_e$ and $\vec{p}$ is 180°, $\vec{E}$ and $\vec{p}$ is (θ + α) (where $\tan\alpha = \frac{1}{2}\tan\theta$) Electric Potential – At a $V_a = \frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2}$, At e $V = 0$ At g $V = \frac{1}{4\pi\varepsilon_0}.\frac{p\cos\theta}{r^2}$	If a, e and g are three points on axial, equatorial and general position at a distance r from the center of dipole on axial point $B_a = \frac{\mu_0}{4\pi}.\frac{2M}{r^3}$ (directed from S to N) on equatorial point $B_e = \frac{\mu_0}{4\pi}.\frac{M}{r^3}$ (directed from N to S) on general point $B_a = \frac{\mu_0}{4\pi}.\frac{M}{r^3}\sqrt{(3\cos^2\theta+1)}$ Angle between – $\vec{B}_a$ and $\vec{M}$ is 0°, $\vec{B}_e$ and $\vec{M}$ is 180°, $\vec{B}$ and $\vec{M}$ is (θ + α) (where $\tan\alpha = \frac{1}{2}\tan\theta$)

(3) **Dipole (electric/magnetic) in uniform field (electric/magnetic)**

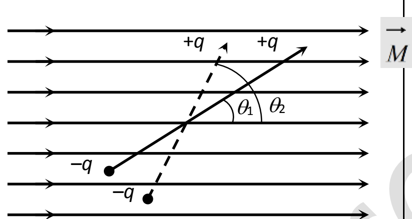
(i) **Torque:** If a dipole is placed in an uniform field such that dipole (i.e. $\vec{p}$ or $\vec{M}$) makes an angle θ with direction of field then two equal and opposite force acting on dipole constitute a couple whose tendency is to rotate the dipole hence a torque is developed in it and dipole tries to align itself in the direction of field.

Consider an electric dipole in placed in an uniform electric field such that dipole (i.e. $\vec{p}$) makes an angle θ with the direction of electric field as shown  	A magnetic dipole of magnetic moment M is placed in uniform magnetic field B by making an angle θ as shown 

Electric dipole	Magnetic dipole
(a) Net force on electric dipole $F_{net} = 0$	(a) Net force on magnetic dipole $F_{net} = 0$
(b) Produced torque τ = pE sinθ $(\vec{\tau} = \vec{P} \times \vec{E})$	(b) torque τ = MB sinθ $(\vec{\tau} = \vec{M} \times \vec{B})$

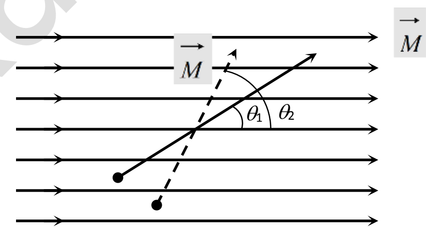
(ii) **Work:** From the above discussion it is clear that in a uniform electric/magnetic field dipole tries to align itself in the direction of electric field (i.e. equilibrium position). To change its angular position some work has to be done.

Suppose an electric/magnetic dipole is kept in an uniform electric/magnetic field by making an angle θ_1 with the field, if it is again turn so that it makes an angle θ_2 with the field, work done in this process is given by the formula



$W = pE(\cos\theta_1 - \cos\theta_2)$

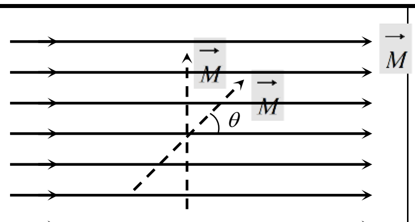
If $\theta_1 = 0^\circ$ and $\theta_2 = \theta$ i.e. initially dipole is kept along the field then it turn through θ so work done $W = pE(1 - \cos\theta)$

