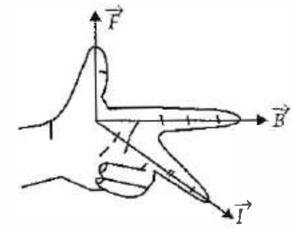


Force Between Two Parallel Current Carrying Conductors

- Two parallel conductors carrying currents in the same direction attract each other while those carrying currents in the opposite direction repel each other.
- When two parallel conductors separated by a distance r carry currents I_1 and I_2, the magnetic field of one will exert a force on the other. The force per unit length on either conductor is

$$f = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$$

- The force of attraction or repulsion acting on each conductor of length l due to currents in two parallel conductor is

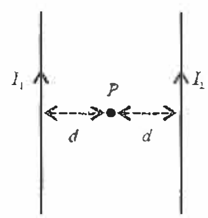
$$F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}l.$$

- If two linear current carrying conductors of unequal length are held parallel to each other, then the force on a long conductor is due to magnetic field interaction due to currents of short conductor and long conductor. If l, L be the lengths of short and long conductor and I_1, I_2 are the currents through short and long conductors respectively and r is the separation between these two parallel conductors, then the force on long conductor is equal to the force on short conductor $= \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}l$.

Illustration 4

Two parallel, long wires carry currents I_1 and I_2 with $I_1 > I_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of I_2 is reversed, the field becomes 30 μT. Find the value of $\frac{I_1}{I_2}$.

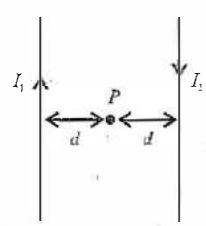
Soln.: Case 1



At P, $B_1 = \frac{\mu_0 I_1}{2\pi d} - \frac{\mu_0 I_2}{2\pi d}$

$B_1 = \frac{\mu_0(I_1 - I_2)}{2\pi d} = 10\ \mu\text{T}$

Case 2



At P, $B_2 = \frac{\mu_0 I_1}{2\pi d} + \frac{\mu_0 I_2}{2\pi d}$

$= \frac{\mu_0}{2\pi d}(I_1 + I_2) = 30\ \mu\text{T}$

$\frac{B_1}{B_2} = \frac{I_1 - I_2}{I_1 + I_2} = \frac{10\,\mu\text{T}}{30\,\mu\text{T}}$

$\Rightarrow \frac{I_1 - I_2}{I_1 + I_2} = \frac{1}{3}$

Using Componendo and Dividendo rule,

$$\frac{(I_1 - I_2) + (I_1 + I_2)}{(I_1 - I_2) - (I_1 + I_2)} = \frac{1+3}{1-3} \Rightarrow \frac{I_1}{-I_2} = \frac{4}{-2} \text{ or } \frac{I_1}{I_2} = 2$$

Torque on a Current Carrying Coil Placed in a Uniform Magnetic Field

- When a current carrying coil is placed in a uniform magnetic field, the net force on it is always zero but different parts of the coil experience forces in different directions. Due to it, the coil may experience a torque or couple.
- When a coil of area A having N turns, carrying current I is placed in a uniform magnetic field B, it will experience torque which is given by

  $$\tau = NIAB\sin\theta = MB\sin\theta$$

  where magnetic dipole moment $M = NIA$ and θ is the angle between the direction of magnetic field and normal to the plane of the coil.
- If the plane of the coil is perpendicular to the direction of magnetic field *i.e.* $\theta = 0°$, then

  $$\tau = 0 \text{ (minimum)}$$

- If the plane of the coil is parallel to the direction of magnetic field *i.e.* $\theta = 90°$, then

  $$\tau = NIAB \text{ (maximum)}$$

- If α is the angle between plane of the coil and the magnetic field, then torque on the coil is

  $$\tau = NIAB\cos\alpha = MB\cos\alpha$$

  Potential energy of the coil is

  $$U = -\vec{M}\cdot\vec{B}$$

- Workdone in rotating the coil through an angle θ from the field direction is

  $$W = MB(1 - \cos\theta)$$

MOVING COIL GALVANOMETER

- It is an instrument used for the detection and measurement of small currents.

Principle of a Moving Coil Galvanometer

- When a current carrying coil is placed in a magnetic field, it experiences a torque.
- In moving coil galvanometer the current I passing through the galvanometer is directly proportional to its deflection (θ).

