- When current passes through breadth side, the resistance offered by cube $= \frac{\rho b}{lh}$
- When current passes through the thickness side, the resistance offered by cube $= \frac{\rho h}{lb}$.

ELECTRIC ENERGY

- It is defined as the total electric work done or energy supplied by the source of emf in maintaining the current in an electric circuit for a given time.

  Electric energy = Electric power × time = $P \times t$.
- The SI unit of electrical energy is joule (J).
- The commercial unit of electric energy is kilowatt-hour (kWh),

  1 kWh = 1000 Wh = 3.6×10^6 J = one unit of electricity consumed.
- The number of units of electricity consumed is

  n = (total wattage × time in hour)/1000.
- Bill of electricity *i.e.*, the cost of consumption of electricity in a house = no. of units of electricity consumed × amount for one unit of electricity.

ELECTRIC POWER

- It is defined as the rate at which work is done by the source of emf in maintaining the current in the electric circuit.

$$\text{Electric power } P = \frac{\text{Electric work done}}{\text{time taken}}$$

$$P = VI = I^2R = \frac{V^2}{R}.$$

- The SI unit of power is watt (W).
- The practical unit of power is kilowatt (kW) and horse power (hp).
- 1 kilowatt = 1000 watt.
- 1 hp = 746 watt.

CARBON RESISTORS

- Resistors in the higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuits. Carbon resistors are small in size.

Colour Code of Resistors

- A colour code is used to indicate the resistance value and its percentage accuracy. Every resistor has a set of coloured rings on it. The first two coloured rings from the left end indicate the first two significant figures of the resistance in ohms. The third colour ring indicates the decimal multiplier and the last colour ring stands for the tolerance in percent. The colour code of a resistor is as shown in the table.

Colour	Number	Multiplier	Tolerance(%)
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour			20

- Suppose a resistor has yellow, violet, brown and gold rings as shown in the figure below. The resistance of the resistor is $(47 \times 10\ \Omega) \pm 5\ \%$.

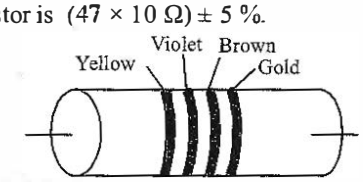


Illustration 2

The resistance of a wire is 10 Ω. It is drawn so that its length increases by 10%. Find the final resistance.

Soln.: Volume of material of wire remains constant.

Volume V = (Area) × (Length) = AL

$$\text{Now } R = \frac{\rho L}{A} = \frac{\rho L^2}{V} = \left(\frac{\rho}{V}\right)L^2 \qquad \left(\because A = \frac{V}{L}\right)$$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{L_1}{L_2}\right)^2 = \left(\frac{L}{1.1L}\right)^2 = \frac{1}{1.21}$$

or $R_2 = R_1 \times 1.21 = 10 \times 1.21 = 12.1\ \Omega$

∴ final resistance = 12.1 Ω.

Illustration 3

A potential difference of 100V is applied to the ends of a copper wire one metre long. Calculate the average drift velocity of the electrons. Compare it with thermal velocity at 27°C. Given, the number of free electrons per unit volume is $8.45 \times 10^{28}\ m^{-3}$ and conductivity of copper is $5.81 \times 10^7 (\Omega\text{-m})^{-1}$.

Soln.: Since $\Delta V = 100$ V, $l = 1$ m,

$$\therefore \quad \text{electric field} = \frac{\Delta V}{l} = \frac{100}{1} = 100\ \text{V m}^{-1}$$

Also, conductivity $\sigma = 5.81 \times 10^7\ \Omega^{-1}\ m^{-1}$

$n = 8.45 \times 10^{28}\ m^{-3}$

$$v_d = \frac{\sigma}{en}E = \frac{5.81 \times 10^7 \times 100}{1.6 \times 10^{-19} \times 8.45 \times 10^{28}}$$

$$= 0.43\ \text{m s}^{-1}$$

Thermal velocity can be calculated considering electrons as ideal gas particles.