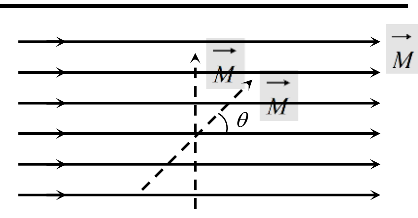


$W = MB(\cos\theta_1 - \cos\theta_2)$

If $\theta_1 = 0^\circ$ and $\theta_2 = \theta$ then W = MB(1 – cosθ)

(iii) **Potential energy:** In case of a dipole (in a uniform field), potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e. if $\theta_1 = 90^\circ$ and $\theta_2 = \theta$ then –





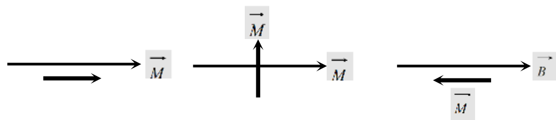
$W = U = pE(\cos 90 - \cos\theta) \Rightarrow U = -pE\cos\theta$	$W = U = MB(\cos 90 - \cos\theta) \Rightarrow U = -MB\cos\theta$

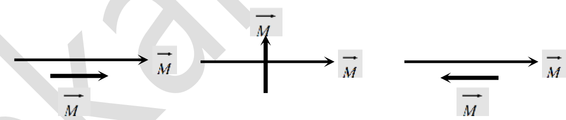
(iv) **Equilibrium of dipole:** We know that, for any equilibrium net torque and net force on a particle (or system) should be zero.

We already discussed when a dipole is placed in a uniform electric/magnetic field net force on dipole is always zero. But net torque will be zero only when $\theta = 0^o$ or 180^o

When $\theta = 0^o$ i.e. dipole is placed along the electric field it is said to be in stable equilibrium, because after turning it through a small angle, dipole tries to align itself again in the direction of electric field.

When $\theta = 180^o$ i.e. dipole is placed opposite to electric field, it is said to be in unstable equilibrium.





$\theta = 0^o$	$\theta = 90^O$	$\theta = 180^o$	$\theta = 0^o$	$\theta = 90^O$	$\theta = 180^o$
Stable equilibrium		Unstable equilibrium	Stable equilibrium		Unstable equilibrium
$\tau = 0$	$\tau_{max} = pE$	$\tau = 0$	$\tau = 0$	$\tau_{max} = MB$	$\tau = 0$
$W = 0$	$W = pE$	$W_{max} = 2pE$	$W = 0$	$W = MB$	$W_{max} = 2MB$
$U_{min} = -pE$	$U = 0$	$U_{max} = pE$	$U_{min} = -MB$	$U = 0$	$U_{max} = MB$

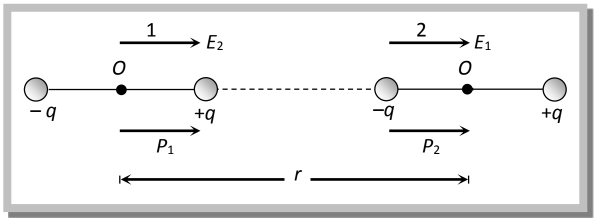
(v) **Angular SHM:** In a uniform electric/magnetic field (intensity E/B) if a dipole (electric/magnetic) is slightly displaced from its stable equilibrium position it executes angular SHM having period of oscillation. If I = moment of inertia of dipole about the axis passing through its center and perpendicular to its length.

For electric dipole: $T = 2\pi\sqrt{\frac{I}{pE}}$ and For Magnetic dipole: $T = 2\pi\sqrt{\frac{I}{MB}}$

(vi) **Dipole-point charge interaction:** If a point charge/isolated magnetic pole is placed in dipole field at a distance r from the midpoint of dipole then force experienced by point charge/pole varies according to the relation $F \propto \frac{1}{r^3}$

(vii) **Dipole-dipole interaction:** When two dipoles placed closed to each other, they experiences a force due to each other. If suppose two dipoles (1) and (2) are placed as shown in figure then

