### ENERGY DENSITY

- The energy stored per unit volume in the electric field between the plates is called energy density ($u$). It is given by

$$u = \frac{1}{2}\varepsilon_0 E^2$$

### SHARING OF CHARGES

- When two capacitors charged to different potentials are connected by a conducting wire, charge flows from the one at higher potential to the other at lower potential till their potentials become equal. The equal potential is called common potential. It is given by

$$V = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

- In sharing charges, there is absolutely no loss of charge. Some energy is, however, lost in the process in the form of heat etc which is given by

$$U_1 - U_2 = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

**Illustration 13**

The plates of a parallel plate capacitor have an area of 90 cm$^2$ each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

**Soln.:** $A = 90\text{ cm}^2 = 90 \times 10^{-4}\text{ m}^2, d = 2.5\text{ mm} = 2.5 \times 10^{-3}\text{ m}$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 9 \times 10^{-3}}{2.5 \times 10^{-3}}$$

or $C = 32$ pF

(a) $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 32 \times 10^{-12} \times 400^2$

or $U = 2.56\ \mu J$

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{\varepsilon_0 A}{d} \times (E.d)^2 = \frac{1}{2}\varepsilon_0 AE^2 d$

or $\frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$

or Energy per unit volume $u = \frac{1}{2}\varepsilon_0 E^2$

### EFFECT OF DIELECTRIC

- When a dielectric slab of dielectric constant $K$ is introduced between the plates of a charged parallel plate capacitor and the charging battery remains connected, then
  - Potential difference between the plates remains constant *i.e.*, $V = V_0$
  - Capacitance $C$ increases *i.e.*, $C = KC_0$
  - Charge on a capacitor increases *i.e.*, $Q = KQ_0$
  - Electric field between the plates remains unchanged *i.e.*, $E = E_0$
  - Energy stored in a capacitor increases *i.e.*, $U = KU_0$
- When a dielecric slab of dielectric constant $K$ is introduced between the plates of a charged parallel plate capacitor and the charging battery is disconnected, then
  - Charge remains unchanged *i.e.*, $Q = Q_0$
  - Capacitance increases *i.e.*, $C = KC_0$
  - Potential difference between the plates decreases *i.e.*, $V = \frac{V_0}{K}$
  - Electric field between the plates decreases *i.e.*, $E = \frac{E_0}{K}$
  - Energy stored in the capacitor decreases *i.e.*, $U = \frac{U_0}{K}$

  where $Q_0$, $C_0$, $V_0$, $E_0$ and $U_0$ represents the charge, capacitance, potential difference, electric field and energy stored in the capacitor of a charged air filled parallel plate capacitor.

### VAN DE GRAAFF GENERATOR

- A Van de Graaff generator consists of a large spherical conducting shell (a few metres in diameter). By means of a moving belt and suitable brushes, charge is continuously transferred to the shell, and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.