UNDAMPED AND DAMPED OSCILLATIONS

- **Undamped oscillations :** When a system oscillates with a constant amplitude which does not change with time, its oscillations are called undamped oscillations.
- The energy of the system executing undamped oscillations remains constant and is independent of time.
- The dissipative forces (*i.e.*, frictional or viscous forces) are not present in the system executing undamped oscillations.
- **Damped oscillations :** When a system oscillates with a decreasing amplitude with time, its oscillations are called damped oscillations.
- The energy of the system executing damped oscillations will go on decreasing with time but the oscillations of the system remain periodic.
- The dissipative forces or damping forces are active in the oscillating system which are generally the frictional or viscous forces.
- The damping force is given by $F_d = -bv$, where, v is the velocity of the oscillator and b is damping constant. Negative sign shows that damping force acts opposite to the velocity at every moment.
- The SI unit of b is kg s^{-1}.
- The differential equation of damped harmonic oscillator is given by

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$$

- The displacement of the damped oscillator at any instant t is given by

$$x = Ae^{-bt/2m}\cos(\omega' t+\phi)$$

where ω' is the angular frequency of the damped oscillator is given by

$$\omega' = \sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$$

- If the damping constant b is small, then $\omega' \approx \omega$, where ω is the angular frequency of the undamped oscillator.
- The mechanical energy E of the damped oscillator at any instant t is given by

$$E=\frac{1}{2}kA^2e^{-bt/m}$$

- **Maintained oscillations :** Due to the damping forces, the amplitude of oscillator will go on decreasing with time. If we can feed the energy to the damped oscillatory system at the same rate at which it is dissipating the energy, then the amplitude of such oscillations would become constant. Such oscillations are called maintained oscillations.

FORCED OSCILLATIONS AND RESONANCE

- **Free oscillations :** When a system oscillates with its own natural frequency, without the help of any external periodic force, its oscillations are called free oscillations.
- **Forced or driven oscillations :** When a system oscillates with the help of an external periodic force, other than its own natural angular frequency, its oscillations are called forced or driven oscillations.
- The differential equation of forced damped harmonic oscillator is given by

$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=F_0\cos\omega_d t$$

where ω_d is the angular frequency of the external force.
- The displacement of the forced damped harmonic oscillator at any instant t is given by

$$x = A\cos(\omega_d t+\phi)$$

where $A = \dfrac{F_0}{\{m^2(\omega^2-\omega_d^2)^2+\omega_d^2b^2\}^{1/2}}$

$$= \frac{F_0}{m\left[(\omega^2-\omega_d^2)^2+\left(\frac{\omega_d b}{m}\right)^2\right]^{1/2}}$$

and $\tan\phi = \dfrac{-v_0}{\omega_d x_0}$

where ω is the natural angular frequency of the oscillator, x_0 and v_0 are the displacement and velocity of the oscillator at time $t = 0$, when the periodic force is applied.
- **Resonance :** It is the phenomenon in which a system is made to oscillate by external force whose frequency is equal to the natural frequency of the system. At resonance, the amplitude of the system is maximum. It is a special case of forced oscillation.

  Condition for resonance is $\omega = \omega_d$.

WAVE MOTION

- It is a kind of disturbance which travels through a material medium (having properties of elasticity and inertia) on account of repeated periodic vibrations of the particles of the medium about their mean position, the disturbance being handed on from one particle to the adjoining particle and so on, without any net transport of the medium. It transports both, energy and momentum without the transport of matter.

Types of Waves

- Waves can be classified into following three categories :
  - Mechanical waves
  - Electromagnetic waves
  - Matter waves
- **Mechanical waves :** These waves require a medium for their propagation. *e.g.*, water waves, sound waves, seismic waves, etc. Sometimes mechanical waves are also called elastic waves.
- **Electromagnetic waves :** These waves do not require any medium for their propagation. *e.g.*, light waves, X-rays, micro waves, etc. All electromagnetic waves travel through vacuum at the same speed c ($c = 3\times10^8$ m s^{-1})
- **Matter waves :** These waves are associated with moving particles of matter like electrons, protons, neutrons, atoms and molecules. These waves arise in quantum mechanical description of nature. These waves are also known as **de Broglie waves**.

TRANSVERSE AND LONGITUDINAL WAVES

- Mechanical waves are of two types :
  - Transverse waves
  - Longitudinal waves
- **Transverse waves :** Those waves in which the particles of the medium oscillate perpendicular to the direction of wave propagation are called transverse waves. *e.g.* vibrations of a string in violin, electromagnetic waves etc.
- **Longitudinal waves :** Those waves in which the particles of the medium oscillate along the direction of wave propagation are called longitudinal waves. *e.g.* ultrasonic waves in air produced by a vibrating quartz crystal, waves produced in a cylinder containing a liquid by moving its piston back and forth etc.
- A transverse wave travels through a medium in the form of crests and troughs.
- A longitudinal wave travels through a medium in the form of compressions and rarefactions.
- Transverse waves can be polarised whereas longitudinal waves cannot be polarised. Hence, transverse or longitudinal nature of a wave can be decided on the basis of polarisation.
- Transverse waves can propagate only in medium with shear modulus of elasticity such as solids and strings, but not in fluids.
- Longitudinal waves need bulk modulus of elasticity and therefore possible in all media, solids, liquids and gases.
- In the case of a vibrating tuning fork the waves in the prongs are transverse while in the stem are longitudinal.
- In case of seismic waves produced by earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds [$v_s = 5$ km s^{-1} while $v_p = 9$ km s^{-1}]. S-waves are transverse while P-waves are longitudinal.
- Some waves in nature are neither transverse nor longitudinal but a combination of the two. *e.g.* waves produced by a motorboat sailing in water is a combination of both longitudinal and transverse waves.
- Waves may be one dimensional, two dimensional or three dimensional according as they propagate energy in just one, two or three dimensions. Thus, transverse waves along a string or longitudinal waves along a spring are one dimensional, surface waves or ripples on water are two dimensional and sound waves proceeding radially from a point source are three dimensional.

VARIOUS TERMS RELATED TO WAVE MOTION

- **Amplitude:** It is defined as the maximum displacement of an oscillating particle of the medium from the mean position. It is denoted by symbol A.
- **Wavelength :** It is defined as the distance travelled by the wave during the time, the particle of the medium completes one oscillation about its mean position. It may also be defined as the distance between two consecutive points in the same phase of wave motion. It is denoted by symbol λ.
- In case of **transverse wave**
  λ = distance between two consecutive crests or troughs.
- In case of **longitudinal waves**
  λ = distance between two consecutive compressions or rarefactions.
- **Time period :** It is defined as the time taken by a particle to complete one oscillation about its mean position. It is denoted by symbol T.
- **Frequency:** It is defined as the number of oscillations made by the particle in one second. It is denoted by symbol υ.

$$\upsilon = \frac{1}{T}$$

- **Wave speed or speed of a wave :** It is defined as the distance travelled by the wave in one second. It is denoted by symbol v and is given by

$$v = \upsilon\lambda \qquad \text{....(i)}$$

As the speed of a wave is related to its wavelength and frequency by the equation (i) but it is determined by the properties of the medium.