Both the dipoles are placed in the field of one another hence potential energy dipole (2) is



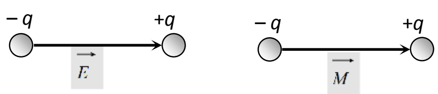
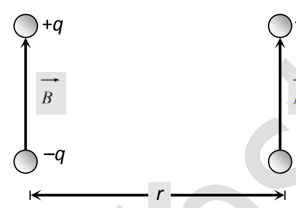
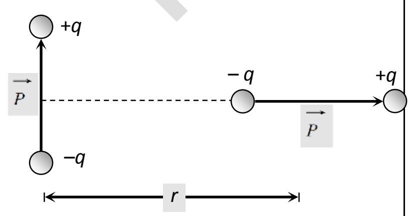
$$U_2 = -p_2E_1\cos 0 = -p_2E_1 = -p_2 \times \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p_1}{r^3}$$

Then by using $F = -\frac{dU}{dr}$, Force on dipole (2) is $F_2 = -\frac{dU_2}{dr}$

$$\Rightarrow \; F_2 = -\frac{d}{dr}\left\{\frac{1}{4\pi\varepsilon_0}\cdot\frac{2p_1p_2}{r^3}\right\} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{6p_1p_2}{r^4}$$

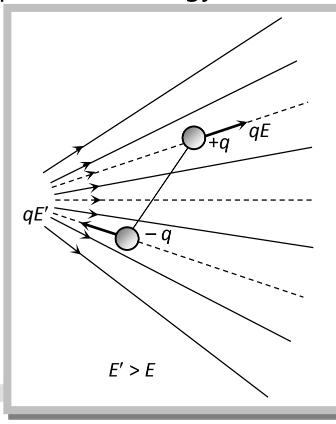
Similarly force experienced by dipole (1) $F_1 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{6p_1p_2}{r^4}$ so $F_1 = F_2 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{6p_1p_2}{r^4}$

Negative sign indicates that force is attractive. $|F| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{6p_1p_2}{r^4}$ and $\boldsymbol{F} \propto \frac{\boldsymbol{1}}{\boldsymbol{r^4}}$

S. No.	Relative position of dipole	Force	Potential energy
(i)		$\frac{1}{4\pi\varepsilon_0}\cdot\frac{6p_1p_2}{r^4}$ (attractive)	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2p_1p_2}{r^3}$
(ii)		$\frac{1}{4\pi\varepsilon_0}\cdot\frac{3p_1p_2}{r^4}$ (repulsive)	$\frac{1}{4\pi\varepsilon_0}\cdot\frac{p_1p_2}{r^3}$
(iii)		$\frac{1}{4\pi\varepsilon_0}\cdot\frac{3p_1p_2}{r^4}$ (perpendicular to r)	0

Note: Same result can also be obtained for magnetic dipole.

(4) **Electric dipole in non-uniform electric field:** When an electric dipole is placed in a non-uniform field, the two charges of dipole experiences unequal forces, therefore the net force on the dipole is not equal to zero. The magnitude of the force is given by the negative derivative of the potential energy w.r.t. distance along the axis of the dipole i.e. $\vec{F} = -\frac{dU}{dr} = -\vec{p}\cdot\frac{d\vec{E}}{dr}$.



Due to two unequal forces, a torque is produced which rotate the dipole so as to align it in the direction of field. When the dipole gets aligned with the field, the torque becomes zero and then the unbalanced force acts on the dipole and the dipole then moves linearly along the direction of field from weaker portion of the field to the stronger portion of the field. So in non-uniform electric field

(i) Motion of the dipole is translatory and rotatory

(ii) Torque on it may be zero.

