At
$$P$$
, $E_1 = E_2$
or $\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1^2} = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2^2}$ $q_1 \oplus \stackrel{\overrightarrow{E_2}}{\xrightarrow{P}} \stackrel{\overrightarrow{E_1}}{\xrightarrow{P}} q_2$
 $\frac{r_1}{r_2} = \sqrt{\frac{q_1}{q_2}} = \sqrt{\frac{16}{4}} = 2$

Also, $r_1 + r_2 = 3.0 \text{ m}$ \therefore $r_1 = 2 \text{ m}$, $r_2 = 1 \text{ m}$ Hence, the point P is at a distance of 2 in from q_1 and 1 m from q_2 .

Illustration 5

Two points masses, *m* each carrying charge -q and +q are attached to the ends of a massless rigid non-conducting rod of length *l*. The arrangement is placed in a uniform electric field *E* such that the rod makes a small angle $\theta = 5^{\circ}$ with the field direction, show that the minimum time needed by the rod to align itself along the field (after it is set free) is

$$t = \frac{\pi}{2} \sqrt{\frac{ml}{2qE}}$$

The torque on rod *AB* is given by *A*

Soln.: The torque on rod *AB* is given by $\tau = qE(l\sin\theta)$

 $= qEl\theta$

The moment of inertia of the rod AB about θ is given by

$$I = m\left(\frac{l}{2}\right)^2 + m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{2}$$

We known that,

$$\tau = I \alpha$$
 or $\alpha =$

$$\therefore \quad \alpha = \frac{qEl\theta}{(ml^2/2)} = \frac{2qE\theta}{ml} = \omega^2 \theta \quad \text{where} \quad \omega^2 = \frac{2qE}{ml}$$

 $\frac{\tau}{I}$

As acceleration is directly proportional to θ , hence the motion of rod is SHM. The time period T is given by

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{ml}{2qE}}$$

The rod will become parallel to E in a time

$$t = \frac{T}{4} = \frac{2\pi}{4} \sqrt{\left(\frac{ml}{2qE}\right)} = \frac{\pi}{2} \sqrt{\left(\frac{ml}{2qE}\right)}$$

ELECTRIC FLUX

• If the lines of force pass through a surface then the surface is said to have flux linked with it. Mathematically it can be formulated as follows :

The flux linked with small area element on the surface of the body

 $d\phi = \bar{E} \cdot d\bar{s}$

where $d\overline{s}$ is the area vector of the small area element. The area vector of a closed surface is always in the direction of outward drawn normal. The total flux linked with whole of the body

 $\phi = \oint \overline{E} \cdot d\overline{s}$

• Gauss's Theorem : The total flux linked with a closed

surface is $\frac{1}{\varepsilon_0}$ times the charge enclosed by the closed surface (Gaussian surface) *i.e.*,

$$\oint \bar{E} \cdot d\bar{s} = \frac{q}{\varepsilon_0}$$

This law is suitable for symmetrical charge distribution and valid for all vector fields obeying inverse square law.

Gaussian Surface :

- (a) It is imaginary surface.
- (b) It is spherical for infinite sheet of charge, conducting and non conducting spheres.
- (c) It is cylindrical for infinite sheet of charge, conducting charge surfaces, infinite line of charges charged cylindrical conductors etc.

APPLICATIONS OF GAUSS'S LAW

• Electric field due to an infinitely long thin uniformly charged straight wire

Electric field due to thin infinitely long straight wire of uniform linear charge density λ is

$$E = \frac{\lambda}{2\pi\varepsilon_{\bullet}r}$$

where r is the perpendicular distance of the observation point from the wire.

Electric field due to a uniformly charged thin spherical shell

Electric field due to uniformly charged thin spherical shell of uniform surface charge density σ and radius Rat a point distant r from the centre of the shell is given as follows :

$$\circ$$
 At a point outside the shell *i.e.*, $r > R$

$$E = \frac{1}{4\pi\varepsilon_0}$$

• At a point on the surface of the shell *i.e.*, r = R $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$

• At a point inside the shell *i.e.*, r < R

$$E=0$$

Here, $q = 4\pi R^2 \sigma$

•

The variation of E with r for a uniformly charged thin spherical shell is as shown in the figure.