- **Intensity of a wave :** It is defined as the amount of energy flow per unit area per unit time in a direction perpendicular to the propagation of wave. It is denoted by the symbol I and is given by

$$I = 2\pi^2\upsilon^2A^2\rho v$$

where υ is the frequency, A is the amplitude, v is the velocity of the wave, ρ is the density of the medium.

- The SI unit of intensity is W m^{-2}.
- Dimensional formula of intensity of a wave is $[ML^0T^{-3}]$.
- **Energy density :** It is defined as amount of energy flow per unit volume. It is denoted by symbol u and is given by

$$u = 2\pi^2A^2\upsilon^2\rho$$

where υ is the frequency, A is the amplitude and ρ is the density of the medium.

- The SI unit of energy density is J m^{-3}.
- Dimensional formula of energy density is $[ML^{-1}T^{-2}]$.

EQUATION OF PLANE PROGRESSIVE WAVE

- Equation of plane progressive wave travelling along the +ve direction of x-axis is given by

$$y = A\sin(\omega t - kx + \phi)$$

where y = displacement of a particle at (x, t)

A = amplitude of the wave

ω = angular frequency = $2\pi\upsilon = 2\pi\frac{1}{T}$

k = propagation constant or angular wave number = $\frac{2\pi}{\lambda}$

ϕ = phase constant or initial phase of the wave.

- Phase of the wave is the argument $(\omega t - kx + \phi)$ of the oscillatory term $\sin(\omega t - kx + \phi)$.
- Wave speed, $v = \frac{\omega}{k}$
- It depends only on the nature of the medium in which the wave propagates.
- **Slope of the wave,** $\frac{dy}{dx} = -kA\cos(\omega t - kx + \phi)$
- **Particle speed,**

$$v_{particle} = \frac{dy}{dt} = \omega A\cos(\omega t - kx + \phi)$$

$$= -\left(\frac{\omega}{k}\right)\frac{dy}{dx}$$

or $v_{particle}$ = –wave speed × slope of the wave

- **Particle acceleration,** $a = \frac{d^2y}{dt^2} = -\omega^2 y$
- Equation of plane progressive wave travelling along negative direction of x-axis is given by

$$y = A\sin(\omega t + kx + \phi)$$

- A plane progressive wave can be written in many forms such as

$$y = A\sin(\omega t - kx) \quad ...(i)$$

or $y = A\sin 2\pi[\upsilon t - \frac{x}{\lambda}]$ ...(ii)

[As $\omega = 2\pi\upsilon$ and $k = (2\pi/\lambda)$]

or $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$ $\left(\because \upsilon = \frac{1}{T}\right)$ ...(iii)

or $y = A\sin\frac{2\pi}{\lambda}(vt - x)$ $\left(\because \frac{\lambda}{T} = v\right)$ ...(iv)

or $y = A\sin k(vt - x)$ $\left(\because k = \frac{2\pi}{\lambda}\right)$ ...(v)

or $y = A\sin\frac{2\pi}{T}\left(t - \frac{x}{v}\right) = A\sin\omega\left(t - \frac{x}{v}\right)$ ...(vi)

- The differential equation of one dimensional progressive wave is given by

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$$

Relationship between Phase Difference, Path Difference and Time Difference

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$\text{Phase difference} = \frac{2\pi}{T} \times \text{time difference}$$

- A path difference of λ corresponds to a phase difference of 2π radian.
- To calculate phase difference between two waves, the equation of both waves must be in sine form or in cosine form.

DISPLACEMENT WAVE AND PRESSURE WAVE

- A longitudinal wave in a fluid (liquid or gas) can be described either in terms of the longitudinal displacement suffered by the particles of the medium (called displacement wave) or in terms of the excess pressure generated due to the compression and rarefaction (called pressure wave).

Equation of displacement wave

$$s = s_m \sin(\omega t - kx)$$

Equation of pressure wave,

$$P = P_m \cos(\omega t - kx)$$

where P_m = pressure amplitude

Since pressure wave is 90° out of phase with respect to the displacement wave. Therefore displacement is minimum when pressure is maximum or vice versa.

Illustration 14

The displacement of a particle of a string carrying a travelling wave is given by $y = (4 \text{ cm}) \sin 2\pi(0.5x - 100t)$, where x is in cm and t is in seconds. The speed of the wave is

(a) 50 cm/s (b) 100 cm/s
(c) 200 cm/s (d) 250 cm/s

Soln. (c) : Comparing the equation with the standard equation of a wave:

$y = A\sin(kx - \omega t)$

Here $k = 2\pi(0.5)$ and $\omega = 2\pi(100)$

$$\text{Speed of wave} = \frac{\omega}{k} \Rightarrow c = \frac{2\pi(100)}{2\pi(0.5)} = 200 \text{ cm/s}$$

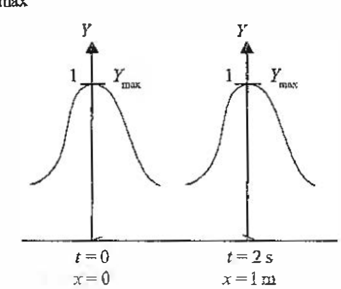
Illustration 15

The amplitude of wave disturbance propogating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time $t = 0$ and by $y = \frac{1}{1+(x-1)^2}$ at $t = 2$ s. where x and y are in metre. The shape of the wave does not change during the propagation. Find the velocity of the wave.

Soln.: At $t = 0$, $y = \frac{1}{(1+x)^2}$

For y to be maximum, $x = 0$,

$\therefore y_{max} = 1$



$y = \frac{1}{1+(x-1)^2}$ at $t = 2$ s

or $y_{max} = 1$ when $x = 1$.

It means that in 2 sec, the wave has moved a distance of 1 m.

$\therefore$ Velocity $= \dfrac{\text{distance}}{\text{time}} = \dfrac{1}{2} = 0.5\ \text{m s}^{-1}$

Illustration 16

The displacement of a particle varies according to the relation $x = 4(\cos\pi t + \sin\pi t)$. The amplitude of the particle is

(a) -4 (b) 4

(c) $4\sqrt{2}$ (d) 8.

Soln. (c) : $x = 4(\cos\pi t + \sin\pi t)$

$$= 4\times\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\pi t + \frac{1}{\sqrt{2}}\sin\pi t\right]$$

or $$x = 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos\pi t + \cos\frac{\pi}{4}\sin\pi t\right]$$

$$= 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$$

Hence amplitude $= 4\sqrt{2}$.

