UNIT 3

Magnetic Effects of Current and Magnetism

MAGNETIC FIELD

- It is a region or space around a magnet or current carrying conductor or a moving charge, in which its magnetic effect can be felt.
- The magnetic effect of electric current was first discovered by Oersted in 1820.

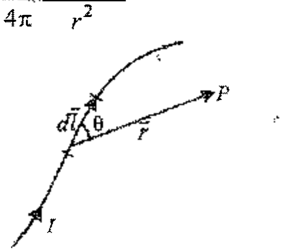
  The sources of magnetic fields are:
  - a current carrying conductor
  - changing electric field
  - moving charged particle
  - permanent magnet and electromagnet etc.
- Magnetic field is a vector quantity and its dimensional formula is $[ML^0T^{-2}A^{-1}]$.
- The SI unit of magnetic field is tesla (T) or weber/metre^2 (Wb/m^2). The CGS unit of magnetic field is gauss (G).

  1 tesla = 10^4 gauss
- Conventionally the direction of the field perpendicular to the plane of the paper is represented by $\otimes$ if into the page, and by $\odot$ if out of the page.

BIOT-SAVART LAW

- A current carrying wire produces a magnetic field around it. Biot-Savart law states that magnitude of intensity of small magnetic field $d\vec{B}$ due to current I carrying element $d\vec{l}$ at any point P at distance r from it is given by

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$



where θ is the angle between $\vec{r}$ and $d\vec{l}$ and $\mu_0 = 4\pi \times 10^{-7}$ T mA^{-1} is called permittivity of free space.

In vectorial form

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3}$$

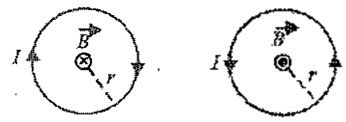
So the direction of $d\vec{B}$ is perpendicular to the plane containing $\vec{r}$ and $d\vec{l}$.

S.I. unit of magnetic field strength is Tesla denoted by T and CGS unit is Gauss denoted by G, where 1 T = 10^4G.

Applications of Biot-Savart Law

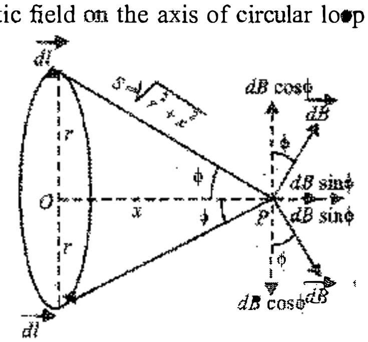
- Magnetic field strength at any point at centre of circular loop carrying current I and radius r is

$$B = \frac{\mu_0 I}{4\pi r^2}\int_0^{2\pi r} dl\sin 90^\circ$$



or $\quad B = \frac{\mu_0 I}{4\pi r^2}\times 2\pi r$ or $B = \frac{\mu_0 I}{2r}$

  - Directed inwards if the current is flowing in the clockwise direction.
  - Directed outwards if the current is flowing in the anticlockwise direction.
- Magnetic field on the axis of circular loop



Small magnetic field due to current element $Id\vec{l}$ of circular loop of radius r at point P at distance x from its centre is

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90^\circ}{s^2} = \frac{\mu_0}{4\pi}\frac{Idl}{\left(r^2+x^2\right)}$$

Component $dB\cos\phi$ due to current element at point P is cancelled by equal and opposite component $dB\cos\phi$ of another diagonally opposite current element, whereas the sine components $dB\sin\phi$ add up to give net magnetic field along the axis. So net magnetic field at point P due to entire loop is

$$B = \oint dB\sin\phi = \int_0^{2\pi r}\frac{\mu_0}{4\pi}\frac{Idl}{\left(r^2+x^2\right)}\frac{r}{\left(r^2+x^2\right)^{1/2}}$$

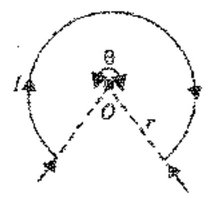
$$B = \frac{\mu_0 Ir}{4\pi\left(r^2+x^2\right)^{3/2}} \int_0^{2\pi r} dl$$

or $$B = \frac{\mu_0 Ir}{4\pi\left(r^2+x^2\right)^{3/2}} . 2\pi r$$

or $$B = \frac{\mu_0 Ir^2}{2\left(r^2+x^2\right)^{3/2}}$$ directed along the axis,

- towards the loop if current in it is in clockwise direction.
- away from the loop if current in it is in anticlockwise direction.

