ELECTRIC POTENTIAL ENERGY

- Electric potential energy of a system of charges is the total amount of work done in bringing the various charges to their respective positions from infinitely large mutual separations.
- The SI unit of electrical potential energy is joule.
- Electric potential energy of a system of two charges is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

- Electric potential energy of a system of n point charges

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

Note in this summation, we should include only one term for each pair of charges.

CONDUCTORS AND INSULATORS

- Those substances which can easily allow electricity to pass through them are called conductors. They have a large number of free charge carriers that are free to move inside the material. Metals, human and animal bodies and earth are conductors.

  Those substances which not allow electricity to pass through them one called insulators also called dielectrics.

Basic Electrostatics Properties of a Conductor

- Inside a conductor, electric field is zero.
- At the surface of a charged conductor, electric field must be normal to the surface at every point.
- The interior of a conductor can have no excess charge in the static situation.
- Electric potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.
- Electric field at the surface of a charged conductor

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

- **Electrostatic shielding :** It is the phenomenon of protecting a certain region of space from external electric field.

Dielectrics and Polarisation

- **Dielectrics :** Dielectrics are non conducting substances. In contrast to conductors, they have no (or negligible number of) charge carriers.
- **Polar molecule :** A polar molecule is one in which the centres of positive and negative charges are separated (even when there is no external field). A polar molecule has a permanent dipole moment *e.g.*, water (H_2O) and HCl.
- **Non-polar molecule :** A non-polar molecule is one in which the centres of positive and negative charges are coincide. A non polar molecule has no permanent dipole moment. *e.g.*, oxygen (O_2) and hydrogen (H_2).
- **Polarisation :** The dipole moment per unit volume is called polarisation and is denoted by For linear isotropic dielectrics $\vec{P} = \chi_e\vec{E}$

  where χ_e is a constant characteristic of the dielectric and is called the electric susceptibility of the dielectric medium.

CAPACITANCE

- Capacitance (C) of a capacitor is the ratio of charge(Q) given and the potential (V) to which it is raised.

$$C = \frac{Q}{V}$$

- The SI unit of capacitance is farad (F).

  1 millifarad (mF) = 10^{-3} farad
  1 microfarad (μF) = 10^{-6} farad
  1 picofarad (pF) = 10^{-12} farad.
- Capacitance is a scalar quantity.
- The dimensional formula of capacitance is $[M^{-1}L^{-2}T^4A^2]$.

Capacitance of spherical conductor

- Capacitance of a spherical conductor of radius R is

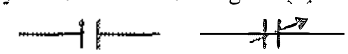
$$C = 4\pi\varepsilon_0 R$$

Taking earth to be a conducting sphere of radius 6400 km, its capacitance will be

$$C = 4\pi\varepsilon_0 R = \frac{6.4\times10^6}{9\times10^9} = 711\ \mu F$$

CAPACITOR

- A condenser or a capacitor is a device that stores electric charge. It consists of two conductors separated by an insulator or dielectric. The two conductors carry equal and opposite charges $\pm Q$.
- In an electrical circuit, a capacitor of fixed capacitance is represented by the symbol as shown in figure (a) while a capacitor of variable capacitance is represented by the symbol as shown in figure (b).



(a) (b)

TYPES OF CAPACITORS

- Depending on their geometry, capacitors are classified as :
  - Parallel plate capacitor
  - Cylindrical capacitor
  - Spherical capacitor
- **Parallel plate capacitor :** It consists of two similar flat conducting plates, arranged parallel to one another, separated by a distance. Its capacitance is given by

$$C = \frac{\varepsilon_0 A}{d} \text{ (when air is between the plates)}$$

$$C = \frac{K\varepsilon_0 A}{d} \text{ (when dielectric is between the plates)}$$

where A is area of each plate and d is separation between the two plates.

- When a dielectric slab of thickness t and dielectric constant K is introduced between the plates, then the capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

- When a metallic conductor of thickness t is introduced between the plates, then capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d - t}$$

- **Cylindrical capacitor** : It consists of two co-axial cylinders of same length.
- Capacitance of an air filled cylindrical capacitor is

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

where a and b are the inner and outer radii and L is the length.
- **Spherical capacitor** : It consists of two concentric spherical shells.
- Capacitance of an air filled spherical capacitor is

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

where a and b are the inner and outer radii.