Electric field due to a uniformly charged nonconducting solid sphere

Electric field due to a uniformly charged non conducting solid sphere of uniform volume charge density ρ and radius *R* at a point distant *r* from the centre of the sphere is given as follows :

• At a point outside the sphere *i.e.*, r > R

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

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• At a point on the surface of the sphere *i.e.*, r = R

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}$$

• At a point inside the sphere *i.e.*, r < R

$$E = \frac{\mathbf{p}r}{3\varepsilon_0} = \frac{1}{4\pi\varepsilon_0} \frac{qr}{R^3}, \text{ for } r < R$$

Here,





 Electric field due to a aniformly charged infinite thin plane sheet

Electric field due to a infinite thin plane sheet of uniformly charge surface density σ is

$$E = \frac{\sigma}{2\epsilon}$$

E is independent of r, distance of the point from sheet.

 Electric field due to two thin infinite parallel sheets of equal and opposite charges

Electric field due to two thin infinite parallel sheets of uniform surface density $+\sigma$ and $-\sigma$, is given as follows :

• At a point anywhere in the space between the two sheets

$$E = \frac{\sigma}{r}$$

• At point outside the sheets, E = 0.

Illustration 6

An electric dipole is placed at the centre of a sphere. Find the electric flux passing through the sphere.

Soln.: Net charge inside the sphere $q_{in} = 0$. Therefore, according to Gauss's law net flux passing through the sphere is zero.

ELECTRIC POTENTIAL

 Electric potential at a point can be physically interpreted as the work done by the field in displacing a unit positive charge from some reference point to the given point

$$V = \frac{\gamma}{a_0}$$

Potential at a point is analytically defined as a scalar function of position whose negative gradient at a point gives electric intensity

$$\vec{E} = -\operatorname{grad} V = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

Potential at earth is assumed to be zero as it is a large conductor and its potential is approximately constant with respect to given charge to it.

Potential Difference :

$$V_{ab} = -\int \vec{E} \cdot d\vec{r}$$



Its value does not depend on frame of reference, hence it is an absolute quantity.

As electric field is conservative, work done and hence potential difference between two points is path independent and depends only on the position of points.

$$W_1 = W_2 = W_3$$

Illustration 7

The electric potential at point A is 20 V and at B is -40 V. Find the work done by an external force and electrostatic force in moving an electron slowly from B to A. Soln : Here, the test charge is an electron,

i.e., $q_0 = 1.6 \times 10^{-19} \text{ C}$

$$V_A = 20 \text{ V} \text{ and } V_B = -40 \text{ V}$$

Work done by external force $(W_{Bd})_{\text{external force}} = q_0(V_A - V_B)$ = $(-1.6 \times 10^{-19})[(20) - (-40)] = -9.6 \times 10^{-18} \text{ J}$

$$(W_{B-A})_{\text{electric force}} = -(W_{B-A})_{\text{external force}}$$

= -(-9.6 × 10⁻¹⁸) J = 9.6 × 10⁻¹⁸ J

- Electric potential due to a point charge : Electric potential at a point distant r from a point charge q is $V = \frac{q}{4\pi \epsilon_0 r}$
- Electric potential due to system of charges : The electric potential at a point due to a system of charges is equal to the algebraic sum of the electric potentials due to individual charges at that point.

