ELECTROMAGNETIC INDUCTION

- It is the phenomenon of generating an emf by changing the number of magnetic lines of force (*i.e.* magnetic flux) associated with the circuit. The emf so generated is known as induced emf. If the circuit is closed the current which flows in it due to induced emf is known as induced current.

Faraday's Laws of Electromagnetic Induction

- **First law :** Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit. This induced emf persists as long as the change in magnetic flux continues.
- **Second law :** The magnitude of the induced emf is equal to the time rate of change of magnetic flux. Mathematically, induced emf is given by

$$\varepsilon = -\frac{d\phi}{dt}$$

where negative sign indicates the direction of ε.

Lenz's law

- This law gives us the direction of induced emf. According to this law, the direction of induced emf in a circuit is such that it opposes the change in magnetic flux responsible for its production. Lenz's law is in accordance with the principle of conservation of energy.

Fleming's Right Hand Rule

- Fleming's right hand rule also gives us the direction of induced emf or current, in a conductor moving in a magnetic field. According to this rule, if we stretch the forefinger, central finger and thumb of our right hand in mutually perpendicular directions such that forefinger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current or emf. The direction of induced current or emf given by Lenz's law and Fleming's right hand rule is the same.

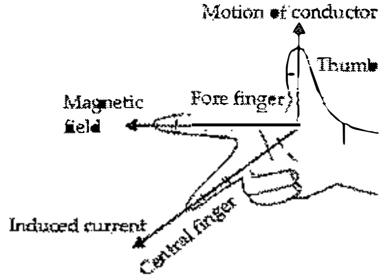
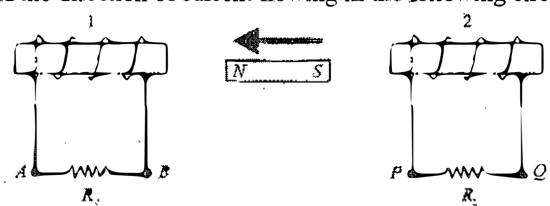
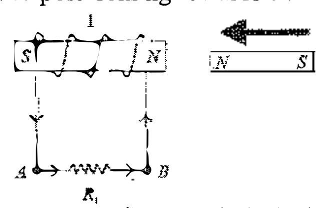


Illustration 1

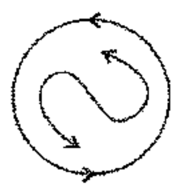
Find the direction of current flowing in the following circuits.



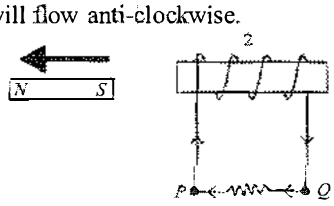
Soln.: Consider the solenoid 1. As the N-pole of the magnet approaches it, the flux through the solenoid increases. The side facing the magnet becomes a N-pole as indicated, as it must oppose the N-pole coming towards it.



N-poles means the current flows anti-clockwise.
Hence the current as seen from the right side of 1 flows anti-clockwise as shown. It means current in R_1 flows from A to B.



- A similar argument shows that left side of solenoid 2, becomes N-pole as it must oppose the movement of S-pole of the magnet going away from it. The left side of solenoid 2 becomes N-pole or current if viewed from that side will flow anti-clockwise.

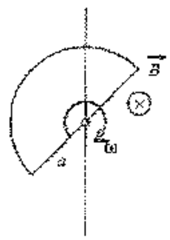


Hence current flows as shown, or from Q to P via R_2.

Illustration 2

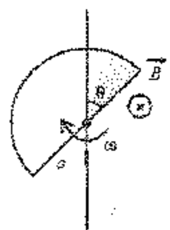
A wire loop enclosing a semi-circle of radius a is located on the boundary of a uniform magnetic field of induction B.

At the moment $t = 0$, the loop of resistance R set into rotation with angular speed ω about an axis coinciding with a line on the boundary. Find the emf induced in the loop as a function of time.



Soln.: Assume the loop to have entered the field region, and let angle be θ.

As the area of the sector shaded is $\frac{1}{2}a^2\theta$,



$$\phi = B\left(\frac{1}{2}a^2\theta\right)\cos 0$$

$$\frac{d\phi}{dt} = \frac{1}{2}a^2 B\left(\frac{d\theta}{dt}\right);$$

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{a^2 B\omega}{2}$$

The Lenz's law indicates that the current flows in the direction indicated.

EDDY CURRENTS

- Eddy currents are basically the currents induced in the body of a conductor due to change in magnetic flux linked with the conductor.
- The direction of eddy currents is given by Lenz's law, or Fleming's right hand rule.

INDUCTOR

- An inductor is a device for storing energy in a magnetic field. An inductor is generally called as inductance. In usual practice a coil or solenoid is treated as inductor. It is denoted by symbol .