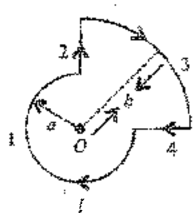
- If the point P is far away from the centre of loop *i.e.*, $x >>> r$, then magnetic field strength at point P is
$$B = \frac{\mu_0 Ir^2}{2x^3} = \frac{\mu_0 I\pi r^2}{2\pi x^3} \text{ or } B = \frac{\mu_0 IA}{2\pi x^3}$$
where $A = \pi r^2$ is area of circular loop.
- If the circular loop has N turns then magnetic field strength at its centre is $B = \frac{\mu_0 NI}{2r}$ and at any point on the axis of circular loop is
$$B = \frac{\mu_0 NIr^2}{2\left(r^2+x^2\right)^{3/2}}$$
- Magnetic field strength at the centre O of circular arc of angle θ carrying current I is



$$B = \frac{\mu_0 I\theta}{4\pi r}$$

Illustration 1

Find the magnetic field intensity at the point O in the figure, when current I flows in the loop as shown.



Soln.: $\vec{B}_O = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$

The magnetic field at O is the vector sum of the magnetic field at O due to the segments 1, 2, 3 and 4.

$\vec{B}_2 = \vec{B}_4 = 0$ as $\vec{dl} \parallel \vec{r}$ for the elements in the segments 2 and 4.

Hence $d\vec{B} = \left(\frac{\mu_0 I}{4\pi}\right)(\vec{dl} \times \vec{r}) = 0$ for them.

Now, $\vec{B}_1 = \frac{\mu_0 I}{4\pi a}\left(\frac{3\pi}{2}\right) \otimes$ and $\vec{B}_3 = \frac{\mu_0 I}{4\pi b}\left(\frac{\pi}{2}\right) \otimes$

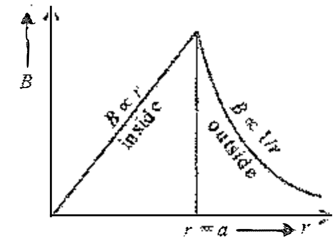
$$\Rightarrow \quad \vec{B}_O = \frac{\mu_0 I}{8}\left[\frac{3}{a} + \frac{1}{b}\right] \otimes$$

AMPERE'S CIRCUITAL LAW

- It states that the line integral of magnetic field around any closed path in vacuum is equal to μ_0 times the total current passing the closed path.
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$
- Ampere's circuital law is analogous to Gauss's law in electrostatics.

Applications of Ampere's Circuital Law

- **Magnetic field due to an infinitely long straight solid cylindrical wire of radius a, carrying current I**
  - Magnetic field at a point outside the wire *i.e.* $(r > a)$ is
  $$B = \frac{\mu_0 I}{2\pi r}$$
  - Magnetic field at a point inside the wire *i.e.* $(r < a)$ is
  $$B = \frac{\mu_0 Ir}{2\pi a^2}$$
  - Magnetic field at a point on the surface of a wire *i.e.* $(r = a)$ is
  $$B = \frac{\mu_0 I}{2\pi a}$$
  - The variation of magnetic field B and the distance r from axis is as shown in the figure.



Note : If the cylindrical wire is hollow *i.e.* it is in the form of pipe, then the magnetic field inside the wire is zero. The variation of magnetic field and distance r from the axis is as shown in the figure.