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} - \frac{q_3}{r_3} + \dots + \frac{q_n}{r_n} \right) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i}$$

Electric potential at any point due to an electric dipole



The electric potential at point P due to an electric dipole is $1 \quad a\cos\theta$

$$V = \frac{p \cos \theta}{4\pi\varepsilon_0} r^2$$

Special cases :

• When the point P lies on the axial line of dipole *i.e.*,
$$\theta = 0^{\bullet}$$
.

$$\frac{V}{4\pi\varepsilon_0 r^2}$$

- When the point P lies on the equatorial line of the dipole, i.e., 0 = 90° ∴ V = 0.
- Electric petential due to uniformly charged thin spherical shell

Electric potential due to a uniformly charged spherical shell of uniform surface charge density σ and radius R at a point distant r from the centre of the shell is given as follows :

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- At a point outside the shell *i.e.*, r > R
- At a point on the surface of the shell *i.e.*, r = R0

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

0 At a point inside the shell *i.e.*, r > R

$$V = \frac{1}{4\pi\varepsilon_0} \frac{1}{I}$$

Here, $q = 4\pi R^2 \sigma$

The variation of V with r for a uniformly charged thin spherical shell is shown in the figure.



Electric potential due to a uniformly charged non conducting solid sphere

Electric potential due to a uniformly charged nonconducting solid sphere of uniform volume charge density ρ and radius R at a point distant r from the centre of sphere is given as follows :

• At a point outside the sphere *i.e.*, r > R

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

At a point on the surface of the sphere *i.e.*, r = R0 9

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

At a point inside the sphere *i.e.*, r < R $V = \frac{1}{4\pi\epsilon_0} \frac{q(3R^2 - r^2)}{2R^3}$ \circ

$$V = \frac{1}{4\pi\varepsilon_0}$$

Here $q = \frac{4}{3}\pi R^3 \rho$

Illustration 8

A spherical conducing shell of inner radius r_1 and outer radius r_2 has a charge Q.

- (a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?
- (b) Is the electric field inside a cavity (with no charge) zero, even if the shall is not spherical, but has any irregular shape? Explain.

Soln.: (a) Surface charge density on the inner surface of shell is

$$\sigma_{\rm in} = \frac{-q}{4\pi r_1^2}$$

and on the outer surface of shell is

$$\sigma_{\rm out} = \frac{Q+q}{4\pi r_2^2}$$

(b) The electric flux linked with any closed surface Sinside the conductor is zero as electric field inside conductor is zero.

So, by Gauss's theorem net charge enclosed by closed surface S is also zero i.e.,



So, if there is no charge inside the cavity, then there cannot be any charge on the inner surface of the shell and hence electric field inside the cavity will be zero, even though the shall may not be spherical.

Illustration 9

A charge Q is distributed over two concentric hollow spheres of radii r and R (> r) such that the surface densities are the same. Find the potential at the centre of the two spheres.

Soln. : If q_1 and q_2 are the charges on two spheres of radii r and R respectively, then the surface charge density is given by

$$\sigma = \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2} \text{ or } \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

$$\frac{q_1}{q_2} + 1 = \frac{r^2}{R^2} + 1 \implies \frac{q_1 + q_2}{q_2} = \frac{r^2 + R^2}{R^2}$$

$$R$$

But $q_1 + q_2 =$ total charge Q.

$$\frac{Q}{q_2} = \frac{r^2 + R^2}{R^2} \text{ or } \frac{QR}{q_2} = \frac{r^2 + R^2}{R} \text{ or } \frac{q_2}{R} = \frac{QR}{r^2 + R^2}$$

Similarly, $\frac{q_1}{r} = \frac{QR}{r^2 + R^2}$ Potential at the common centre $= \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{R}$

$$= \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1}{r} + \frac{q_2}{R} \right) = \frac{1}{4\pi\varepsilon_0} \left(\frac{Qr}{r^2 + R^2} + \frac{QR}{r^2 + R^2} \right)$$

EQUIPOTENTIAL SURFACE

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- An equipotential surface is a surface with a constant value of potential at all points on the surface. Properties of an equipotential surface
- Electric field lines are always perpendicular to an . equipotential surface.
- Work done in moving an electric charge from one point to another on an equipotential surface is zero.
- Two equipotential surfaces can never intersect one another.

Relationship between \vec{E} and \vec{V}

$$\bar{E} = -\bar{\nabla}V$$
where $\bar{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$

-ve sign shows that the direction of \overline{E} is the direction of decreasing potential.