MAGNETIC DIPOLE MOMENT

- It is defined as the product of strength of either pole (m) and the magnetic length ($2\vec{l}$) of the magnet. It is denoted by $\vec{M}$.
  Magnetic dipole moment = Strength of either pole × magnetic length
  $$\vec{M} = m(2\vec{l})$$
- Magnetic dipole moment is a vector quantity and it is directed from south to north pole of the magnet.
- The SI unit of magnetic dipole moment is A m^2.
- If a magnet of moment M and pole strength m is cut into two equal parts along its length, then pole strength of each part is $m/2$ and the magnetic moment of each part is $M/2$.
- If a magnet of magnetic moment M and pole strength m is cut into two equal halves along perpendicular to its length, the pole strength of each part is m and magnetic moment of each part is $M/2$.

Magnetic Field at a Point due to Magnetic Dipole (bar magnet)

- The magnetic field due to a bar magnet at any point on the axial line (end on position) is
  $$B_{axial} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2 - l^2)^2}$$
  where r = distance between the centre of the magnet and the given point on the axial line, $2l$ = magnetic length of the magnet and M = magnetic moment of the magnet.
- For short magnet $l^2 \ll r^2$
  $$B_{axial} = \frac{\mu_0 2M}{4\pi r^3}$$
  The direction of B_{axial} is along SN.
- The magnetic field due to a bar magnet at any point on the equatorial line (board-side on position) of the bar magnet is, $B_{equatorial} = \dfrac{\mu_0 M}{4\pi(r^2 + l^2)^{3/2}}$
  For short magnet, $B_{equatorial} = \dfrac{\mu_0 M}{4\pi r^3}$
  The direction of $B_{equatorial}$ is parallel to NS.

Torque on a Magnetic Dipole placed in a Uniform Magnetic Field

- When a magnetic dipole of dipole moment $\vec{M}$ is placed in a uniform magnetic field $\vec{B}$, it will experience a torque and is given by
  $$\vec{\tau} = \vec{M} \times \vec{B} \quad \text{or} \quad \tau = MB\sin\theta$$
  where θ is the angle between $\vec{M}$ and $\vec{B}$
- Torque acting on a dipole is maximum ($\tau_{max} = MB$) when dipole is perpendicular to the field and minimum ($\tau = 0$) when dipole is parallel or antiparallel to the field.
- When a dipole is placed in a uniform magnetic field, it will experience only torque and the net force on the dipole is zero while when it is placed in a non uniform magnetic field, it will experience both torque and net force.

Work done in Rotating the Magnetic Dipole in a Uniform Magnetic Field

- Work done in rotating the magnetic dipole from θ_1 to θ_2 with respect to uniform magnetic field is
  $$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = -MB(\cos\theta_2 - \cos\theta_1) = MB(\cos\theta_1 - \cos\theta_2)$$
- If the dipole is rotated from field direction *i.e.* $\theta_1 = 0°$ to position θ *i.e.* $\theta_2 = \theta$
  $\therefore \quad W = MB(1 - \cos\theta)$.

Potential Energy of a Magnetic Dipole

- Potential energy of a magnetic dipole in a uniform magnetic field is
  $$U = -\vec{M}\cdot\vec{B} = -MB\cos\theta$$
- The potential energy of the dipole will be minimum ($= -MB$) when $\theta = 0°$, *i.e.*, the dipole is parallel to the field, and maximum ($= MB$) when $\theta = 180°$, *i.e.*, the dipole is antiparallel to the field.

Current Loop as a Magnetic Dipole

- A current loop behaves as a magnetic dipole whose magnetic dipole moment is $M = IA$ where A is the area enclosed by loop and I is the current flowing in the loop.
- If there are N turns in a loop, then $M = NIA$.

Illustration 7

The apparent dips in two mutually perpendicular planes are δ_1 and δ_2. Find true dip δ.

Soln.: Here $\tan\delta = \dfrac{B_V}{B_H}$ or $\cot\delta = \dfrac{B_H}{B_V}$

In a plane, at θ with true dip plane/mag. meridian

$$\cot\delta_1 = \frac{B_H\cos\theta}{B_V}$$

In a plane, perpendicular to above plane,

$$\cot\delta_2 = \frac{B_H\sin\theta}{B_V}$$

$$\therefore \cot^2\delta_1 + \cot^2\delta_2 = \frac{B_H^2\cos^2\theta}{B_V^2} + \frac{B_H^2\sin^2\theta}{B_V^2}$$

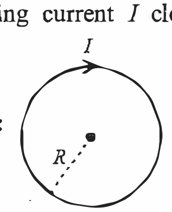
$$\therefore \cot^2\delta_1 + \cot^2\delta_2 = \frac{B_H^2}{B_V^2}(\cos^2\theta + \sin^2\theta)$$

or $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$

Illustration 8

What is the magnetic moment of a circular loop of radius R carrying current I clockwise?

Soln.:

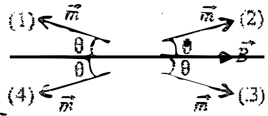


$$\vec{m} = I \times \vec{A} \Rightarrow \vec{m} = I(\pi R^2)\otimes$$

Since, the area enclosed in the loop is πR^2, and applying right hand thumb rule, gives the direction of $\vec{m}$ as $\otimes$ or into the page.