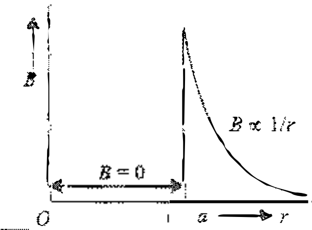
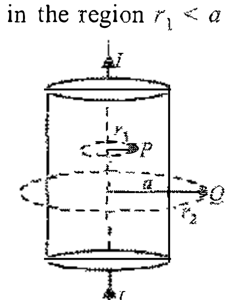


Illustration 2

Figure shows a long straight wire of a circular cross-section with radius 'a' carrying steady current I. The current I is uniformly distributed across this cross-section. Calculate the magnetic field in the region $r_1 < a$ and $r_2 > a$.



Soln.: The current I is uniformly distributed in the wire, current per unit area can be calculated as $\frac{I}{\pi a^2}$

(a) To calculate magnetic field at P, let us consider a loop of radius r_1 and apply Ampere's law.

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{net}$$

$$B2\pi r_1 = \mu_0\left[\frac{I}{\pi a^2}\pi r_1^2\right]$$

$$B = \frac{\mu_0 I r_1}{2\pi a^2} \text{ or } B \propto r_1$$

(b) To calculate magnetic field at Q, let us consider a loop of radius r_2 and apply Ampere's law.

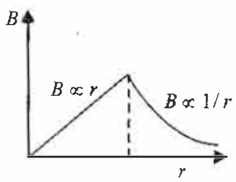
$$\oint B\cdot dl = \mu_0 I_{net}$$

$$B \times 2\pi r_2 = \mu_0 I$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r_2}$$

$$B \propto \frac{1}{r_2}$$

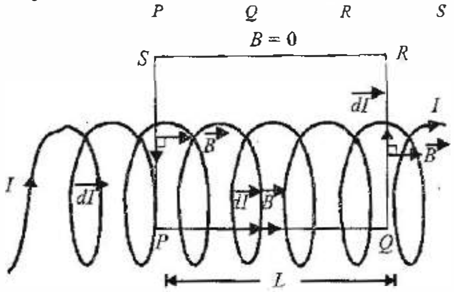
A graph can be plotted showing variation of magnetic field B with distance r from centre of wire.



- **Magnetic field strength at centre of long solenoid**

Let a solenoid consists of N turns per unit length and carry current I. Then magnetic field lines inside the solenoid are parallel to its axis, whereas outside the solenoid, magnetic field is zero. Line integral of magnetic field over closed loop $PQRS$ shown in figure is

$$\oint \vec{B}\cdot d\vec{l} = \int_P^Q \vec{B}\cdot d\vec{l} + \int_Q^R \vec{B}\cdot d\vec{l} + \int_R^S \vec{B}\cdot d\vec{l} + \int_S^P \vec{B}\cdot d\vec{l}$$



$$= \int_P^Q B\,dl\cos 0 + \int_Q^R B\,dl\cos 90^\circ + 0 + \int_S^P B\,dl\cos 90^\circ$$

$$= B\int_{PQRS} dl + 0 + 0 + 0 = B\cdot L \qquad \text{...(i)}$$

But by Ampere's circuital law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \times \text{Total current threading loop } PQRS$$

$= \mu_0 \times$ number of turns of solenoid in loop $PQRS \times I$

$= \mu_0 NLI$ ...(ii)

So, by equations (i) and (ii)

$$B\cdot L = \mu_0 NLI \text{ or } B = \mu_0 NI$$

Which gives magnetic field strength inside straight current carrying solenoid, directed along the axis of solenoid.