Illustration 17

The displacement of a particle executing simple harmonic motion is given by: $y = 10\sin\left(6t + \frac{\pi}{3}\right)$. Here, y is in metre and t is in second. The initial displacement and velocity of the particle are respectively

(a) $5\sqrt{3}$ m and 30 m s^{-1} (b) $20\sqrt{3}$ m and 30 m s^{-1}

(c) $15\sqrt{3}$ m and 30 m s^{-1} (d) 15 m and $5\sqrt{3}$ m s^{-1}

Soln. (a) : At $t = 0$, $y = 10\sin\left(6(0) + \frac{\pi}{3}\right)$

$$= 10\times\frac{\sqrt{3}}{2} = 5\sqrt{3}\ \text{m};\ v = \frac{dy}{dt} = 60\cos\left(6t + \frac{\pi}{3}\right)$$

At $t = 0$, $v = 60\cos\left(0 + \frac{\pi}{3}\right) = 60\times\frac{1}{2} = 30$ m/s.

Illustration 18

A transverse wave is described by the equation $y = y_0\sin 2\pi\left(\upsilon - \frac{x}{\lambda}\right)$. Find the value of λ so that the maximum particle velocity is equal to four times the wave velocity.

Soln.: Wave velocity, $v = \dfrac{\text{Coefficient of } t}{\text{Coefficient of } x}$

$\therefore\ v = \dfrac{2\pi\upsilon}{2\pi/\lambda} = \lambda\upsilon$

Maximum particle velocity, $V = \omega a = 2\pi\upsilon y_0$ $[\because a = y_0]$

Now, as per question,

$V = 4v$ or $2\pi\upsilon y_0 = \lambda\upsilon\times 4$

or $\lambda = \dfrac{\pi y_0}{2}$

SPEED OF TRANSVERSE WAVE

- Speed of a transverse wave on a stretched string is given by

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string, μ is the mass per unit length of the string called **linear mass density.**

- Speed of a transverse wave in a solid is given by

$$v = \sqrt{\frac{\eta}{\rho}}$$

where η is the modulus of rigidity, ρ is the density of a solid.

SPEED OF LONGITUDINAL WAVE

- Speed of a longitudinal wave in a medium is given by

$$v = \sqrt{\frac{E}{\rho}}$$

where E is the modulus of elasticity and ρ is the density of the medium.

- Speed of a longitudinal wave in a solid bar is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y is the Young's modulus and ρ is the density of material of the bar.

- Speed of a longitudinal wave in a fluid is given by

$$v = \sqrt{\frac{B}{\rho}}$$

where B is the bulk modulus and ρ is density of a fluid.

ULTRASONIC AND INFRASONIC WAVES

- **Ultrasonic waves :** The human ear can hear the sound waves between 20 Hz to 20 kHz. This range is called audible range. The sound waves having frequencies above the audible range are called ultrasonic waves.
- **Infrasonic waves :** The sound waves which have frequencies less than the audible range are called infrasonic waves.

NEWTON'S FORMULA

- Newton assumed that propagation of sound wave in gas is an isothermal process. Therefore, according to Newton, speed of sound in gas is given by

$$v = \sqrt{\frac{P}{\rho}}$$

where P is the pressure of the gas and ρ is the density of the gas.

- According to the Newton's formula, the speed of sound in air at STP is 280 m s^{-1}. But the experimental value of the speed of sound in air at STP is 332 m s^{-1}. Newton could not explain this large difference. Newton's formula was corrected by Laplace.

LAPLACE'S CORRECTION

- Laplace assumed that propagation of sound wave in gas is an adiabatic process. Therefore, according to Laplace, speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{m}}$$

- According to Laplace's correction the speed of sound in air at STP is 331.3 m s^{-1}. This value agrees fairly well with the experimental values of the speed of sound in air at STP.

FACTORS AFFECTING SPEED OF SOUND

- Factors affecting speed of sound in a gaseous medium as are follows :
  - **Effect of temperature :** Speed of sound in a gas is directly proportional to the square root of its absolute temperature. *i.e.,* $v \propto \sqrt{T}$.

    Speed of sound in a gas at t°C,

    $$v_t = v_0\left[1+\frac{t}{546}\right]$$

    where v_0 is the speed of sound in the gas at 0°C.

    For air $v_0 = 332\text{ m s}^{-1}$

    $\therefore\ v_t = v_0 + 0.61t\text{ m s}^{-1}$

    Speed of sound in air increases by 0.61 m s^{-1} for every 1°C rise in temperature.
  - **Effect of pressure :** The speed of sound in a gas is given by

    $$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

    Speed of sound in gas is independent of the pressure of the gas, provided temperature remains constant.
  - **Effect of humidity :** With increase in humidity, density of air decreases

    $$v = \sqrt{\frac{\gamma P}{\rho}}$$

    Therefore, with rise in humidity, speed of sound increases. The speed of sound in moist air is more than the speed of sound in dry air, since the density of moist air is less than that of dry air.
  - **Effect of wind :** If the wind is blowing, the speed of sound changes. The speed of sound is increased if the wind is blowing in the direction of the propagation of sound wave. But if the wind is blowing opposite to the direction of propagation, the speed of sound is decreased.

Illustration 19

A uniform rope of length 10 m and mass 3 kg hangs vertically from a rigid support. A block of mass 1 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.05 m is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is

(a) 0.10 m (b) 0.12 m
(c) 0.16 m (d) 0.18 m

Soln. (a) : Since the rope has a mass, the tension along its length is variable. At the top end its tension is (3 + 1)g N or 4g N. At the bottom end, its tension is 1g N. Now, speed of the transverse wave on the string,

$$c = \sqrt{\frac{T}{\mu}}$$

$$\upsilon\lambda = \sqrt{\frac{T}{\mu}} \qquad (\because\ c = \upsilon\lambda)$$

As frequency υ and μ, the linear density are constant, so $\frac{\sqrt{T}}{\lambda}$ is a constant.

$$\frac{\sqrt{T_{Top}}}{\lambda_{Top}} = \frac{\sqrt{T_{Bottom}}}{\lambda_{Bottom}};\ \frac{\sqrt{4g}}{\lambda_{top}} = \frac{\sqrt{1g}}{0.05\text{ m}}$$

$\Rightarrow \lambda_{top} = 0.05 \times 2\text{ m} = 0.1\text{ m}.$

Illustration 20

If the speed of sound in oxygen is 470 m/s, then the speed of sound in hydrogen at the same temperature is

(a) 1540 m/s (b) 1670 m/s
(c) 1880 m/s (d) 2050 m/s

Soln. (c): As both oxygen and hydrogen molecules are diatomic, the adiabatic ratio is same for both. Now, the speed of sound in gases

$$v = \sqrt{\frac{\gamma RT}{M}};\ \frac{v_H}{v_O} = \sqrt{\frac{M_{O_2}}{M_{H_2}}}$$

$$\frac{v_H}{470\text{ m/s}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4$$

$\Rightarrow v_H = 4 \times 470\text{ m/s} = 1880\text{ m/s}.$

Illustration 21

The extension in a wire, obeying Hooke's law, is x. The speed of sound in the stretched wire is v. If the extension in the wire is increased to $1.5x$, what will be the new speed of sound in the wire?