Self Induction

- Whenever the current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, an emf is induced in the coil or the circuit which opposes the change that causes it. This phenomenon is known as self induction and the emf induced is known as self induced emf or back emf.
- When a current I flows through a coil and ϕ is the magnetic flux linked with the coil, then

  $\phi \propto I$ or $\phi = LI$

  where L is coefficient of self induction or self inductance of the coil.
- The self induced emf is

  $$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$$
- The SI unit of L is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.
- Self inductance of a solenoid is $L = \mu_0 n^2 lA$ where l is length of the coil solenoid, n is number of turns per unit length of a solenoid and A is area of cross section of the solenoid.
- Self inductance of a circular coil is

  $$L = \frac{\mu_0 N^2 \pi R}{2}$$

  where R is the radius of a coil and N is the number of turns

Mutual Induction

- Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is known as mutual induction. The coil or circuit in which the current changes is known as primary while the other in which emf is set up is known as secondary.
- Let I_P be the current flowing through primary coil at any instant. If ϕ_S is the flux linked with secondary coil then

  $\phi_S \propto I_P$ or $\phi_S = MI_P$

  where M is coefficient of mutual inductance of two coils.
- The emf induced in the secondary coil is given by

  $$\varepsilon_S = -M\frac{dI_P}{dt}.$$
- The SI unit of M is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.
- **Coefficient of coupling (K) :** Coefficient of coupling of two coils is a measure of the coupling between the two coils and is given by

  $$K = \frac{M}{\sqrt{L_1 L_2}}$$

  where L_1 and L_2 are coefficients of self inductance of the two coils and M is coefficient of mutual inductance of the two coils.
- The coefficient of mutual inductance of two long co-axial solenoids, each of length l, area of cross section A, wound on air core is

  $$M = \frac{\mu_0 N_1 N_2 A}{l}$$

  where N_1, N_2 are total number of turns of the two solenoid.

Combination of Inductances

- Two inductors of self-inductances L_1 and L_2 are kept so far apart that their mutual inductance is zero. These are connected in series. Then the equivalent inductance is

  $$L = L_1 + L_2$$
- Two inductors of self-inductance L_1 and L_2 are connected in series and they have mutual inductance M. Then the equivalent inductance of the combination is

  $$L = L_1 + L_2 \pm 2M$$
- The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.

- Two inductors of self-inductance L_1 and L_2 are connected in parallel. The inductors are so far apart that their mutual inductance is negligible. Then their equivalent inductance is

$$L = \frac{L_1 L_2}{L_1 + L_2} \quad \text{or} \quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Energy Stored in an Inductor

- When a current I flows through an inductor, the energy stored in it and is given by

$$U = \frac{1}{2} L I^2$$

- The energy stored in an inductor is in the form of magnetic energy.

Illustration 3

What is the self-inductance of a long solenoid of N-turns and cross-sectional area A and length l.

Soln.: The magnetic field inside a long solenoid.

$$B = \mu_0 n I = \mu_0 \left(\frac{N}{l}\right) I$$

The flux associated with each turn $\phi_{turn} = B \cdot A$

$$\phi_{turn} = \mu_0 \left(\frac{N}{l}\right) IA$$

$$\phi_{total} = N \cdot \phi_{turn} = \left(\frac{\mu_0 N^2 A}{l}\right) I$$

$$L = \frac{\phi_{Total}}{I} \Rightarrow L = \left(\frac{\mu_0 N^2 A}{l}\right)$$

- The self-inductance depends purely on geometric parameters.

Illustration 4

A solenoid S_1 is placed coaxially inside another solenoid S_2. The radii of the inner and outer solenoids are r_1 and r_2 respectively. If N_1 and N_2 are the number of turns of coil in solenoid S_1 and S_2 respectively and l is the length of solenoid S_2 carrying current I, then calculate the mutual inductance between the two solenoids.

Soln.: Magnetic field of the outer solenoid is $B = \dfrac{\mu_0 N_2 I}{l}$, when current I passes through it.

The flux associated with the inner coil is $\phi_1 = N_1 B \pi r_1^2$

$$\Rightarrow \quad \phi_1 = N_1 \left(\frac{\mu_0 N_2}{l}\right) I \pi r_1^2$$

$\therefore$ Mutual inductance between the two solenoid to given by

$$M = \frac{\phi_1}{I} = \left(\frac{\mu_0 N_1 N_2}{l}\right) \pi r_1^2$$

ALTERNATING CURRENTS

- It is that current which changes continuously in magnitude and in direction periodically. It can be represented by a sine curve or a cosine curve

$$I = I_0 \sin \omega t \quad \text{or} \quad I = I_0 \cos \omega t$$

- Here, I_0 is peak value of current and is known as amplitude of ac, I is instantaneous value of alternating current.

$$\omega = 2\pi/T$$
$= 2\pi\upsilon$ where T is period of ac and υ is frequency of ac.