- At the ends of the long current carrying solenoid, magnetic field strength is,

$$B = \frac{\mu_0 NI}{2}$$

- **Magnetic field inside the toroid**

When current I is passed through a toroid having N turns per unit length and of mean radius r, then magnetic field lines set up inside the toroid in the form of concentric circles. Let one such loop be of radius r, then line integral of magnetic field over that closed loop is

$$\oint \vec{B}\cdot d\vec{l} = \oint \vec{B}\cdot d\vec{l}\cos 0 = B\oint dl = B\cdot 2\pi r \qquad \text{...(i)}$$

But by Ampere's circuital law

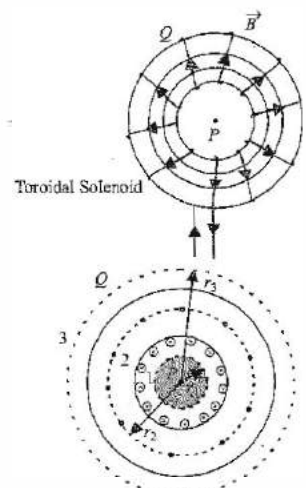
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \times \text{total current threading the toroid}$$

$= \mu_0 \times$ total number of turns in toroid $\times I$

$= \mu_0 N 2\pi r I$ ...(ii)

By equations (i) and (ii) $B.2\pi r = \mu_0 N\, 2\pi r I$ or $B = \mu_0 NI$

This gives magnetic field at any point inside the toroid, directed at any point along the tangent to concentric circular magnetic line of force at that point inside the toroid.



Cross sectional view of the toroidal solenoid

FORCE ON A CHARGED PARTICLE IN A UNIFORM ELECTRIC FIELD

- When a charged particle of charge q moving into a uniform electric field $\vec{E}$, the force acting on it is given by $\vec{F} = q\vec{E}$.
- The direction of $\vec{F}$ is same as that of $\vec{E}$ is positive if q is +ve and $-\vec{E}$ if q is –ve.

FORCE ON A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

- When a charged particle of charge q, moving with velocity $\vec{v}$ is subjected to a uniform magnetic field $\vec{B}$, the force acting on it is

$\vec{F} = q(\vec{v} \times \vec{B})$ or $F = qvB \sin\theta$

where θ is the angle between $\vec{v}$ and $\vec{B}$.

- The direction of this force is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.

  $\vec{F} = 0$ if $\vec{v} = 0$, *i.e.* a charge at rest does not experience any magnetic force.

  $\vec{F} = 0$ if $\theta = 0$ or $180°$ *i.e.*, the magnetic force vanishes if $\vec{v}$ is either parallel or antiparallel to the direction of $\vec{B}$.
- Force will be maximum if $\theta = 90°$, *i.e.*, if $\vec{v}$ is perpendicular to $\vec{B}$, the magnetic force has a maximum value and is given by $F_{max} = qvB$.

Motion of a Charged Particle in a Uniform Magnetic Field

- When a charged particle of charge q and mass m moves with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$, the force acting on it is $F = qvB \sin\theta$. The following two case arise :
- **Case I :** When the charged particle is moving perpendicular to the field *i.e.* $\theta = 90°$.
  - In this case path is circular.
  - Radius of circular path is

$$R = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{qB}.$$

  where K is the kinetic energy of a charged particle.

  - Time period of revolution is $T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$
  - The frequency, $\upsilon = \frac{1}{T} = \frac{qB}{2\pi m}$
  - The angular frequency, $\omega = 2\pi\upsilon = \frac{qB}{m}$
- **Case II :** When the charged particle is moving at an angle θ to the field (other than $0°$, $90°$ or $180°$).

  In this case, path is helical.
  - Due to perpendicular component of $\vec{v}$, to $\vec{B}$, *i.e.*, $v_\perp = v\sin\theta$, the particle describes a circular path of radius R, such that

$$\frac{mv_\perp^2}{R} = qv_\perp B \quad \text{or} \quad R = \frac{mv_\perp}{qB} = \frac{mv\sin\theta}{qB}$$