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{kx}{\mu}}$ where k = constant

$$\therefore\ \frac{v_2}{v_1} = \sqrt{\frac{1.5x \times k}{m} \times \frac{m}{x \times k}}$$

$$\therefore\ \frac{v_2}{v_1} = \sqrt{1.5} = 1.22 \text{ or } v_2 = 1.22v_1 = 1.22v$$

or new speed of sound = $1.22v$.

Illustration 22

The speed of longitudinal wave is 100 times the speed of transverse wave in a brass wire. What is the stress in wire? Young's modulus of brass = $1.0 \times 10^{11}\text{ N m}^{-2}$.

Soln.: Speed of longitudinal wave in wire $v_1 = \sqrt{\frac{Y}{d}}$

Speed of transverse wave in wire $v_2 = \sqrt{\frac{T}{\mu}}$

Given: $v_1 = 100\ v_2$

$$\therefore\ \sqrt{\frac{Y}{d}} = 100\sqrt{\frac{T}{\mu}} \text{ or } \sqrt{\frac{Y}{d}} = 100\sqrt{\frac{T}{Ad}}$$

$$\left[\because \mu = \frac{\text{volume} \times \text{density}}{\text{length}} = Ad\right]$$

or $Y = (100)^2 \times \frac{T}{A} = (100)^2 \times \text{stress}$

$$\left[\because \frac{T}{A} = \text{stress}\right]$$

or $\text{Stress} = \frac{Y}{(100)^2} = \frac{1.0 \times 10^{11}}{10^4} = 1.0 \times 10^7 \text{ N m}^{-2}$

$\therefore$ Stress = 1.0×10^7 N m^{-2}.

Illustration 23

If the bulk modulus of water is 2100 MPa what is the speed of sound in water?

Soln. : $v = \sqrt{\frac{B}{\rho}}$

$\therefore$ $v = \sqrt{\frac{2100 \times 10^6}{1000}} = \sqrt{2.1 \times 10^6}$

or $v = 1.45 \times 10^3 = 1450$ m/s.

REFLECTION OF WAVES

- The reflection of waves at a boundary or interface between two media occurs as follows :
- A travelling wave, at a rigid boundary or a closed end is reflected with a phase reversal of π but the reflection at an open boundary takes place without any phase change.
- Let the incident wave be represented by
  $y_i = A\sin(\omega t - kx)$
  For reflection at a rigid boundary, the reflected wave is represented by
  $y_r = A\sin(\omega t + kx + \pi) = -A\sin(\omega t + kx)$
  For reflection at an open boundary, the reflected wave is represented by
  $y_r = A\sin(\omega t + kx)$
- An echo can be cited as an example of reflection of sound from a distant object such as hill or cliff. If there is a sound reflector at a distance d from the source, the time interval between original sound and its echo at the site of source will be
  $$t = \frac{d}{v} + \frac{d}{v} = \frac{2d}{v}.$$
  Now as persistence of ear is (1/10) s, echo of a sharp or momentary sound (such as clap) will be heard if
  $$t > \frac{1}{10} \text{ or } \frac{2d}{v} > \frac{1}{10}, \text{ i.e., } d > \frac{v}{20}$$
- If a person standing between two parallel hills fires a gun and hears the first echo after t_1 s, the second echo after t_2 s, and v is the velocity of sound, then the distance between the two hills is given by
  $$s_1 + s_2 = (vt_1/2) + (vt_2/2) = [v(t_1 + t_2)/2]$$

PRINCIPLE OF SUPERPOSITION OF WAVES

- It states that when two or more waves travel in a medium in such a way that each wave represents its separate motion individually, then the resultant displacement of particle of the medium at any time is equal to the vector sum of the individual displacements. If $y_1, y_2, y_3, \ldots, y_n$ are the displacements at a point due to the n waves, then the resultant displacement at that point is given by
  $$y = y_1 + y_2 + y_3 + \ldots + y_n$$
- **The superposition of waves give rise to following three phenomena :**
  - Interference
  - Stationary waves
  - Beats

INTERFERENCE OF WAVES

- When two waves of same frequency and wavelength having constant phase difference travelling with same speed in the same direction superpose on each other, they give rise to an effect called interference of waves.
- **Condition for constructive interference**
  Phase difference = $2n\pi$ where $n = 0, 1, 2, \ldots$
  $$\text{Path difference} = \frac{\lambda}{2\pi} \times \text{phase difference} = n\lambda$$
  where $n = 0, 1, 2, \ldots$
- **Condition for destructive interference**
  Phase difference = $(2n + 1)\pi$ where $n = 0, 1, 2, \ldots$
  $$\text{Path difference} = \frac{\lambda}{2\pi} \times \text{Phase difference} = \left(n + \frac{1}{2}\right)\lambda$$
  where $n = 0, 1, 2, \ldots$
- The phenomenon of interference is based on conservation of energy.

STATIONARY WAVES

- When two waves of same frequency, wavelength and amplitude travel in opposite directions at the same speed, their superposition gives rise to a new type of waves called stationary waves or standing waves. Energy does not propagate in this type of wave hence, it is named as stationary wave.

Stationary Waves are of Two Types

  - Longitudinal stationary waves
  - Transverse stationary waves
- **Longitudinal stationary waves :** It is produced in
  - Organ pipe
  - Resonance tube
- **Transverse stationary waves :** It is produced in
  - Stretched string
  - Sonometer
- Equation of a stationary wave is given by
  $$y = 2A\cos(kx)\sin(\omega t)$$
- Stationary waves are characterised by nodes and antinodes.
- Nodes are the points for which the amplitude is minimum whereas antinodes are the points for which the amplitude is maximum.
- In a stationary wave nodes and antinodes are formed alternately and distance between them is $\lambda/4$.

- At antinodes, displacement and velocity is maximum.
- At nodes, displacement and velocity is zero.
- Distance between two consecutive nodes or antinodes is $\lambda/2$. Distance between a node and adjoining antinode is $\lambda/4$.

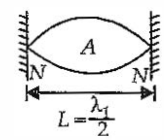
VIBRATIONS IN A STRETCHED STRING OF LENGTH L FIXED AT BOTH ENDS

- Speed of transverse waves in a stretched string is given by

$$v=\sqrt{\frac{T}{\mu}}$$

where T is the tension of the string, μ is the mass per unit length of the string.