Illustration 9

Figure shows four orientations at angle θ of its magnetic moment $\vec{m}$ in a magnetic field B. Rank the orientations according to the magnitude of the torque on the dipole.



Soln.: $\tau = \vec{m} \times \vec{B}$

$\tau = mB \sin\theta$ where θ is the angle between $\vec{m}$ and $\vec{B}$, when their tails are together.

$\tau_1 = mB \sin(180 - \theta) = mB \sin\theta$

$\tau_2 = mB \sin\theta$

$\tau_3 = mB \sin\theta$

$\tau_4 = mB \sin(180 - \theta) = mB \sin\theta$

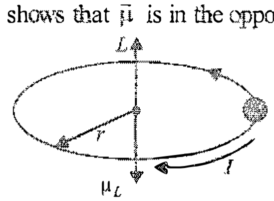
$\tau_1 = \tau_2 = \tau_3 = \tau_4$

Magnetic Dipole Moment of a Revolving Electron

- An electron revolving around the central nucleus in an atom has a magnetic moment and it is given by

$$\vec{\mu}_L = -\frac{e}{2m}\vec{L}$$

- The –ve sign shows that $\vec{\mu}$ is in the opposite direction to $\vec{L}$.



- In magnitude $\mu_L = \frac{e}{2m_e}L$ where L is the magnitude of the angular momentum of the revolving electron.
- **Bohr magneton :** The smallest value of μ_L is known as Bohr magneton.

$$\mu_B = \frac{eh}{2m_e} = 9.274 \times 10^{-24}\ \text{J/T} = 5.788 \times 10^{-5}\ \text{eV/T}.$$

where m_e is the mass of the electron and $\hbar = \frac{h}{2\pi}$.

- **Gyromagnetic ratio :** The ratio $\frac{\mu_L}{L}$ is known as gyromagnetic ratio

$$\frac{\mu_L}{L} = \frac{e}{2m_e}$$

- It is a constant and its value is 8.8×10^{10} C/kg.

GAUSS'S LAW FOR MAGNETISM

- Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero.

$$\phi = \sum_{\substack{\text{all area}\\ \text{elements } \Delta\vec{S}}} \vec{B} \cdot \Delta\vec{S} = 0$$

- This law establishes that isolated magnetic poles do not exist.

EARTH'S MAGNETIC FIELD AND MAGNETIC ELEMENTS

- Three quantities are needed to specify the magnetic field of the earth on its surface – the horizontal component, the magnetic declination and the magnetic dip. These are known as elements of the earth's magnetic field or magnetic elements.
- **Geographical meridian and magnetic meridian**
  The vertical plane passing through the geographical north pole and south pole at given place is known as the geographical meridian of that place. And a vertical plane passing through the axis of a freely suspended or pivoted magnet is known as magnetic meridian.
- **Magnetic declination :** Magnetic declination at a place is defined as the angle between the geographic meridian and magnetic meridian.
- **Magnetic dip or inclination :** Magnetic dip at a place is defined as the angle made by the earth's magnetic field with the horizontal in the magnetic meridian. It is denoted by δ.
- **Horizontal component :** It is component of earth's magnetic field along the horizontal direction in the magnetic meridian. It is denoted by B_H.
- If B is intensity of earth's total magnetic field, then the horizontal component of earth's magnetic field is given by

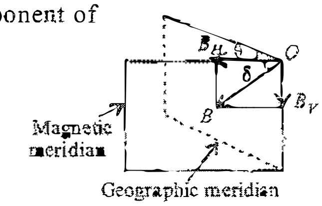
$$B_H = B\cos\delta$$

Also, the vertical component of earth's magnetic field,

$$B_V = B\sin\delta$$

$$\therefore \quad B = \sqrt{B_H^2 + B_V^2}$$

and $\tan\delta = \frac{B_V}{B_H}$



- The earth always has a vertical component except at equator.
- The earth always has a horizontal component except at the poles.
- In a vertical plane at an angle θ to magnetic meridian

$$B'_H = B_H\cos\theta \text{ and } B'_V = B_V$$

$$\therefore \quad \tan\delta' = \frac{B'_V}{B'_H} = \frac{B_V}{B_H\cos\theta} = \frac{\tan\delta}{\cos\theta}$$

$$\tan\delta' = \frac{\tan\delta}{\cos\theta}$$

- If at a given place δ_1 and δ_2 are angles of dip in two arbitrary vertical planes which are perpendicular to each other, the true angle of dip δ is given by

$$\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$$

- Angle of dip δ at a plane is related to its magnetic latitude λ through the relation

$$\tan\delta = 2\tan\lambda$$

CLASSIFICATION OF MAGNETIC MATERIALS

- Magnetic Intensity (H) in vacuum is defined as the ratio of applied magnetic field B_0 to the permeability of free space μ_0 *i.e.*,

$$H = \frac{B_0}{\mu_0}$$

Its S.I. unit is A m^{-1}

- Intensity of magnetisation (I) is defined as the magnetic moment developed per unit volume, when a magnetic specimen is subjected to the magnetising field *i.e.*,