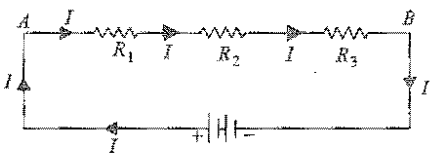
$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3\times(1.38\times10^{-23})300}{9.1\times10^{-31}}}$$

(where k_B is the Boltzmann constant)

$v_{rms} = 1.17 \times 10^5$ m s^{-1}

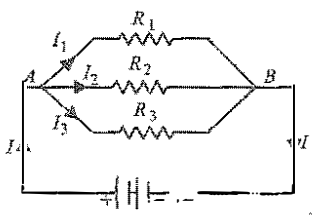
RESISTORS IN SERIES AND PARALLEL

- **Resistors in series :** The various resistors are said to be connected in series if they are connected as shown in the figure.



- The current through each resistor is the same.
- The equivalent resistance of the combination of resistors is

  $R_s = R_1 + R_2 + R_3$
- Series combinations of resistors are used in resistance box and decorative bulbs.
- If n wires each of resistance R are connected in series, the equivalent resistance is nR.
- **Resistors in parallel :** The various resistors are said to be connected in parallel if they are connected as shown in figure.



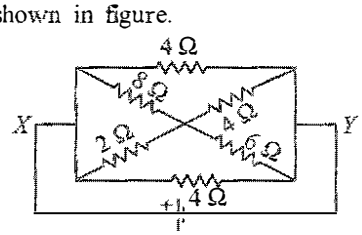
- The potential difference is same across each resistor.
- The equivalent resistance of the combination of resistors is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

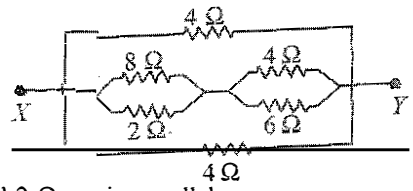
- All the domestic appliances in a house are usually connected in parallel combinations.

Illustration 4

Calculate resultant resistance between the points X and Y in the circuit shown in figure.



Soln.: The equivalent circuit is drawn in the figure:



8 Ω and 2 Ω are in parallel

$$\therefore \quad R_1 = \frac{8\times2}{8+2} = 1.6\ \Omega$$

4 Ω and 6 Ω are in parallel

$$\therefore \quad R_2 = \frac{4\times6}{4+6} = 2.4\ \Omega$$

R_1 and R_2 are in series

$\therefore \quad R_s = R_1 + R_2 = 1.6 + 2.4 = 4.0\ \Omega$

Now the network is reduced to 4 Ω, 4 Ω and 4 Ω in parallel.

$$\therefore \quad \frac{1}{R'} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \quad \text{or} \quad R' = \frac{4}{3}\ \Omega$$

EFFECT OF TEMPERATURE ON RESISTANCE

- The resistance of a metallic conductor increases with increase in temperature.
- The resistance of a conductor at temperature t°C is given by

  $R_t = R_0(1 + \alpha t + \beta t^2)$

  where R_t is the resistance at t°C, R_0 is the resistance at 0°C and α and β are the characteristics constants of the material of the conductor. If the temperature t°C is not sufficiently large, β is negligible, the above relation can be written as

  $R_t = R_0(1 + \alpha t)$

  where α is the temperature coefficient of resistance. Its unit is K^{-1} or °C^{-1}.
- For metals α is positive *i.e.* resistance increases with rise in temperature.
- For insulators and semiconductors α is negative *i.e.*, resistance decreases with rise in temperature.

Illustration 5

A silver wire has a resistance of 2.1 ohm at 27.5°C, and a resistance of 2.7 Ω at 100° C. Determine the temperature coefficient of resistivity of silver.

Soln.: Resistance of a conductor changes with the temperature, if α is taken as temperature coefficient of resistivity then the relation is

$R_2 = R_1[1 + \alpha(t_2 - t_1)]$

$2.7 = 2.1\ [1 + \alpha(100 - 27.5)]$

$0.6 = 2.1\ \alpha\ [72.5]$

$\alpha = 0.00394$°C^{-1}

$\alpha = 39.4 \times 10^{-4}$°C^{-1}

ELECTROCHEMICAL CELL

- It is a device which, by converting chemical energy into electrical energy, maintains the flow of charge in a circuit.