- Fundamental mode or first mode,



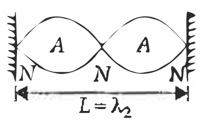
$\lambda_1=2L$

Fundamental frequency

$$\upsilon_1=\frac{v}{\lambda_1}=\frac{v}{2L}=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

This frequency is called **first harmonic.**

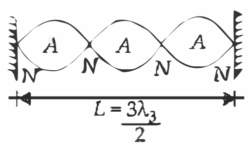
- Second mode, $\lambda_2=L$



Frequency $\upsilon_2=\frac{v}{\lambda_2}=\frac{v}{L}=2\upsilon_1$

This frequency is called **second harmonic or first overtone.**

- Third mode, $\lambda_3=\frac{2L}{3}$



Frequency $\upsilon_3=\frac{v}{\lambda_3}=\frac{3v}{2L}=3\upsilon_1$

This frequency is called **third harmonic or second overtone.**

- For n^{th} mode, $\lambda_n=\frac{2L}{n}$
- Frequency of n^{th} mode,

$$\upsilon_n=\frac{v}{\lambda_n}=\frac{nv}{2L}=n\upsilon_1=\frac{n}{2L}\sqrt{\frac{T}{\mu}}\text{ where } n=1,2,3,....$$

This frequency is called **n^{th} harmonic or $(n-1)^{th}$ overtone.**

Note : In general, $\upsilon_p=\frac{p}{2L}\sqrt{\frac{T}{\mu}}$,

where p = number of loops.

- **Laws of vibrating stretched string :**

The fundamental frequency of a stretched string is given by

$$\upsilon=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

  - Law of length, $\upsilon\propto\frac{1}{L}$

    when T and μ are constants.
  - Law of tension, $\upsilon\propto\sqrt{T}$

    when L and μ are constants.
  - Law of mass, $\upsilon\propto\frac{1}{\sqrt{\mu}}$

    when L and T are constants.

Note : If ρ is the density of the material of the string and D is the diameter of string, then mass per unit length, $\mu=\frac{\pi D^2\rho}{4}$

$$\therefore\quad \upsilon=\frac{1}{2L}\sqrt{\frac{4T}{\pi D^2\rho}}=\frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}.$$

- Laws of vibration of stretched string can be verified experimentally by using a sonometer.

Illustration 24

The vibration of a string fixed at both ends are given by the following equation:

$y=(5\text{ mm})\sin((1.57\text{ cm}^{-1})x)\cdot\cos[(314\text{ s}^{-1})t]$.

If the length of the string is 10 cm, the number of loops formed in the vibration are

(a) 2 (b) 3
(c) 4 (d) 5

Soln. (d) : This is an equation of a stationary wave. The coefficient of x represents k.

$$\Rightarrow k=\frac{2\pi}{\lambda}=1.57\text{ or }\lambda=\frac{2(3.14)}{1.57}=4\text{ cm}$$

Since each loop in a stationary wave exists between consecutive nodes, the length of the loop is $\frac{\lambda}{2}$ or 2 cm.

$$l=n\left(\frac{\lambda}{2}\right)\text{ or } n=\frac{2l}{\lambda}=\frac{2(10\text{ cm})}{4\text{ cm}}=5.$$

Illustration 25

A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M, the wire resonates, with the same tuning fork forming three antinodes for the same positions of the bridges. Determine the value of M.

Soln. : In the first case, five antinodes are formed between the two bridges. It corresponds to fifth harmonic. No of loops, $p=5$.

$$\therefore\text{ frequency }\upsilon=\frac{p}{2L}\sqrt{\frac{T}{m}}\text{ or }\upsilon=\frac{5}{2L}\sqrt{\frac{9g}{m}}$$

In second case, the frequency corresponds to third harmonic No. of loops, $p=3$.

$$\therefore\text{ frequency}n=\frac{3}{2L}\sqrt{\frac{Mg}{m}}$$

$$\therefore\ \frac{5}{2L}\sqrt{\frac{9g}{m}}=\frac{3}{2L}\sqrt{\frac{Mg}{m}}\text{ or }5\sqrt{9}=3\sqrt{M}$$

or $25\times9=9M$ or $M=25$ kg.

Illustration 26

The equation of a stationary wave is given by

$y = 6 \sin \frac{2\pi x}{3} \cos 40\pi t$, where y and x are in cm and time t is in s. In respect of the component progressive waves, calculate

(a) amplitude
(b) wavelength
(c) frequency

Soln.: The standard equation of a stationary wave is

$$y = 2a \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} t.$$

(a) $2a = 6 \quad \Rightarrow a = 3$ cm

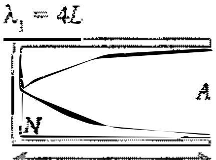
(b) $\frac{2\pi}{\lambda} = \frac{2\pi}{3} \Rightarrow \lambda = 3$ cm

(c) $\frac{2\pi}{T} = 2\pi\upsilon = 40\pi \Rightarrow \quad \upsilon = 20$ Hz.

$\therefore \quad \upsilon = 20$ Hz.

CLOSED ORGAN PIPE

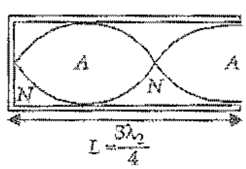
- In a closed organ pipe, one end is closed and other end is open.
- In a closed organ pipe, the closed end is always a node while the open end is always an antinode.
- Fundamental mode, or first mode $\lambda_1 = 4L$ where L is the length of the pipe.



  Fundamental frequency,

  $$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

  where v is the speed of sound in air.
  This frequency is called **first harmonic.**
- Second mode,

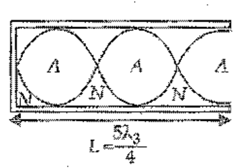


  $$\lambda_2 = \frac{4L}{3}$$

  Frequency,

  $$\upsilon_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3\upsilon_1$$

  This frequency is called **third harmonic or 1st overtone.**
- Third mode, $\lambda_3 = \frac{4L}{5}$



  Frequency, $\upsilon_3 = \frac{v}{\lambda_3} = \frac{5v}{4L} = 5\upsilon_1$

  This frequency is called **fifth harmonic or second overtone.**

  $\upsilon_1 : \upsilon_2 : \upsilon_3 = 1 : 3 : 5$

  Only odd harmonics are present.
- For n^{th} mode, $\lambda_n = \frac{4L}{(2n-1)}$

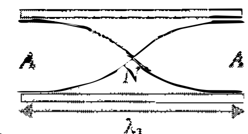
  Frequency, $\upsilon_n = \frac{v}{\lambda_n} = \frac{v(2n-1)}{4L} = (2n-1)\upsilon_1$

  where $n = 1, 2, 3, ...$

  This frequency is called **$(2n-1)^{th}$ harmonic or $(n-1)^{th}$ overtone.**

OPEN ORGAN PIPE

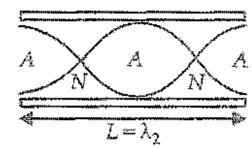
- In an open organ pipe, both ends are open.
- In an open organ pipe, at both ends there will be antinodes.
- Fundamental mode or first mode, $\lambda_1 = 2L$ where L is the length of the pipe.



