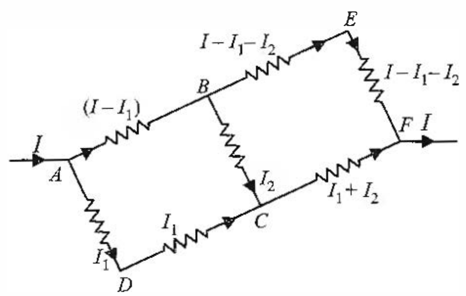
- According to sign convention while traversing a closed loop (in clockwise or anti-clock wise direction), if negative pole of the cell is encountered first then its emf is positive, otherwise negative. The product of resistance and current in an arm of the circuit is taken positive if the direction of current in that arm is in the same sense as one moves in a closed loop and is taken negative if the direction of current in that arm is opposite to the sense as one moves in the closed loop.

Applications of Kirchhoff's Laws

- If 12 wires each of resistance R ohm, are connected to form a skeleton cube, then the total resistance between two diagonally opposite corners of the cube = $5R/6$ ohm,
- If 12 wires; each of resistance R ohm are connected to form a skeleton cube, then the total resistance between the corners of the same edge of cube = $7R/12$ ohm.
- If 11 wires each of resistance R ohm, are connected to form a skeleton cube, then the total resistance from one end of vacant edge to the other end is = $7R/5$ ohm.

Illustration 8

Calculate the current in the arm AD as shown in figure. Each resistance is of 10 Ω.



Soln.: The current distribution is shown in the figure. By using Kirchhoff's laws, for mesh $ABCDA$,

$$10(I - I_1) + 10I_2 - 20I_1 = 0$$

or $\quad -3I_1 + I_2 = -I$ ...(i)

For mesh $BEFCB$,

$$20(I - I_1 - I_2) - 10I_2 - 10(I_1 + I_2) = 0$$

or $2(I - I_1 - I_2) - I_2 - I_1 - I_2 = 0$

or $-3I_1 - 4I_2 = -2I$ ...(ii)

Solving equation (i) and (ii), we get

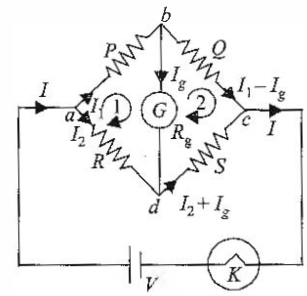
$$\therefore \quad I_1 = \frac{2I}{5},\ I_2 = \frac{I}{5}$$

Current in arm $AD = I_1$

$$I_{AD} = \frac{2I}{5}.$$

WHEATSTONE BRIDGE

It is an arrangement of four resistors P, Q, R and S, such that if we know the value of the resistances of any three of them, we can obtain the value of fourth unknown resistance.



In loop-1, $abda$

$$I_1P + I_gR_g - I_2R = 0 \qquad \text{...(i)}$$

In loop-2, $bcdb$

$$(I_1 - I_g)Q - (I_2 + I_g)S - I_gR_g = 0 \qquad \text{...(ii)}$$

Wheatstone bridge is said to be in balanced condition if electric current I_g flowing through the galvanometer becomes zero. So in balanced Wheat stone bridge, $I_g = 0$ and equations (i) and (ii) become

$$I_1P - I_2R = 0 \text{ or } I_1P = I_2R \qquad \text{...(iii)}$$

$$I_1Q - I_2S = 0 \text{ or } I_1Q = I_2S \qquad \text{...(iv)}$$

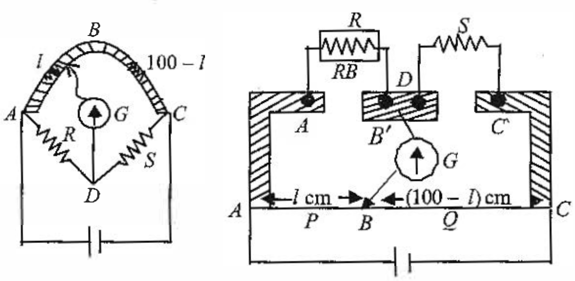
Dividing equation (iii) and (iv), we get

$$\frac{P}{Q} = \frac{R}{S}$$

This gives us equation of balanced Wheatstone bridge.