Electromotive Force (emf) of a Cell

- It is defined as the potential difference between the two terminals of a cell in an open circuit *i.e.*, when no current flows through the cell. It is denoted by symbol ε.
- The SI unit of emf is joule/coulomb or volt and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.
- The emf of a cell depends upon the nature of electrodes, nature and the concentration of electrolyte used in the cell and its temperature.

Terminal Potential Difference

- It is defined as the potential difference between two terminals of a cell in a closed circuit *i.e.* when current is flowing through the cell.

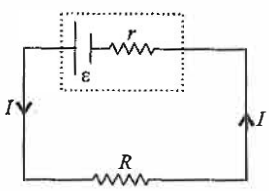
Internal Resistance of a Cell

- It is defined as the resistance offered by the electrolyte and electrodes of a cell when the current flows through it.

  Internal resistance of a cell depends upon the following factors:
  - Distance between the electrodes
  - The nature of the electrolyte
  - The nature of electrodes
  - Area of the electrodes, immersed in the electrolyte

Relationship between ε, V and R

- When a cell of emf ε and internal resistance r is connected to an external resistance R as shown in the figure.



The voltage across R is

$$V = IR$$

$$= \frac{\varepsilon}{R+r}R$$

or $\varepsilon = IR + Ir$ or $V = \varepsilon - Ir$ or $\varepsilon = V + Ir$

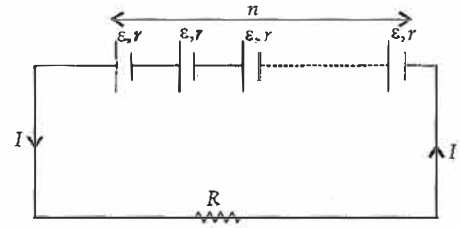
or $r = R\left(\frac{\varepsilon}{V} - 1\right)$

- During discharging of a cell terminal potential difference = emf of a cell – voltage drop across the internal resistance of a cell. *i.e.* terminal potential difference across it is lesser than emf of the cell. The direction of current inside the cell is from negative terminal to positive terminal.
- During charging of a cell, terminal potential difference = emf of a cell + voltage drop across internal resistance of a cell *i.e.* terminal potential difference becomes greater than the emf of the cell. The direction of current inside the cell is from positive terminal to negative terminal.

Grouping of Cells

- Cells can be grouped in the following three ways:
  - Series grouping
  - Parallel grouping
  - Mixed grouping

- **Series grouping :** If n identical cells each of emf ε and internal resistance r are connected to the external resistor of resistance R as shown in the figure, they are said to be connected in series grouping.

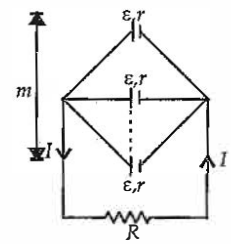


$\varepsilon_{eq} = n\varepsilon$ and $r_{eq} = nr$

$\therefore$ Current in the circuit, $I = \frac{n\varepsilon}{R + nr}$

- **Special cases :**
  - If $R << nr$, then $I = \frac{n\varepsilon}{nr} = \frac{\varepsilon}{r}$.
  - If $R >> nr$, then $I = \frac{n\varepsilon}{R}$.

- **Parallel grouping :** If m identical cells each of emf ε and internal resistance r are connected to the external resistor of resistance R as shown in figure, they are said to be connected in parallel grouping.



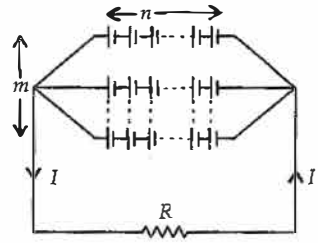
$\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{m}$.

$\therefore$ The current in the circuit, $I = \frac{\varepsilon}{R + \left(\frac{r}{m}\right)}$.

Special cases :

- If $\frac{r}{m} << R$, then $I = \frac{\varepsilon}{R}$.
- If $\frac{r}{m} >> R$, then $I = m\frac{\varepsilon}{r}$.

- **Mixed grouping :** If the cells are connected as shown in figure they are said to be connected in mixed grouping. Let there be n cells in series in one row and m such rows of cells in parallel. Suppose all the cells are identical. Let each cell be of emf ε and internal resistance r.