  Fundamental frequency,

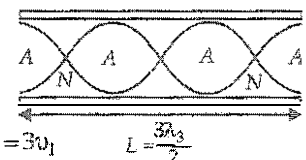
  $$\upsilon_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

  where v is the speed of sound in air.
  This frequency is called **first harmonic.**
- Second mode, $\lambda_2 = L$



  Frequency, $\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\upsilon_1$

  This frequency is called **second harmonic or first overtone.**
- Third mode, $\lambda_3 = \frac{2L}{3}$



  Frequency, $\upsilon_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3\upsilon_1$

  This frequency is called **third harmonic or second overtone.**

  $\upsilon_1 : \upsilon_2 : \upsilon_3 = 1 : 2 : 3$

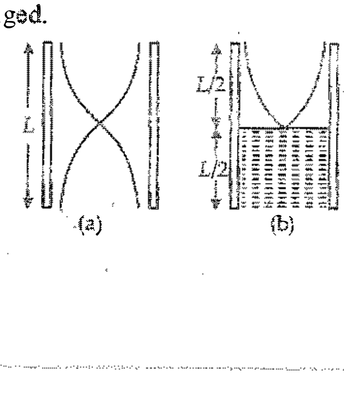
  Hence in open organ pipe all harmonics are present, whereas in a closed organ pipe only odd harmonics are present.
- For n^{th} mode, $\lambda_n = \frac{2L}{n}$

  Frequency, $\upsilon_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = n\upsilon_1$, where $\upsilon = 1, 2, ...$

  This frequency is called **n^{th} harmonic or $(n-1)^{th}$ overtone.**
- The fundamental frequency of an open organ pipe is twice that of a closed organ pipe of the same length.
- If an open pipe of length L is half submerged in water, it will become a closed pipe of length half that of open pipe as shown in figures (a) and (b). So its frequency will become

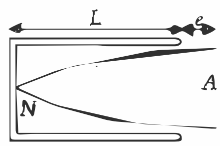
  $$\upsilon_C = \frac{\upsilon}{4(L/2)} = \frac{\upsilon}{2L} = \upsilon_O.$$

  i.e., equal to that of open pipe, *i.e.*, frequency will remain unchanged.



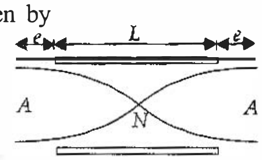
END CORRECTION

- The antinode at the open end of a pipe is not formed exactly at the open end but a little outside. This is called the end correction.
  This is denoted by e and is given by $e = 0.6r$.
  where r is the radius of the pipe. If L is the length of pipe then for closed organ pipe L is replaced by $L + e$ while for open organ pipe L is replaced by $L + 2e$.
- Due to the end correction the fundamental frequency of a closed organ pipe is given by



$$\upsilon_C = \frac{v}{4[L+e]} = \frac{v}{4[L+0.6r]}$$

- Due to the end correction, the fundamental frequency of an open organ pipe is given by



$$\upsilon_O = \frac{v}{2[L+2e]} = \frac{v}{2[L+1.2r]}$$

Speed of Sound in Air by Resonance Tube

- Speed of sound in air at room temperature by using resonance tube is given by
  $v = 2\upsilon(L_2 - L_1)$
  where,
  υ = frequency of the tuning fork
  L_1 = first resonance length
  L_2 = second resonance length
  End correction, $e = \frac{L_2 - 3L_1}{2}$

Illustration 27

An open organ pipe has a length of 5 cm. The highest harmonic of such a tube that is in the audible range (20 - 20000 Hz) is (speed of sound in air is 340 m/s)

(a) 4 (b) 5
(c) 6 (d) 7

Soln.: (b) : For an open pipe,

$$\upsilon = \frac{n \cdot c}{2L} = \frac{n \cdot (340 \text{ m/s})}{2 \times (5 \times 10^{-2} \text{ m})} = n \cdot (3400 \text{ Hz})$$

$$3400 \cdot n < 20000; \therefore n < \left(\frac{20000}{3400}\right); n < 5.9$$

$\Rightarrow$ The greater integral value of $n = 5$.

Illustration 28

An open pipe is suddenly closed at one end with the result that the frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. Determine the fundamental frequency of the open pipe.

Soln.: Let the length of the organ pipe = l

When open, fundamental frequency $(\upsilon_1) = v/2l$

When closed, frequency of third harmonic,

$$\upsilon_2 = 3\left(\frac{v}{4l}\right)$$

Given that, $\upsilon_2 = \upsilon_1 + 100$

$$\therefore \frac{3v}{4l} = \frac{v}{2l} + 100 \quad \text{or} \quad v = 4l \times 100$$

$$\therefore \upsilon_1 = \frac{v}{2l} = \frac{4l \times 100}{2l} = 200 \text{ Hz}$$

$\therefore$ Fundamental frequency of open pipe = 200 Hz.

Illustration 29

A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. Find the length of the open organ pipe.

Soln.: Frequency of first overtone in closed pipe $= 3\left(\frac{v_c}{4L}\right)$

Let length of open pipe = l_0

$\therefore$ Frequency of first overtone in open pipe $= 2\left(\frac{v_0}{2l_0}\right)$

The two frequencies are given to be equal.

$$\therefore \frac{3v_c}{4L} = \frac{2v_o}{2l_o} \quad \text{or} \quad l_o = \frac{4}{3}\left(\frac{v_o}{v_c}\right)L$$

Now, velocity of sound in a gas $v = \sqrt{\frac{K}{\rho}}$

$$\therefore \frac{v_o}{v_c} = \sqrt{\frac{\rho_c}{\rho_o}}$$

$$\therefore l_o = \frac{4L}{3}\sqrt{\frac{\rho_c}{\rho_o}} = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

$$\therefore \text{Length of open organ pipe} = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}.$$

BEATS

- When two waves of nearly equal (but not exactly equal) frequencies travelling with same speed in the same direction superpose on each other, they give rise to beats.
- **Beat frequency :** It is defined as number of beats heard per second.
  Beat frequency = no. of beats/second = $(\upsilon_1 \sim \upsilon_2)$
  = difference in frequencies.
- Phenomenon of beats can be used to determine the frequency of a tuning fork as follows:
  - If υ_A is the known frequency of tuning fork A and υ_B is the unknown frequency of tuning fork B. When both tuning forks A and B are sounded together, and produced beats of frequency υ, then $\upsilon_B = \upsilon_A \pm \upsilon$. Here, the $\pm$ sign of υ is decided either by loading or by filing any one of the tuning forks. If the tuning fork B is loaded with wax, and sounded with the tuning fork A, if the beat frequency increases, then $\upsilon_B = \upsilon_A - \upsilon$. If beat frequency decreases, then $\upsilon_B = \upsilon_A + \upsilon$. If instead of loading the tuning fork B with wax tuning fork B is filed and sounded with the tuning fork A, if the beat frequency increases,

then $\upsilon_B = \upsilon_A + \upsilon$. If the beat frequency decreases, then $\upsilon_B = \upsilon_A - \upsilon$.

