SERIES *LCR* CIRCUIT

- Let $V = V_0 \sin\omega t$

  Then, $I = I_0 \sin(\omega t - \phi)$

  where $I_0 = \frac{V_0}{Z}$

  Here Z is the impedance of the series LCR circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

- The alternating current lags behind the voltage by a phase angle ϕ

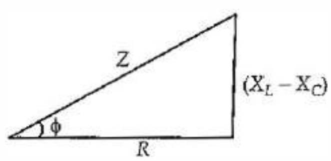
$$\tan\phi = \frac{X_L - X_C}{R}$$

- When $X_L > X_C$, $\tan\phi$ is positive. Therefore, ϕ is positive. Hence current lags behind the voltage by a phase angle ϕ. The ac circuit is inductance dominated circuit.

  When $X_L < X_C$, $\tan\phi$ is negative. Therefore, ϕ is negative. Hence current leads the voltage by a phase angle ϕ. The ac circuit is capacitance dominated circuit.

Impedance Triangle

- It is a right angled triangle, whose base represents ohmic resistance (R), perpendicular represents reactance ($X_L - X_C$) and hypotenuse represents impedance (Z) of the series LCR circuit as shown in figure.



- Impedance of circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Admittance

- The reciprocal of the impedance of an ac circuit is known as admittance. It is represented by Y.

$$\therefore \text{ Admittance} = \frac{1}{\text{Impedance}} \quad \text{or} \quad Y = \frac{1}{Z}$$

- The unit of admittance is $(\text{ohm})^{-1}$ or siemen.

Susceptance

- The reciprocal of the reactance of an ac circuit is known as susceptance. It is represented by S.

$$\therefore \text{ Susceptance} = \frac{1}{\text{Reactance}}$$

- The unit of susceptance is $(\text{ohm})^{-1}$ or siemen.

- $\text{Inductive susceptance} = \frac{1}{\text{Inductive reactance}}$

$$\text{or} \quad S_L = \frac{1}{X_L} = \frac{1}{\omega L}$$

- $\text{Capacitive susceptance} = \frac{1}{\text{Capacitive reactance}}$

$$\text{or} \quad S_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C$$

RESONANT SERIES *LCR* CIRCUIT

- When the frequency of ac supply is such that the inductive reactance and capacitive reactance become equal ($X_L = X_C$), the impedance of the series LCR circuit is equal to the ohmic resistance in the circuit. As such, the current in the circuit becomes maximum. Such a series LCR circuit is known as resonant series LCR circuit and the frequency of the ac supply is known as resonant frequency (υ_r). The resonant frequency is

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

  The series resonance circuit is known as acceptor circuit. It is used in radio and TV receivers sets for tuning a particular radio station/TV channel.

- Resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Then only voltages across L and C cancel each other. We cannot have resonance in a LR or RC circuit.

Illustration 7

An LCR series circuit with $L = 100$ mH, $C = 100$ μF and $R = 120\ \Omega$ is connected to an ac source of emf, $\varepsilon = (30\text{ V})\sin(100\text{ s}^{-1})t$. Find the impedance and peak current.

Solution: Here $\omega = 100$ rad s^{-1}

$$X_L = \omega L = (100\text{ rad s}^{-1}) \cdot (100 \times 10^{-3}\text{ H}) = 10\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(100\text{ rad s}^{-1}) \cdot (100 \times 10^{-6}\text{ F})} = 100\ \Omega$$

Now,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(120)^2 + (10 - 100)^2}$$

$$Z = \sqrt{(120)^2 + (90)^2} = 150\ \Omega$$

$$I_0 = \frac{\varepsilon_0}{Z} = \frac{30\text{ V}}{150\ \Omega} = 0.2\text{ A}.$$

QUALITY FACTOR

- It is a measure of sharpness of resonance. It is defined as the ratio of reactance of either the inductance or capacitance at the resonant angular frequency to the total resistance of the circuit.

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$Q = \frac{X_C}{R} = \frac{1}{\omega_r CR}$$

$$\therefore Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

  Quality factor is also expressed in terms of bandwidth

$$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

POWER IN AN AC CIRCUIT

- In an ac circuit we may define three types of power.