$\varepsilon_{eq} = n\varepsilon$ and $r_{eq} = \frac{nr}{m}$ $\therefore I = \frac{n\varepsilon}{R + \left(\frac{nr}{m}\right)}$

- In case of mixed grouping of cells, current in the circuit will be maximum, when

$$R = \frac{nr}{m}$$

i.e. external resistance = total internal resistance of all cells

- When one cell is wrongly connected in series of n identical cells, each of emf ε, it will reduce the total emf by 2ε

i.e. effective emf $= n\varepsilon - 2\varepsilon$.

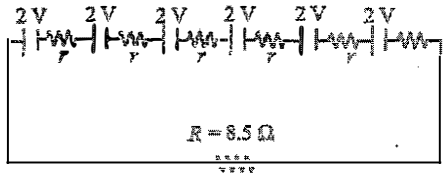
- When n cells each of internal resistance r are wrongly connected in series, the total internal resistance of cells $= nr$ *i.e.* there is no effect on the total internal resistance of the cells.

Illustration 6

(a) Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. Find the current in 8.5 Ω resistance and terminal voltage of the battery.

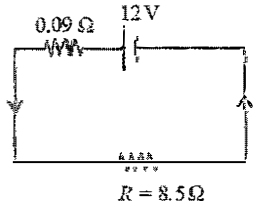
(b) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?

Soln.: (a) Six cells are joined in series.



Equivalent emf is $2 \times 6 = 12$ V

equivalent internal resistance is $0.015 \times 6 = 0.09\ \Omega$



Current drawn from the battery

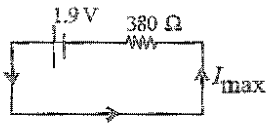
$$I = \frac{12}{8.5 + 0.09} = \frac{12}{8.59} = 1.4\ \text{A}$$

Terminal voltage of the battery

$$V = \varepsilon - Ir = 12 - 1.4 \times 0.09$$

$$V = 12 - 0.126 = 11.89\ \text{V}$$

(b) Maximum current is drawn from a battery when external resistance is treated to be zero.



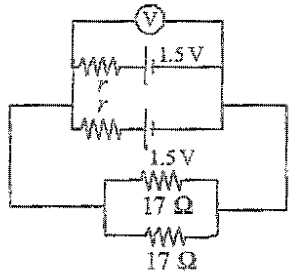
$$I_{max} = \frac{1.9}{380} = 0.005\ \text{A}.$$

To start a car a current of the order of 100 A is needed, so the battery mentioned above can not drive the starting motor.

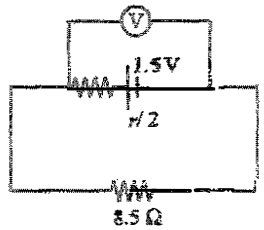
Illustration 7

Two identical cells of emf 1.5 V each are joined in parallel to provide supply to an external circuit consisting of two resistors of 17 Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cells to be 1.4 V. What is the internal resistance of each cell?

Soln.:



Let the internal resistance of each cell is 'r' on equivalent simplified circuit can be drawn



Current, $I = \dfrac{1.5}{\frac{r}{2} + 8.5}$

Terminal Potential under the given condition will be

$V = \varepsilon - Ir$

$$1.4 = 1.5 - \frac{1.5}{\left(\frac{r}{2} + 8.5\right)} \frac{r}{2}$$

$$\frac{1.5r}{r + 17} = 0.1$$

$$15r = r + 17$$

$$14r = 17$$

$$r = \frac{17}{14} = 1.21\ \Omega$$

KIRCHHOFF'S LAWS

- Kirchhoff in 1942 put forward the following two laws to solve the complicated circuits. These two laws are stated as follows:
- **Kirchhoff's first law or Kirchhoff's junction law or Kirchhoff's current law:** It states that the algebraic sum of the currents meeting at a junction is zero.
- Kirchhoff's first law supports law of conservation of charge.

  According to sign convention the current flowing towards a junction is taken as positive and the current flowing away from the junction is taken as negative.
- **Kirchhoff's second law or Kirchhoff's loop law or Kirchhoff's voltage law :** It states that in a closed loop, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistance and the respective currents flowing through them.

  $\Sigma\varepsilon = \Sigma IR$
- Kirchhoff's second law supports the law of conservation of energy.