of a concave mirror, ($u = R = 2f = 2(-10) = -20$ cm) the image is also formed there itself. $m = -1 \Rightarrow$ image is same size and inverted.

(c) $u = -15$ cm and $f = -10$ cm

$$\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10} \Rightarrow v = -30 \text{ cm}$$

Image is real, $m = \frac{-v}{u} = -\left(\frac{-30}{-15}\right) = -2$

Image is inverted and enlarged.

(d) $u = -10$ cm and $f = -10$ cm

$$\frac{1}{v} + \frac{1}{-10} = \frac{1}{-10}$$

$v = \infty$

When the object is at the focus, image is formed at infinity. This means, the reflected rays travel parallel to each other.

$$m = \frac{-v}{u} = -\left(\frac{\infty}{-10}\right) = \infty$$

(e) $u = -5$ cm and $f = -10$ cm

$$\frac{1}{v} + \frac{1}{-5} = \frac{1}{-10} \Rightarrow v = +10 \text{ cm}$$

Image is virtual. It is formed inside the mirror.

$$m = \frac{-v}{u} = -\left(\frac{10}{-5}\right) = +2$$

The image is erect and enlarged.

REFRACTION OF LIGHT

- When a ray of light passes from one medium to another, in which it has a different velocity, there occurs a change in the direction of propagation of light except when it strikes the surface of separation of two media normally. This bending of a ray of light is known as refraction.
- The angles made by the incident ray and the refracted ray with the normal to the separating surface at the point of incidence are known as the angles of incidence and of refraction respectively.

Laws of Refraction

The two laws of refraction are as follows:

- The incident ray, the normal and the refracted ray all lie in the same plane.
- The ratio of the sine of angle of incidence to the sine of angle of refraction for any two media is constant for a light of definite colour. This constant is denoted by ${}^1\mu_2$ or μ_{21} called the refractive index of the second medium with respect to the first, the subscripts 1 and 2 indicating that the light passes from medium 1 to medium 2.

$$\frac{\sin i}{\sin r} = {}^1\mu_2$$

This is also known as Snell's law.

where i = angle of incidence, r = angle of refraction.

- ${}^1\mu_2$ is a characteristic of the pair of media and also depends on the wavelength of light but is independent of the angle of incidence.

$$^2\mu_1 = \frac{1}{^1\mu_2} \quad \text{or} \quad {}^2\mu_1 \times {}^1\mu_2 = 1$$

- Absolute refractive index : Refractive index of a medium with respect to vacuum (or in practice air) is known as absolute refractive index of the medium

$$\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

- General expression for Snell's law

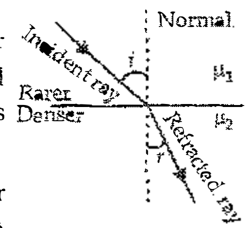
$$^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\left(\frac{c}{v_2}\right)}{\left(\frac{c}{v_1}\right)} = \frac{v_1}{v_2}$$

where c is the speed of light in air, v_1 and v_2 be the speeds of light in medium 1 and medium 2 respectively.

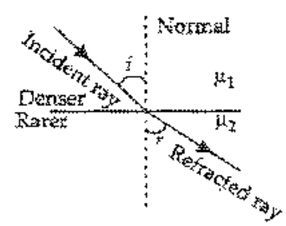
- According to Snell's law,

$$^1\mu_2 = \frac{\sin i}{\sin r}; \quad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} \quad \text{or} \quad \mu_1 \sin i = \mu_2 \sin r$$

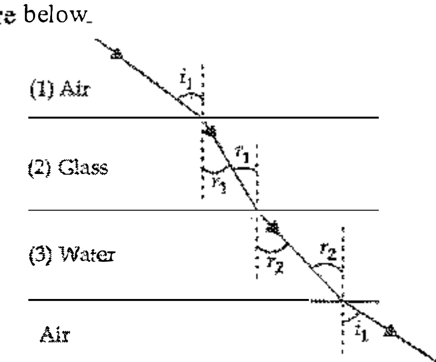
- When a light travels from one medium to another, its frequency remains constant but its wavelength as well as velocity changes.
- When a light passes from a rarer to denser medium ($\mu_2 > \mu_1$), it will bend towards the normal as shown in the figure.



- When a light passes from a denser medium to rarer medium ($\mu_1 > \mu_2$) it will bend away from the normal as shown in the figure.



- The deviation of the incident ray when it is refracted is given by an angle $\delta = |i - r|$
- If a light ray passes through a number of parallel media and if the first and the last medium are same. The emergent ray is parallel to the incident ray as shown in figure below.



$$^1\mu_2 = \frac{\sin i_1}{\sin r_1}$$

$$^2\mu_3 = \frac{\sin r_1}{\sin r_2} \text{ and } ^3\mu_1 = \frac{\sin r_2}{\sin i_1}$$

Hence,

$$^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = \frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i_1} = 1$$

Optical Density

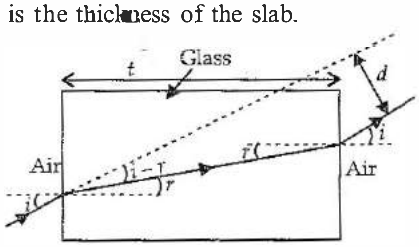
- It is a measure of a refractive index of a medium. A medium with a relatively high refractive index is said to have a high optical density while one with a low refractive index is said to have a low optical density.
- Optical density should not be confused with mass density, which is mass per unit volume. It is possible that mass density of an optically denser medium may be less than that of an optically rarer medium. For example, turpentine and water. Mass density of turpentine is less than that of water but its optical density is higher than water.