- **Instantaneous power :** The power in the ac circuit at any instant of time is known as instantaneous power. It is equal to the product of values of alternating voltage and alternating current at that time.
- **Average power (P_{av}) :** The power averaged over one full cycle of ac is known as average power. It is also known as true power.

$$P_{av} = V_{rms}\, I_{rms} \cos\phi = \frac{V_0 I_0}{2} \cos\phi$$

- **Apparent power :** The product of virtual voltage (V_{rms}) and virtual current (I_{rms}) in the circuit is known as virtual power.

$$P_v = V_{rms}\, I_{rms} = \frac{V_0 I_0}{2}$$

Power Factor

- It is defined as the ratio of true power to apparent power of an ac circuit

$$\cos\phi = \frac{\text{True power}}{\text{Apparent power}}$$

- Power factor is also defined as the ratio of the resistance to the impedance of an ac circuit

$$\cos\phi = \frac{R}{Z}$$

- It is unitless and dimensionless quantity.
  In pure resistive circuit,
  $\phi = 0°$; $\cos\phi = 1$.
- In pure inductive or capacitive circuit

$$\phi = \frac{\pi}{2};\ \cos\phi = 0.$$

- In RL circuit,

$$Z = \sqrt{R^2 + X_L^2} \text{ and } \cos\phi = \frac{R}{Z}$$

- In RC circuit,

$$Z = \sqrt{R^2 + X_C^2} \text{ and } \cos\phi = \frac{R}{Z}$$

- In series LCR circuit,

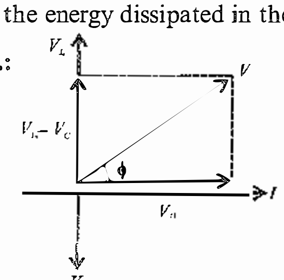
$$Z = \sqrt{R^2 + (X_L - X_C)^2} \text{ and } \cos\phi = \frac{R}{Z}$$

- At resonance, $X_L = X_C$
  $\therefore\ Z = R$ and $\phi = 0°$
  $\cos\phi = 1$

Illustration 8

An ac circuit contains an inductor (20 mH), a capacitor (100 μF), a resistor (50 Ω) and an ac source of 12 V, 50 Hz. Find the energy dissipated in the circuit in 1000 s.

Soln.:



$P_{av} = V_{rms} I_{rms} \cos\phi$

In LCR,

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ}$$

$$\Rightarrow\ P_{av} = V_{rms}\left(\frac{V_{rms}}{Z}\right)\frac{R}{Z} = \frac{V_{rms}^2 \cdot R}{Z^2}$$

Here $V_{rms} = 12$ V;
$R = 50\ \Omega$, $X_L = \omega L = 2\pi\upsilon L$
$= 2\pi(50\text{ Hz})(20 \times 10^{-3}\text{ H})$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} = \frac{1}{2\pi(50\text{ Hz})(100 \times 10^{-6}\text{ F})}$$

$$= \left(\frac{100}{\pi}\right)\Omega$$

$$\Rightarrow\ Z = \sqrt{R^2 + (X_L - X_C)^2} = 56\ \Omega$$

Energy used in 1000 s is, $P_{Av}t = \left(\frac{V_{rms}^2 \cdot R}{Z^2}\right)t$

$$= \left(\frac{12^2 \times 50}{56^2}\right) \times 1000 = 2.3 \times 10^3\text{ J.}$$

Illustration 9

An emf $\varepsilon = 100\sin 314t$ volt is applied to a condenser of capacity 500 μF. Calculate

(a) the instantaneous current in the circuit.
(b) the instantaneous power
(c) the frequency of power
(d) the maximum energy stored in capacitor

Sol. (a) Current leads over applied emf by $\frac{\pi}{2}$ in a capacitance.