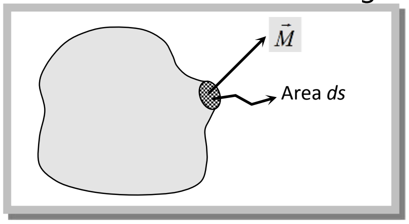
Concepts

☞ For a short dipole, electric field intensity at a point on the axial line is double than at a point on the equatorial line on electric dipole i.e. $E_{axial} = 2E_{equatorial}$

☞ It is interesting to note that dipole field $E \propto \frac{1}{r^3}$ decreases much rapidly as compared to the field of a point charge $\left(E \propto \frac{1}{r^2}\right)$.

23. Electric Flux.

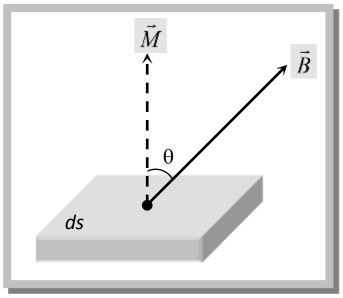
(1) **Area vector:** In many cases, it is convenient to treat area of a surface as a vector. The length of the vector represents the magnitude of the area and its direction is along the outward drawn normal to the area.



(2) **Electric flux:** The electric flux linked with any surface in an electric field is basically a measure of total number of lines of forces passing normally through the surface. **or**

Electric flux through an elementary area $\vec{ds}$ is defined as the scalar product of area of field i.e. $d\phi = \vec{E} \cdot \vec{ds} = E\,ds\,\cos\theta$

Hence flux from complete area (S) $\phi = \int E\,ds\cos\theta$ = ES cosθ



If θ = 0°, i.e. surface area is perpendicular to the electric field, so flux linked with it will be max.

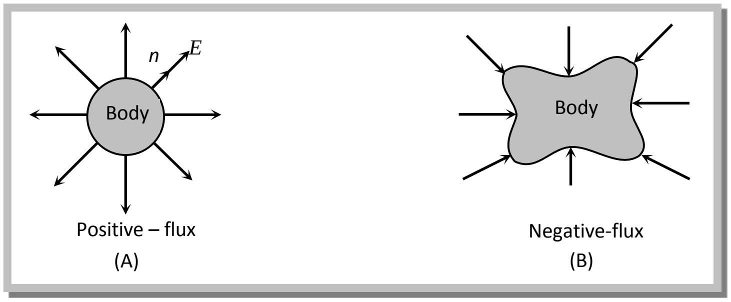
i.e. ϕ_{max} = E ds and if θ = 90°, ϕ_{min} = 0

(3) **Unit and Dimensional Formula**

S.I. unit – (volt × m) or $\frac{N-C}{m^2}$

It's Dimensional formula – $(ML^3T^{-3}A^{-1})$

(4) **Types:** For a closed body outward flux is taken to be positive, while inward flux is to be negative



24. Gauss's Law.

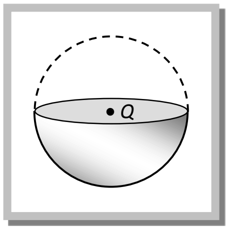
(1) **Definition:** According to this law, total electric flux through a closed surface enclosing a charge is $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed i.e. $\phi = \frac{1}{\varepsilon_0}(Q_{enc.})$

(2) **Gaussian Surface:** Gauss's law is valid for symmetrical charge distribution. Gauss's law is very helpful in calculating electric field in those cases where electric field is symmetrical around the source producing it. Electric field can be calculated very easily by the clever choice of a closed surface that encloses the source charges. Such a surface is called "Gaussian surface". This surface should pass through the point where electric field is to be calculated and must have a shape according to the symmetry of source.

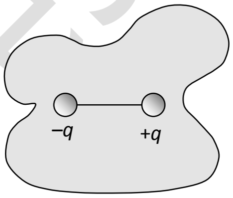
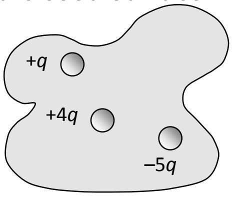
e.g. If suppose a charge Q is placed at the center of a hemisphere, then to calculate the flux through this body, to encloses the first charge we will have to imagine a Gaussian surface. This imaginary Gaussian surface will be a hemisphere as shown.