  - Time period of revolution is

$$T = \frac{2\pi R}{v\sin\theta} = \frac{2\pi m}{qB}$$

  - The frequency is $\upsilon = \frac{qB}{2\pi m}$
  - The angular frequency is $\omega = 2\pi\upsilon = \frac{qB}{m}$
  - The pitch of the helical path is

$$p = v_\parallel \times T = v\cos\theta \times T = \frac{2\pi mv}{qB}\cos\theta = \frac{2\pi R}{\tan\theta}$$

Force on a Charged Particle in Combined Uniform Electric and Magnetic Fields

- When a charged particle of charge q moving with velocity $\vec{v}$ is subjected to an electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force acting on the particle is

$$\vec{F} = \vec{F}_E + \vec{F}_M = q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E} + \vec{v} \times \vec{B})$$

- This force is known as Lorentz force and is named after the Dutch physicist Hendrik Anton Lorentz.

CYCLOTRON

- It is a heavy particle accelerator, invented in 1929 by E.O. Lawrence for accelerating charged particles such as protons, deuterons, or alpha particles to high velocities.

  Cyclotron frequency, $\upsilon = \frac{Bq}{2\pi m}$

  where m and q are mass and charge of the particle and B is the strength of magnetic field.

Illustration 3

A beam of protons enters a uniform magnetic field of 0.3 T, with a velocity of 4×10^5 m s^{-1}, in a direction making an angle of $60°$ with the direction of magnetic field. Determine

(i) the path of motion of the particles

(ii) the radius of the path of the particles

(iii) the pitch of the helix.

Soln.: (i) When the charged particle enters a magnetic field at angle other than $90°$, it follows a helical path.

(ii) Force on proton = Bev

$$\text{Centripetal force} = \frac{mv^2}{r}$$

$$\therefore \quad Bev = \frac{mv^2}{r}$$

$$\text{or} \quad r = \frac{mv}{Be} = \frac{(1.67 \times 10^{-27}) \times (4 \times 10^5)}{0.3 \times (1.6 \times 10^{-19})}$$

Radius of the path, $r = 0.014$ m

(iii) Pitch of the path

It is the linear distance covered by the charged particle, in one time period, in the direction of magnetic field.

$$\text{Pitch} = \frac{2\pi mv\cos\theta}{eB} = (2\pi\cos\theta) \times r$$

$$= 2 \times 3.14 \times \cos 60° \times 0.014 = 0.04 \text{ m.}$$

Force on a Current Carrying Conductor in a Uniform Magnetic Field

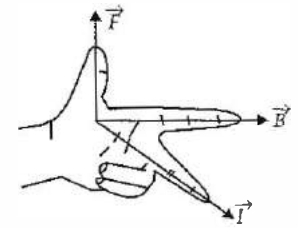
- The force experienced by a straight conductor of length l carrying current I when placed in a uniform magnetic field $\vec{B}$ is

$$\vec{F} = I(\vec{l} \times \vec{B}); \quad F = IlB \sin\theta$$

  where θ is the angle between $\vec{l}$ and $\vec{B}$.

  If $\theta = 0°$, then $F = 0$ (minimum)

  If $\theta = 90°$, then $F = BIl$ (maximum)
- The direction of this force is given by Fleming's left hand rule.
- **Fleming's left hand rule :** Stretch the fore-finger, central finger and thumb of left hand mutually perpendicular. Then if the fore-finger is along the direction of field ($\vec{B}$), the central finger in the direction of current I, the thumb gives the direction of force as shown in the figure.



Force Between Two Parallel Current Carrying Conductors

- Two parallel conductors carrying currents in the same direction attract each other while those carrying currents in the opposite direction repel each other.
- When two parallel conductors separated by a distance r carry currents I_1 and I_2, the magnetic field of one will exert a force on the other. The force per unit length on either conductor is
$$f = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$
- The force of attraction or repulsion acting on each conductor of length l due to currents in two parallel conductor is
$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}l.$$
- If two linear current carrying conductors of unequal length are held parallel to each other, then the force on a long conductor is due to magnetic field interaction due to currents of short conductor and long conductor. If l, L be the lengths of short and long conductor and I_1, I_2 are the currents through short and long conductors respectively and r is the separation between these two parallel conductors, then the force on long conductor is equal to the force on short conductor $= \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}l.$

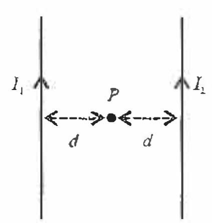
Illustration 4

Two parallel, long wires carry currents I_1 and I_2 with $I_1 > I_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of I_2 is reversed, the field becomes 30 μT. Find the value of $\frac{I_1}{I_2}$.