$\therefore\ I_0 = \frac{\varepsilon_0}{X_C} = \varepsilon_0 \times \omega C$ where $\varepsilon_0 = 100$ V

or $I_0 = (100) \times (314) \times (500 \times 10^{-6})$

or $I_0 = 314 \times 5 \times 10^{-2} = 15.70$ A

$\therefore\ I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ where $\omega = 314$ rad s^{-1}

or $I = (15.70)\cos\omega t$ A $= 15.70\cos 314t$ A

(b) Instantaneous power,
$P = EI$
or $P = (100\sin 314t)(15.70\cos\omega t)$
$= 1570(\sin 314t)(\cos 314t)$
$= 785 \times (2\sin 314t\cos 314t)$
$= 785\sin 628t$

(c) Frequency of power.

$$\upsilon_P = \frac{\omega}{2\pi} = \frac{628}{2 \times 3.14} = 100\text{ Hz}$$

$\therefore\ \upsilon_P = 100$ Hz

(d) Maximum energy stored in capacitor

$$U_0 = \frac{1}{2}C\varepsilon_0^2$$

or $U_0 = \frac{1}{2} \times (500 \times 10^{-6}) \times (100)^2$

or $U_0 = 2.5$ J

Hence

(a) $I = 15.70 \cos \omega t$ A $= 15.70 \cos 314t$ A

(b) $P = 785 \sin 628t$ W

(c) $\upsilon_P = 100$ Hz

(d) $U_B = 2.5$ J.

WATTLESS CURRENT

- The average power associated over a complete cycle with a pure inductor or pure capacitor is zero, even though a current is flowing through them. This current is known as the wattless current or idle current.

TRANSFORMER

- It is a device used for converting a low alternating voltage to a high alternating voltage and vice versa. It is based on phenomenon of mutual induction.
- For ideal transformer, $\frac{V_S}{V_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = k.$

  where k is called transformation ratio.
- For a step-up transformer, $k > 1$. *i.e.* $V_S > V_P$, $I_S < I_P$ and $N_S > N_P$.
- For a step-down transformer, $k < 1$. *i.e.* $V_S < V_P$, $I_S > I_P$ and $N_S < N_P$.
- Efficiency of a transformer,

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{V_S I_S}{V_P I_P}.$$

AC GENERATOR/DYNAMO

- An ac generator/dynamo produces alternating current energy from mechanical energy of rotation of a coil. It is based on the phenomenon of electromagnetic induction. The form of emf induced is $\varepsilon = \varepsilon_0 \sin\omega t$, where $\varepsilon_0 = NAB\omega$, max. emf induced. Here, N is total number of turns in the coil, A is face area of the coil, B is strength of magnetic field applied and ω is angular velocity of the armature coil.

DC GENERATOR/DYNAMO

- A dc generator/dynamo produces direct current energy from mechanical energy of rotation of a coil. Its principle and working are same as those of ac generator. There is only a little change in the design of the generator. Slip ring arrangement used in ac generator is replaced by split ring arrangement in dc generator.

DC MOTOR

- A dc motor converts direct current energy from a battery into mechanical energy of rotation. It is based on the fact that when a coil carrying current is placed in a magnetic field, it experiences a torque, which rotates the coil. The efficiency of a dc motor is given by

$$\eta = \frac{\text{back emf}}{\text{emf of battery}}$$

Illustration 10

A transformer of 100% efficiency has 200 turns in primary and 40000 turns in the secondary. It is connected to a 220 V main supply and the secondary feeds to a 100 kΩ resistance. Calculate

(i) the output potential difference

(ii) the power delivered to load.

Soln.: $N_P = 200$, $N_S = 40000$, $V_P = 220$ V

$$V_S = \left(\frac{N_S}{N_P}\right) V_P = \left(\frac{40000}{200}\right)(220\text{ V})$$

$$= 44000\text{ V} = 44\text{ kV}.$$

Power delivered to load $= V_S I_S$

$$= V_S\left(\frac{V_S}{R_S}\right) = \frac{V_S^2}{R_S} = \frac{(44000\text{V})^2}{100 \times 10^3\ \Omega}$$

$$= 19360\text{ W} = 19.36\text{ kW}.$$