Mean or Average Value of Alternating Current or Voltage over one Complete Cycle

- The mean or average value of alternating current or voltage over one complete cycle is zero.

$$I_m \text{ or } \bar{I} \text{ or } I_{av} = \frac{\int_0^T I_0 \sin \omega t \, dt}{\int_0^T dt} = 0$$

$$V_m \text{ or } \bar{V} \text{ or } V_{av} = \frac{\int_0^T V_0 \sin \omega t \, dt}{\int_0^T dt} = 0$$

- Average value of alternating current for first half cycle is

$$I_{av} = \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637 I_0$$

- Similarly, for alternating voltage, the average value over first half cycle is

$$V_{av} = \frac{\int_0^{T/2} V_0 \sin \omega t dt}{\int_0^{T/2} dt} = \frac{2V_0}{\pi} = 0.637 V_0$$

- Average value of alternating current for second cycle is

$$I_{av} = \frac{\int_{T/2}^{T} I_0 \sin \omega t dt}{\int_{T/2}^{T} dt} = -\frac{2I_0}{\pi} = -0.637 I_0$$

- Similarly, for alternating voltage, the average value over second half cycle is

$$V_{av} = \frac{\int_{T/2}^{T} V_0 \sin \omega t dt}{\int_{T/2}^{T} dt} = -\frac{2V_0}{\pi} = -0.637\, V_0$$

- The average value of alternating current (or voltage) during the first and second half cycles are equal but opposite in sign *i.e.* they are alternately positive and negative so that the average over one cycle is zero.

Mean Value or Average Value of Alternating Current over any Half Cycle

- It is that value of steady current, which would send the same amount of charge through a circuit in the time of half cycle *i.e.* $T/2$ as is sent by ac through the same circuit in the same time.

$$I_{av} = \frac{2I_0}{\pi} = 0.637 I_0$$

- Similarly, for alternating voltage

$$I_{av} = \frac{2I_0}{\pi} = 0.637 I_0$$

Root Mean Square (rms) Value of Alternating Current

- It is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the alternating current, when passed through the same resistance for the same time. The rms value of ac is also known as effective value or virtual value of ac. It is represented by I_{rms}, I_{eff}, or I_v.

$$I_{rms} \text{ or } I_v = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

- Similarly, for alternating voltage

$$V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

- All ac instruments measure rms value of ac.

Form Factors

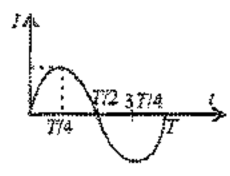
- Form factor is ratio of rms value to average value of alternating current or voltage during half cycle.

$$\text{Form factor} = \frac{I_{rms}}{I_{av}} = \frac{I_0/\sqrt{2}}{2I_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1.11$$

Illustration 5

The peak value of an alternating current is 5 A and its frequency is 60 Hz (a) Find its rms value (b) How long will the current take to reach the peak value starting from zero?

Solution: Here, $I_0 = 5$ A



$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5\text{ A}}{\sqrt{2}} = 3.54\text{A}$$

As is evident from the graph, starting from 0, the current takes a time $\frac{T}{4}$ to reach the peak value.

$$\text{Here } T = \frac{2\pi}{\upsilon} = \left(\frac{2\pi}{60}\right) = \left(\frac{\pi}{30}\right) \text{ s}$$

The required time, $t = \frac{T}{4}$

$$\text{or } t = \left(\frac{\pi}{30 \times 4}\right) = \left(\frac{\pi}{120}\right) \text{ s}$$

AC Circuit Containing Pure Resistance only

Let $V = V_0 \sin\omega t$

- Then, $I = \frac{V}{R} = \frac{V_0}{R}\sin\omega t = I_0 \sin\omega t$

- Here the alternating voltage is in phase with current, when ac flows through a resistor.

AC Circuit Containing Pure Inductor only

- Let $V = V_0 \sin\omega t$

  Then, $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

  where $I_0 = \frac{V_0}{\omega L}$

- Thus, the alternating current lags behind the alternating voltage by a phase angle of $\frac{\pi}{2}$ when ac flows through an inductor.
- Inductive reactance: It is the opposition offered by the inductor to the flow of alternating current through it.

$$X_L = \omega L = 2\pi\upsilon L$$

- The inductive reactance is zero for dc ($\upsilon = 0$) and has a finite value for ac.

AC Circuit Containing Pure Capacitor only

- Let $V = V_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

  where $I_0 = (\omega C)V_0$.

- Thus, the alternating current leads the voltage by a phase angle of $\frac{\pi}{2}$, when ac flows through a capacitor.
- Capacitive reactance : It is the opposition offered by the capacitor to the flow of alternating current through it.

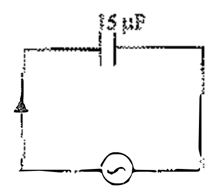
  The capacitive reactance is infinite for dc ($\upsilon = 0$) and has a finite value for ac.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C}$$

- The capacitive reactance is infinite for dc ($\upsilon = 0$) and has a finite value for ac.

Illustration 6

A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?



220 V, 50 Hz

Sol. The capacitive reactance is

$$X_C = \frac{1}{2\pi\upsilon C} = \frac{1}{2\pi(50\text{ Hz})(15.0 \times 10^{-6}\text{F})} = 212\,\Omega$$

The rms current is

$$I_V = \frac{V}{X_C} = \frac{220}{212} = 1.04\text{ A}$$

when the frequency is doubled, the capacitive reactance reduced to half and virtual current increases to double.

$$X_C' = \frac{X_C}{2} = 106\,\Omega$$

$$I_V' = 2I_V = 2.08\text{ A}$$