Note : Loading a tuning fork with wax decreases its frequency while filing a tuning fork increases its frequency.

- Tuning fork is a source of sound of single frequency and frequency of a tuning fork of arm length L and thickness d in the direction of vibration is given by

$$\upsilon = \left[\frac{d}{L^2}\right] v = \frac{d}{L^2}\sqrt{\frac{Y}{\rho}} \qquad \left[\because v = \sqrt{\frac{Y}{\rho}}\right]$$

where Y is the Young's modulus and ρ is the density of the material of the tuning fork.

Illustration 30

A source of sound of frequency 512 Hz is moving towards a huge reflector-wall, with a velocity of 10 m/s. Velocity of sound in air = 330 m/s. How many beats per second will be heard by an observer standing between the source and the wall?

Soln.: No beat is heard. It is because the frequency received by the listener directly from the source and that received on reflection from wall is same. Frequency does not change by reflection.

Each frequency received $= \dfrac{512 \times 330}{330 - 10}$ Hz = 528 Hz

Illustration 31

Three sound waves of equal amplitudes have frequencies $(\upsilon - 1)$, υ, $(\upsilon + 1)$. They superpose to give beats. The number of beats produced per second will be

(a) 4 (b) 3 (c) 2 (d) 1

Soln. (d) : Take three sound waves as

$y_1 = A\cos 2\pi(\upsilon - 1)t$

$y_2 = A\cos 2\pi\upsilon t$ and $y_3 = A\cos 2\pi(\upsilon + 1)t$

Resultant wave is

$y = y_1 + y_2 + y_3$

$= A\cos 2\pi(\upsilon - 1)t + A\cos 2\pi\upsilon t + A\cos 2\pi(\upsilon + 1)t$

$= 2A\cos 2\pi t \cos 2\pi\upsilon t + A\cos 2\pi\upsilon t$

$= A(1 + 2\cos 2\pi t)\cos 2\pi\upsilon t$

The amplitude $A(1 + 2\cos 2\pi t)$ becomes maximum 1 time in 1 sec.

∴ number of beats per second = 1

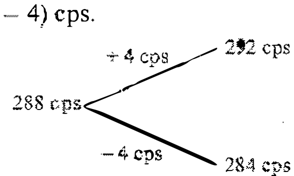
Illustration 32

A tuning fork arrangement (pair) produces 4 beats/sec with one fork of frequency 288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is

(a) 286 cps (b) 292 cps

(c) 294 cps (d) 288 cps.

Soln. (b) : The wax decreases the frequency of unknown fork. The possible unknown frequencies are (288 + 4)cps and (288 – 4) cps.



Wax reduces 284 cps and so beats should increases. It is not given in the question. This frequency is ruled out. Wax reduced 292 cps and so beats should decrease. It is given that the beats decrease to 2 from 4. Hence unknown fork has frequency 292 cps.

DOPPLER'S EFFECT

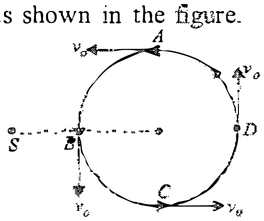
- When a source of sound or an observer or both are in relative motion, there is an apparent change in the frequency of sound as heard by the observer. This phenomenon is called Doppler's effect.
- According to Doppler's effect the apparent frequency heard by the observer is given by

$$\upsilon' = \upsilon\left[\frac{v \pm v_o}{v \mp v_s}\right]$$

where v_s, v_o and v are the speed of source, observer and sound relative to air.

The upper sign on v_s (or v_o) is used when source (observer) moves towards the observer (source) while lower sign is used when it moves away.

- If the wind blows with speed v_w in the direction of sound, v is replaced by $v + v_w$ in the above equation. If the wind blows with speed v_w in a direction opposite to that of sound, v is replaced by $v - v_w$ in the above equation.
- If source and observer both are stationary *i.e.* $v_s = v_o = 0$ then $\upsilon' = \upsilon$. Similarly, if source and observer both are moving in the same direction with same speed, *i.e.* $v_s = v_o$, then $\upsilon' = \upsilon$. Thus it is clear that if there is no relative motion between the source and the observer then there is no Doppler effect.
- When a source is revolving in a circle and observer is stationary outside, as shown in the figure.



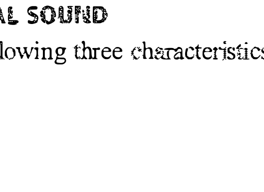
At A, $\upsilon_{max} = \dfrac{v\upsilon}{v - v_s}$

At C, $\upsilon_{min} = \dfrac{v\upsilon}{v + v_s}$

Beat frequency $= \upsilon_{max} - \upsilon_{min}$.

There is no Doppler effect at B and D.

- When an observer is revolving in a circle with a stationary source outside, as shown in the figure.



At A, $\upsilon_{max} = \dfrac{(v + v_o)\upsilon}{v}$

At C, $\upsilon_{min} = \dfrac{(v - v_o)\upsilon}{v}$

Beat frequency $= \upsilon_{max} - \upsilon_{min}$.

There is no Doppler effect at B and D.

CHARACTERISTICS OF MUSCIAL SOUND

- A musical sound has the following three characteristics :
  - Loudness

- Pitch
- Quality or timbre

- **Loudness :** Loudness is a subjective sensation (*i.e.* it depends on the listener) which is determined by the intensity of the sound and by the sensitivity of the observer's ear.
- The loudness of a sound of intensity I is given by

$$L = \log_{10}\left(\frac{I}{I_0}\right)$$

where I_0 = threshold intensity = 10^{-12} W m^{-2}

- A practical and small unit of loudness of sound is decibel (dB).
- $1 \text{ decibel} = \frac{1}{10}$ bel.
- In decibel the loudness of a sound of intensity I is given by

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

- **Pitch :** It determines the sharpness or shrillness of sound. Higher the frequency, higher is the pitch and hence sharper is the sound.
- **Quality :** It is that characteristic of musical sound which differentiates sounds coming from different sources even when they are of same intensity and same pitch.