Illustration 10

A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Soln.: $C_0 = \frac{\varepsilon_0 A}{d} = 8$ pF

$$C = \frac{K\varepsilon_0 A}{d/2} = \frac{2K\varepsilon_0 A}{d} = 2KC_0 = 2 \times 6 \times 8 \text{ pF}$$

or $C = 96$ pF

Illustration 11

A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e. bending of the field lines at the ends).

Sol.: Capacitance of cylindrical capacitor is

$$C = \frac{2\pi\varepsilon_0 L}{2.303\log_{10}\frac{b}{a}} = \frac{2 \times 15 \times 10^{-2}}{9 \times 10^9 \times 2.303 \times \log_{10}\frac{1.5}{1.4}}$$

or $C = 1.21 \times 10^{-10}$ F

Electric potential of the inner cylinder is

$$V = \frac{Q}{C} = \frac{3.5 \times 10^{-6}\text{ C}}{1.21 \times 10^{-10}\text{ F}}$$

or $V = 2.89 \times 10^4$ V.

GROUPING OF CAPACITORS

- **Capacitors in series** : For n capacitors connected in series, the equivalent capacitance C_S is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

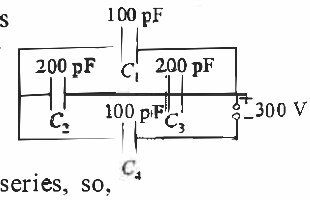
- **Capacitors in parallel** : For n capacitors connected in parallel, the equivalent capacitance C_P is given by

$$C_P = C_1 + C_2 + ... + C_n$$

- When capacitors are connected in series, the charge through each capacitor is same.
- When capacitors are connected in parallel, the potential difference across each capacitor is same.

Illustration 12

Obtain the equivalent capacitors of the network in figure. For a 300 V supply, determine the charge and voltage across each capacitor.



Soln.: $\because$ C_2 and C_3 are in series, so,

$C' = 100$ pF

$\because$ C_1 and C' are parallel,

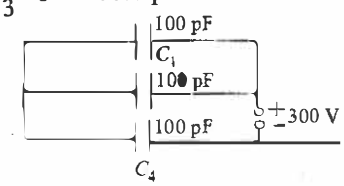
so $C'' = C_1 + C' = 100 + 100$

or $C'' = 200$ pF

$\because$ C_4 and C'' are in series, so net capacitance of the network is

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4} = \frac{1}{200} + \frac{1}{100} = \frac{1+2}{200}$$

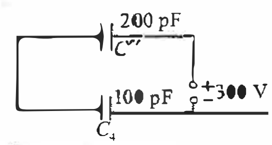
or $C_2 = \frac{200}{3}$ pF $= 66.7$ pF



Net charge stored on the combination is

$$Q = CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8}\text{ C}$$

As C'' and C_4 are in series, so



$Q'' = Q_4 = Q$

or $Q'' = Q_4 = 2 \times 10^{-8}$C

and hence $V'' = \frac{Q''}{C''} = \frac{2 \times 10^{-8}\text{ C}}{200 \times 10^{-12}\text{ F}} = 100$ V

and $V_4 = \frac{Q_4}{C_4} = \frac{2 \times 10^{-8}\text{ C}}{100 \times 10^{-12}\text{ F}} = 200$ V

$\because$ C_1 and C' are in parallel, so

$V_1 = V' = V''$

or $V_1 = V' = 100$V

and hence $Q_1 = C_1V_1 = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8}$ C

and $Q' = C'V' = 100 \times 10^{-12} \times 100 = 1 \times 10^{-8}$ C

$\because$ C_2 and C_3 are in parallel, so

$Q_2 = Q_3 = Q'$

or $Q_2 = Q_3 = 1 \times 10^{-8}$ C

and hence $V_2 = \frac{Q_2}{C_2} = \frac{1 \times 10^{-8}\text{ C}}{200 \times 10^{-12}\text{ F}} = 50$ V

and $V_3 = \frac{Q_3}{C_3} = \frac{1 \times 10^{-8}\text{ C}}{200 \times 10^{-12}\text{ F}} = 50$ V.

ENERGY STORED IN A CAPACITOR

- Work done in charging a capacitor gets stored in the capacitor in the form of its electric potential energy and it is given by

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$