Net flux through this closed body $\phi = \frac{Q}{\varepsilon_0}$

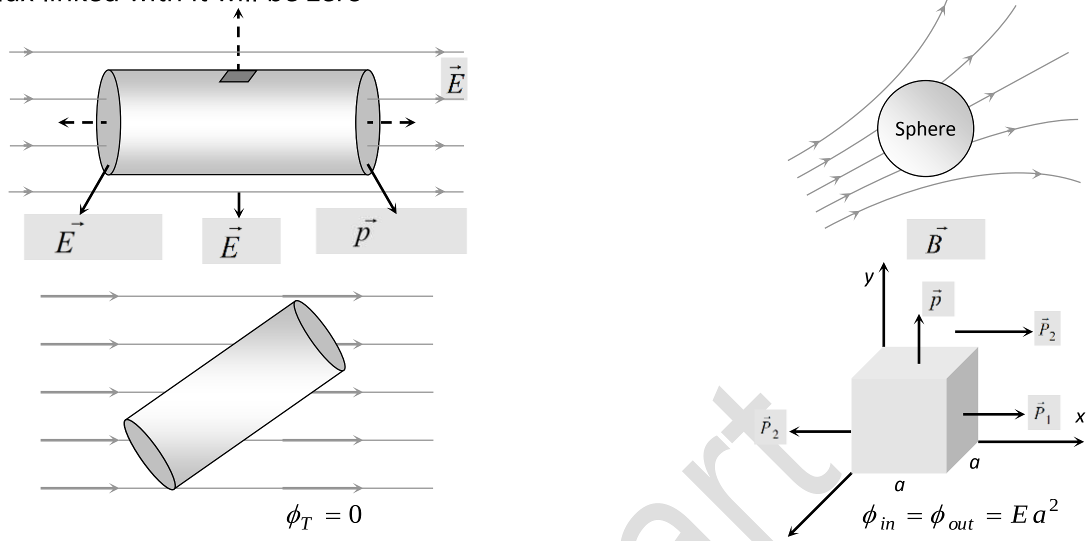
Hence flux coming out from given hemisphere is $\phi = \frac{Q}{2\varepsilon_0}$.



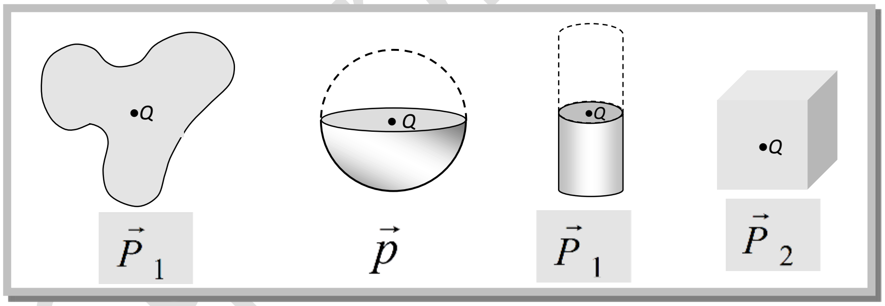
(3) **Zero flux:** The value of flux is zero in the following circumstances

(i) If a dipole is enclosed by a surface	(ii) If the magnitude of positive and negative charges are equal inside a closed surface
$\phi = 0; Q_{enc} = 0$ 	$Q_{enc} = 0$, so, $\phi = 0$ 

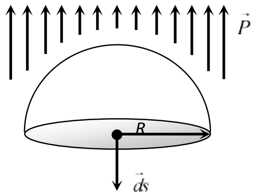
(iii) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non-uniform) total flux linked with it will be zero



(4) **Flux emergence:** Flux linked with a closed body is independent of the shape and size of the body and position of charge inside it