Optical Path

- It is defined as the product of geometrical distance and the refractive index of the medium.
  Optical path = μd
  where d is the distance travelled by the light in the medium.
  The optical path represents the distance light travels in a vacuum in the same time it travels a distance d in the medium.

Lateral Shift

- When the medium is same on both sides of a glass slab, then the deviation of the emergent ray is zero. That is the emergent ray is parallel to the incident ray but it does suffer lateral displacement/shift with respect to the incident ray and is given by
- Lateral shift, $d = t\dfrac{\sin(i-r)}{\cos r}$
  where t is the thickness of the slab.



Real Depth and Apparent Depth

- When one looks into a pool of water, it does not appear to be as deep as it really is. Also when one looks into a slab of glass, the material does not appear to be as thick as it really is. This all happens due to refraction of light.
- If a beaker is filled with water and a point lying at its bottom is observed by someone located in air, then the bottom point appears raised. The apparent depth is less than the real depth. It can be shown that

$$\text{apparent depth} = \frac{\text{real depth}}{\text{refractive index } (\mu)}$$

- If there is an ink spot at the bottom of a glass slab, it appears to be raised by a distance

$$d = t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$$

  where t is the thickness of the glass slab and μ is its refractive index.
- If a beaker is filled with immiscible transparent liquids of refractive indices μ_1, μ_2, μ_3 and individual depth d_1, d_2, d_3 respectively, then the apparent depth of the beaker is

$$= \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}$$

Refraction Effects at Sunrise and Sunset

- The sun is visible a little before the actual sunrise and until a little after the actual sunset due to refraction of light through the atmosphere.

Illustration 4

(a) Find the refractive index of glass with respect to water. Given: $\mu_g = \frac{3}{2}$ and $\mu_w = \frac{4}{3}$.

(b) A source of yellow light placed in air is observed by a person swimming under water. If the wavelength of yellow light in air is 6000 Å, then find its speed, wavelength and colour as observed by the person.

Sol.: (a) $_w\mu_g = \dfrac{\mu_g}{\mu_w} = \dfrac{3}{2} \div \dfrac{4}{3} = \dfrac{3}{2} \times \dfrac{3}{4} = \dfrac{9}{8}$.

(b) $\mu_w = \dfrac{c}{c_w}$

or $c_w = \dfrac{c}{\mu_w} = \dfrac{3 \times 10^8 \text{ m/s}}{4/3} \Rightarrow c_w = \dfrac{9}{4} \times 10^8$ m/s

or $c_w = 2.25 \times 10^8$ m/s

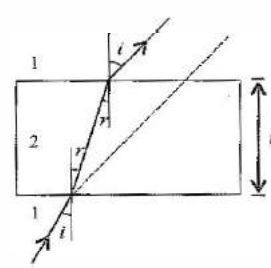
Now, $\mu_w \cdot \lambda_w = \lambda$ or $\lambda_w = \dfrac{\lambda}{\mu_w} = \dfrac{6000\,\text{Å}}{4/3} = \dfrac{6000 \times 3}{4}$ Å

$\Rightarrow \lambda_w = 4500$ Å.

Colour for the human eye depends on the frequency. Since frequency is independent of the media, the light as seen by the person inside water will be yellow.

Illustration 5

When a ray passes through a glass slab of thickness t at an angle i with an angle of refraction r, what is the lateral shift of the emergent ray?



Soln.: Using Snell's law, for ray 1 and its refraction,

$\sin i = \mu \sin r$ and $\sin e = \mu \sin r$

$\Rightarrow \angle i = \angle e$

The emergent ray is parallel to the incident ray.

Further,

$AB = \frac{AN}{\cos r} = \frac{t}{\cos r}$

And $BC = AB \sin (i - r)$

$\Rightarrow$ lateral shift (d)

$= \frac{t}{\cos r} \sin (i - r)$

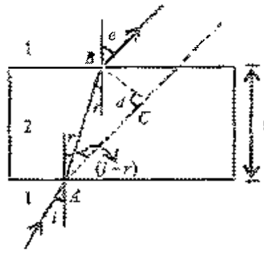


Illustration 6

A stone S is on the bottom of a swimming pool. The depth of the lying pool is d and index of refraction of water is μ. What is the depth of the swimming pool visible to a normal eye? Check the results with $d = 6$ ft and $\mu = 1.5$.