Soln.: Case 1

At P, $B_1 = \frac{\mu_0 I_1}{2\pi d} - \frac{\mu_0 I_2}{2\pi d}$

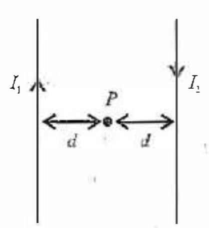
$B_1 = \frac{\mu_0(I_1 - I_2)}{2\pi d} = 10\ \mu T$



Case 2

At P, $B_2 = \frac{\mu_0 I_1}{2\pi d} + \frac{\mu_0 I_2}{2\pi d}$

$= \frac{\mu_0}{2\pi d}(I_1 + I_2) = 30\ \mu T$



$$\frac{B_1}{B_2} = \frac{I_1 - I_2}{I_1 + I_2} = \frac{10\ \mu T}{30\ \mu T}$$

$$\Rightarrow \frac{I_1 - I_2}{I_1 + I_2} = \frac{1}{3}$$

Using Componendo and Dividendo rule,

$$\frac{(I_1 - I_2) + (I_1 + I_2)}{(I_1 - I_2) - (I_1 + I_2)} = \frac{1+3}{1-3} \Rightarrow \frac{I_1}{-I_2} = \frac{4}{-2} \text{ or } \frac{I_1}{I_2} = 2$$

Torque on a Current Carrying Coil Placed in a Uniform Magnetic Field

- When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but different parts of the coil experience forces in different directions. Due to it, the coil may experience a torque or couple.
- When a coil of area A having N turns, carrying current I is placed in a uniform magnetic field B, it will experience torque which is given by
$$\tau = NIAB\sin\theta = MB\sin\theta$$
where magnetic dipole moment $M = NIA$ and θ is the angle between the direction of magnetic field and normal to the plane of the coil.
- If the plane of the coil is perpendicular to the direction of magnetic field *i.e.* $\theta = 0°$, then
$$\tau = 0 \text{ (minimum)}$$
- If the plane of the coil is parallel to the direction of magnetic field *i.e.* $\theta = 90°$, then
$$\tau = NIAB \text{ (maximum)}$$
- If α is the angle between plane of the coil and the magnetic field, then torque on the coil is
$$\tau = NIAB\cos\alpha = MB\cos\alpha$$
Potential energy of the coil is
$$U = -\vec{M}\cdot\vec{B}$$
- Workdone in rotating the coil through an angle θ from the field direction is
$$W = MB(1 - \cos\theta)$$

MOVING COIL GALVANOMETER

- It is an instrument used for the detection and measurement of small currents.

Principle of a Moving Coil Galvanometer

- When a current carrying coil is placed in a magnetic field, it experiences a torque.
- In moving coil galvanometer the current I passing through the galvanometer is directly proportional to its deflection (θ).
$I \propto \theta$ or, $I = G\theta$.
where $G = \frac{k}{NAB}$ = galvanometer constant
A = area of a coil, N = number of turns in the coil,
B = strength of magnetic field, k = torsional constant of the spring *i.e.* restoring torque per unit twist.
- **Current sensitivity :** It is defined as the deflection produced in the galvanometer, when unit current flows through it.
$$I_s = \frac{\theta}{I} = \frac{NAB}{k}$$
The unit of current sensitivity is rad A^{-1} or div A^{-1}.
- **Voltage sensitivity :** It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.
$$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NAB}{kR}.$$