Illustration 33

A vibrating tuning fork, tied to the end of a string 1.988 m long, is whirled round in a circle. If it makes 2 rps, calculate the ratio of the frequencies of highest and lowest notes heard by an observer situated in the plane of the fork. Velocity of sound is 350 m/s.

Soln.: Here source is revolving in a circle.

Let velocity of source = v_s

The frequency heard by observer is maximum when fork/source is approaching the observer/listener. The frequency is minimum when the fork/source is moving away from the observer/listener.

$\therefore$ Maximum frequency, $\upsilon_1 = \frac{v\upsilon}{v - v_s}$

Minimum frequency, $\upsilon_2 = \frac{v\upsilon}{v + v_s}$

$$\therefore \quad \frac{\upsilon_1}{\upsilon_2} = \frac{v + v_s}{v - v_s} \quad \text{...(i)}$$

$v_s = r\omega = r \times 2\pi \times N = 1.988 \times 2 \times 3.14 \times 2 = 25 \text{ m s}^{-1}$.

$$\therefore \quad \frac{v_1}{v_2} = \frac{350 + 25}{350 - 25} = \frac{375}{325} = \frac{15}{13}$$

$$\therefore \quad \frac{\text{Highest frequency}}{\text{Lowest frequency}} = \frac{15}{13}.$$

Illustration 34

How many times more intense is 20 dB sound compared to 10 dB sound?

Soln.: Loudness $L = \log_{10}\frac{I}{I_0}$

or $L = \log_{10} I - \log_{10} I_0$

$\therefore \quad 2 = \log I_1 - \log I_0 \qquad [\because 20 \text{ dB} = 2\text{B}]$

$1 = \log I_2 - \log I_0 \qquad [\because 10 \text{ dB} = 1\text{B}]$

$\therefore \quad (2 - 1) = \log I_1 - \log I_2$

$$1 = \log_{10}\left(\frac{I_1}{I_2}\right) \quad \text{or} \quad \frac{I_1}{I_2} = 10^1 = 10.$$

Illustration 35

A car sounding a horn of frequency 1000 Hz passes an observer. The ratio of frequencies of the horn noted by the observer before and after passing of car is 11 : 9. If the speed of sound is v, find the speed of the car.

Soln.: $\upsilon' = \frac{v\upsilon}{v - v_s}$ (car approaches observer)

$\upsilon'' = \frac{v\upsilon}{v + v_s}$ (car goes past observer)

$$\therefore \quad \frac{\upsilon'}{\upsilon''} = \frac{v + v_s}{v - v_s} \quad \text{or} \quad \frac{11}{9} = \frac{v + v_s}{v - v_s}$$

or $11(v - v_s) = 9v + 9v_s$ or $v_s = \frac{v}{10}$

$\therefore$ Speed of car $= \frac{v}{10}$.

Illustration 36

A sound source is moving towards a stationary listener with $\frac{1}{10}$th of the speed of sound. Find the ratio of apparent to real frequency.

Soln.: $\upsilon' = \frac{v\upsilon}{v - v_s}$ or $\frac{\upsilon'}{\upsilon} = \frac{v}{\left(v - \frac{v}{10}\right)} = \frac{10}{9}$

$$\therefore \quad \frac{\upsilon'}{\upsilon} = \frac{10}{9}.$$

REVERBERATION TIME

- Reverberation is the phenomenon of persistence of sound for sometime after the source stops producing sound. The time for which sound continues to be heard even after the source has stopped producing sound is called reverberation time. According to Sabine's formula the reverberation time of a hall is given by

$$T = \frac{0.16V}{\Sigma a_i s_i}$$

where V is the volume of the hall (in m^3) and $\Sigma a_i s_i = a_1 s_1 + a_2 s_2 + a_3 s_3 + ...$ = total absorption of the hall where $s_1, s_2, s_3, ...$ are the area of surfaces (in m^2) which absorb sound and $a_1, a_2, a_3, ...$ are their respective absorption constants.

CONCEPT MAP

Oscillations

Periodic Motion : It is that motion which repeats itself after regular interval of time.

Time Period : It is defined as the smallest time interval after which oscillation repeats itself. It is denoted by symbol T.
Its SI unit is second.

Frequency : It is defined as the number of oscillations completed in a unit time. It is denoted by symbol υ.

$$\upsilon = \frac{1}{T}$$

Its SI unit is hertz.

- **Free Oscillations :** When a system oscillates with its own natural frequency without the help of an external periodic force, its oscillations are called free oscillations.
- **Forced Oscillations :** When a system oscillates with the help of an external periodic force with a frequency, other than its own natural frequency, its oscillations are called forced oscillations.
- **Resonant Oscillations :** When a system oscillates with its own natural frequency, with the help of an external periodic force whose frequency is the same as that of the natural frequency of the oscillating system, then the oscillations of the system are called resonant oscillations.

Displacement : It is defined as the distance of the oscillating particle from its mean position at any time. It is given by

$$x = A\sin(\omega t + \phi)$$

Amplitude : It is defined as the maximum (positive or negative) value of displacement from the mean position. If S is the span of S.H.M., then amplitude,

$$A = \frac{S}{2}$$

Phase : The argument $(\omega t + \phi)$ is called phase of the motion. The constant ϕ is called initial phase (i.e., phase at $t = 0$) or phase constant.

Velocity : It is defined as time rate of change of the displacement of the particle at the given instant.

or $$\upsilon = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

$$\upsilon = \omega\sqrt{A^2 - x^2}$$

Acceleration : It is defined as the time rate of change of the velocity of the particle at the given instant.

$$a = -\omega^2 x$$

$$a = \frac{d\upsilon}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi)$$

Energy :
Kinetic energy,

$$K = \frac{1}{2}m\omega^2(A^2 - x^2)$$

Potential energy,

$$U = \frac{1}{2}m\omega^2 x^2$$

Total energy,

$$E = K + U = \frac{1}{2}m\omega^2 A^2$$

Simple Harmonic Motion : It is a kind of periodic motion, in which a particle moves to and fro about a mean position under a restoring force, which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant.

Undamped Simple Harmonic Oscillations : When a simple harmonic system oscillates with a constant amplitude which does not change with time, its oscillations are called undamped simple harmonic oscillations. The total energy of the system executing undamped simple harmonic oscillations remains constant and is independent of time.

Damped Simple Harmonic Oscillations : When a simple harmonic system oscillates with a decreasing amplitude with time, its oscillations are called damped simple harmonic oscillations. The energy of the system executing damped simple harmonic oscillations will go on decreasing with time.

System Executing Simple Harmonic Motion

The time period of spring pendulum is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The time period of simple pendulmu is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Symbols Used

- x = linear displacement
- A = amplitude
- υ = frequency
- T = time period
- v = velocity
- ω = angular velocity
- a = acceleration
- m = mass
- k = spring constant or force constant
- L = length of the pendulum
- g = acceleration due to gravity