  $I \propto \theta$ or, $I = G\theta$.

  where $G = \frac{k}{NAB}$ = galvanometer constant

  A = area of a coil, N = number of turns in the coil,

  B = strength of magnetic field, k = torsional constant of the spring *i.e.* restoring torque per unit twist.
- **Current sensitivity :** It is defined as the deflection produced in the galvanometer, when unit current flows through it.

  $$I_s = \frac{\theta}{I} = \frac{NAB}{k}$$

  The unit of current sensitivity is rad A^{-1} or div A^{-1}.
- **Voltage sensitivity :** It is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

  $$V_s = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NAB}{kR}.$$

The unit of voltage sensitivity is rad V^{-1} or div V^{-1}.

$$V_s = \frac{1}{R} I_s.$$

AMMETER

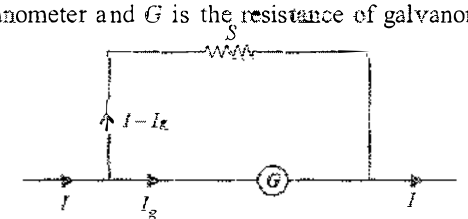
- It is an instrument used to measure current in an electrical circuit.

Conversion of Galvanometer into an Ammeter

- A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer, whose value is given by

$$S = \left(\frac{I_g}{I - I_g}\right) G$$

where I_g is the current for full scale deflection of galvanometer, I is the current to be measured by the galvanometer and G is the resistance of galvanometer.



- Ammeter is a low resistance instrument and it is always connected in series to the circuit. An ideal ammeter has zero resistance.

Illustration 5

A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

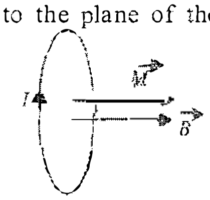
(a) total torque on the coil,

(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, the free electron density in copper is given to be at 10^{29} m^{-3}.)

Soln.: The magnetic field is normal to the plane of the coil, so condition of minimum torque.



(a) Torque on the coil

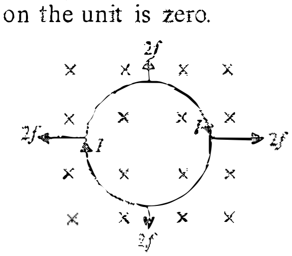
$\tau = NIBA \sin \theta$

Here $\theta = 0°$

$\therefore \quad \tau = 0$

(b) Force on every element of the coil is cancelled by force on corresponding element.

Net force on the unit is zero.



(c) To calculate force on each electron, let us find drift velocity.

$I = Anev_d$

$5 = 10^{-5} \times 10^{29} \times 1.6 \times 10^{-19} v_d$

$v_d = 3.125 \times 10^{-5}$ m s^{-1}

Now force

$F = e v_d B$

$F = 1.6 \times 10^{-19} \times 3.125 \times 10^{-5} \times 0.1$

$F = 5 \times 10^{-25}$ N

VOLTMETER

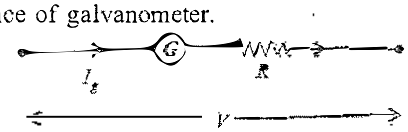
- It is an instrument used to measure potential difference across any element in an electrical circuit.

Conversion of Galvanometer into Voltmeter

- A galvanometer can be converted into voltmeter of given range by connecting a suitable resistance R in series with the galvanometer, whose value is given by

$$R = \frac{V}{I_g} - G$$

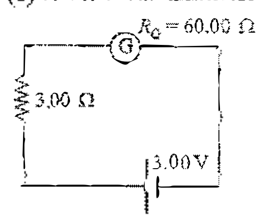
where V is the voltage to be measured, I_g is the current for full scale deflection of galvanometer and G is the resistance of galvanometer.



- Voltmeter is a high resistance instrument and it is always connected in parallel with the circuit element across which potential difference is to be measured. An ideal voltmeter has infinite resistance.
- In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is $S = G/(n - 1)$.
- In order to increase the range of voltmeter n times the value of resistance to be connected in series with galvanometer is $R = (n - 1)G$.

Illustration 6

In the circuit given below the current is to be measured. What is the value of the current if the ammeter shown (a) is a galvanometer with a resistance $R_G = 60.00\ \Omega$; (b) is a galvanometer described in (a) but converted to an ammeter by a shunt resistance $r_s = 0.02\ \Omega$; (c) is an ideal ammeter with zero resistance?



Soln.: (a) The ammeter shown is a galvanometer with resistance

$R_G = 60.00\ \Omega$

Current in the circuit

$$I = \frac{3.00}{63} = 0.048 \text{ A}$$

(b) Now the galvanometer is converted into ammeter by using shunt resistance. So the equivalent resistance of ammeter

$$R_A = \frac{r_s R_G}{r_s + R_G} = \frac{0.02 \times 60}{60.02} \approx 0.02\Omega$$

Now, total resistance in circuit $R = 3.02\ \Omega$

Hence current $I = \dfrac{3}{3.02} = 0.99$ A

(c) For an ideal ammeter the resistance is zero, and the reading of current is accurate.

$I = 3/3 = 1.00$ A