METER BRIDGE

Meter Bridge is a device which works on the principle of wheat stone bridge and is used for measuring an unknown resistance.



A comparison between the connection of wheatstone bridge and meter bridge can be done in the above figures. In the figure we have assumed a 100 cm long wire where l cm length between AB serves as resistance $P = \rho\frac{l}{A}$ and $(100 - l)$ cm length between BC serves as resistance $Q = \rho\left(\frac{100-l}{A}\right)$. Similarly in meter Bridge where null deflection is observed at point B, the length l cm of AB serve as resistance P and length BC $(100 - l)$ cm serve as resistance Q. It is also assumed that in the copper strip the potential is same everywhere.

Measuring an Unknown Resistance

Unknown resistance wire is attached between strip points D and C and some resistance is taken out from the resistance box. Terminal of galvanometer attached with jockey is touched at different points on wire AC for a null point, the position of null point on wire is treated as point B.

At balance condition,

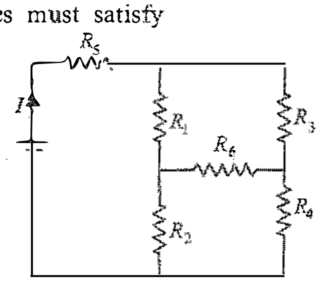
$$\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{\rho\frac{l}{A}}{\rho\frac{100-l}{A}} = \frac{R}{S}$$

So, unknown resistance,

$$S = \frac{R(100-l)}{l}$$

Illustration 9

In the given circuit, it is observed that the current I is independent of the value of the resistance R_6. Then the resistance values must satisfy



(a) $R_1R_2R_5 = R_3R_4R_6$

(b) $\frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}$

(c) $R_1R_4 = R_2R_3$

(d) $R_1R_3 = R_2R_4$

Soln.: (c) In Wheatstone's bridge, while it is balanced, no current flows through the galvanometer arm. Current is independent of galvanometer resistance. Along those lines, it can inferred that I can be independent of R_6 only when R_1, R_2, R_3 and R_4 belong to resistance arms and R_6 belongs to galvanometer arm.

In balanced bridge,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ or } R_1R_4 = R_2R_3.$$

Potentiometer

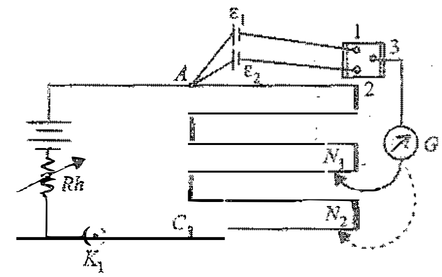
- **Principle of potentiometer :** It is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross-section and a constant current is flowing through it.

  i.e., $V \propto l$ (If I and A are constant)

  or $V = Kl$

  where K is known as potential gradient *i.e.*, fall of potential per unit length of the given wire.

- **Comparison of emfs of two cells by using potentiometer**

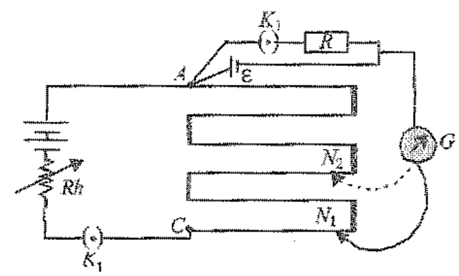


G is galvanometer and Rh is rheostat. 1, 2, 3, are terminals of a two way key

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$

where l_1, and l_2 are the balancing lengths of potentiometer wire for the emfs ε_1 and ε_2 of two cells respectively.

- **Determination of internal resistance of a cell by potentiometer**



$$r = \left(\frac{l_1 - l_2}{l_2}\right)R$$

where l_1 = balancing length of potentiometer wire corresponding to emf of the cell, l_2 = balancing length of potentiometer wire corresponding to terminal potential difference of the cell when a resistance R is connected in series with the cell whose internal resistance is to be determined.

Illustration 10

In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?

Soln.: The potential gradient remain the same, as no change in the setting of standard circuit.

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\frac{1.25}{E_2} = \frac{35}{63}$$

$$E_2 = 2.25 \text{ V}$$