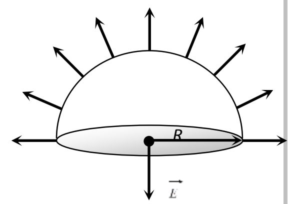


(i) If a hemispherical body is placed in uniform electric field then flux linked with the curved surface



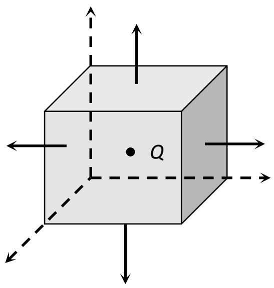
$\phi_{curved} = +\pi R^2 E$

(ii) If a hemispherical body is placed in non-uniform electric field as shown below. Then flux linked with the curved surface.



$\phi_{curved} = 2\pi R^2 E$

(v) If charge is kept at the center of cube

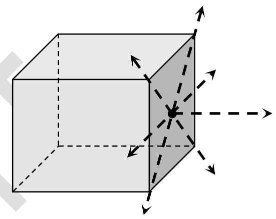


$\phi_{total} = \frac{1}{\varepsilon_0}.(Q)$

$\phi_{face} = \frac{Q}{6\varepsilon_0}$

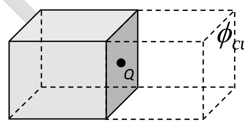
$\phi_{corner} = \frac{Q}{8\varepsilon_0}$ $\phi_{edge} = \frac{Q}{12\varepsilon_0}$

(iv) If charge is kept at the center of a face



First we should enclosed the charge by assuming a Gaussian surface (an identical imaginary cube)

$\phi_{total} = \frac{Q}{\varepsilon_0}$



$\phi_{cube} = \frac{Q}{2\varepsilon_0}$ (i.e. from 5 face only)

$\phi_{face} = \frac{1}{5}\left(\frac{Q}{2\varepsilon_0}\right) = \frac{Q}{10\varepsilon_0}$.

Concept

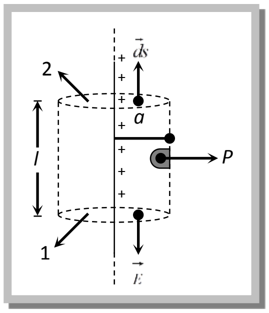
☞ In C.G.S. $\varepsilon_0 = \frac{1}{4\pi}$. Hence if 1C charge is enclosed by a closed surface so flux through the surface will be $\phi = 4\pi$.

25. Application of Gauss's Law.

Gauss's law is a powerful tool for calculating electric field in case of symmetrical charge distribution by choosing a Gaussian surface in such a way that $\vec{E}$ is either parallel or perpendicular to its various faces.

E.g. Electric field due to infinitely long line of charge: Let us consider a uniformly charged wire of infinite length having a constant linear charge density is $\lambda \left(\lambda = \frac{\text{charge}}{\text{length}} \right)$.

Let P be a point distant r from the wire at which the electric field is to be calculated.



Draw a cylinder (Gaussian surface) of radius r and length l around the line charge which encloses the charge Q ($Q = \lambda . l$). Cylindrical Gaussian surface has three surfaces; two circular and one curved for surfaces (1) and (2) angle between electric field and normal to the surface is 90° i.e., $\theta = 90^o$. So flux linked with these surfaces will be zero. Hence total flux will pass through curved surface and it is

$$\phi = \int E\, ds \cos\theta \quad \text{.... (i)}$$

According to Gauss's law

$$\phi = \frac{Q}{\varepsilon_0} \quad \text{.... (ii)}$$

Equating equation (i) and (ii) $\int E\, ds = \frac{Q}{\varepsilon_0}$

$$\Rightarrow \quad E\int ds = \frac{Q}{\varepsilon_0} \Rightarrow Ex2\pi rl = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{Q}{2\pi\varepsilon_0 rl} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r} \quad \left\{K = \frac{1}{4\pi\varepsilon_0}\right\}$$