Soln.: A point source S is observed from air at a small angle α to the normal. Even if you see normal to the surface of water, the image is formed by a cone of light entering the pupil of the eye. ($\alpha \to 0$)

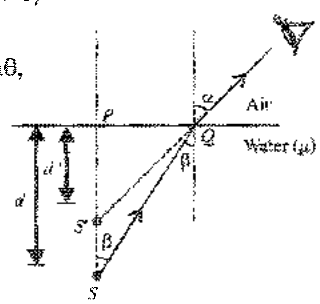
Now, $\mu \sin\beta = 1 \sin\alpha$

When $\theta \to 0$, $\sin\theta = \tan\theta$,

Here, $\tan\alpha = \sin\alpha$

and $\tan\beta = \sin\beta$

$\mu \tan\beta = \tan\alpha$



$\mu \cdot \frac{PQ}{PS} = \frac{PQ}{PS'}$

$\Rightarrow \frac{PS'}{PS} = \frac{1}{\mu}$ or $PS' = \left(\frac{PS}{\mu}\right)$ or $d' = \frac{d}{\mu}$

$\Rightarrow SS' = PS - PS' = PS - \frac{PS}{\mu}$

Apparent shift $= d\left(1 - \frac{1}{\mu}\right)$

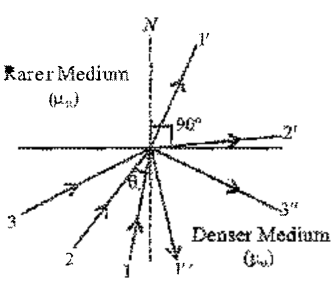
For $d = 6$ feet and $\mu = 1.5$.

Apparent depth, $d' = \frac{d}{\mu} = \frac{6}{1.5} = 4$ feet.

A 6 feet deep pool appears to be 4 feet deep to the eye. Most people are taller than 4 feet and shorter than 6 feet. A person could get into the pool here and expect to find the ground 4 feet. His expectation is behind and the consequences could be dangerous.

TOTAL INTERNAL REFLECTION

When a ray is incident from a denser medium to rarer medium, it bends away from the normal. The incident ray 1, refracts as 1′ away from the normal. It also partially reflects as 1″. The angle of incidence of a certain ray 2, that refracts at an angle of 90° is called the critical angle θ_c.



When incident angle is greater than θ_c, the ray cannot refract, but instead reflects totally. This phenomenon is called total internal reflection.

For θ_c, Snell's law is $\mu_D \sin \theta_c = \mu_R \sin 90°$

$\Rightarrow \sin \theta_c = \frac{\mu_R}{\mu_D}$ or $\theta_c = \sin^{-1}\left(\frac{\mu_R}{\mu_D}\right)$

Usually, $\mu_R = \mu_{air} = 1$ and $\mu_D = \mu$.

Then, $\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$

Illustration 7

In the figure shown for an angle of incidence i at the top of the surface, what is the minimum refractive index for total internal reflection at the vertical surface.

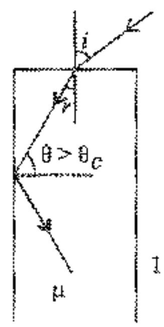
Soln.: The ray will total internally reflect at the vertical surface if $\theta > \theta_c$.

Now, $r = (90° - \theta)$ and

Snell's law is $\sin i = \mu \sin r$

$\frac{\sin i}{\mu} = \sin (90° - \theta)$

$\Rightarrow \cos \theta = \frac{\sin i}{\mu}$

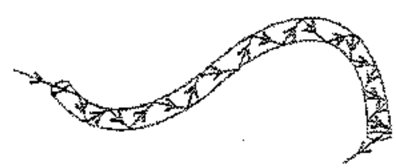


or $\sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{\sin^2 i}{\mu^2}}$

If $\theta > \theta_C$, then $\sin \theta > \sin \theta_C$ (As $\sin \theta$ is an increasing function for $0 < \theta < 90°$)

$\sqrt{1 - \frac{\sin^2 i}{\mu^2}} > \frac{1}{\mu}$

$1 - \frac{\sin^2 i}{\mu^2} > \frac{1}{\mu^2}$



$\mu^2 - \sin^2 i > 1$ or $(\mu^2 - 1) > \sin^2 i$

If total internal reflection has to be larger for all value, the above inequality must be satisfied for all $(\sin^2 i)_{max} = 1$

$\Rightarrow \mu^2 - 1 > 1$ or $\mu > \sqrt{2}$

This total internal reflection phenomenon is used in fibre optics to bend light in a curved path.

REFRACTION FROM A SPHERICAL SURFACE

- The portion of a refracting medium, whose curved surface forms the part of a sphere, is known as spherical refracting surface.
- Spherical refracting surface are of two types :
  Convex refracting spherical surface
  Concave refracting spherical surface
- Sign conventions for spherical refracting surface are the same as those for spherical mirrors.

- When the object is situated in rarer medium, the relation between μ_1 (refractive index of rarer medium), μ_2 (refractive index of the spherical refracting surface) and R (radius of curvature) with the object and image distances is given by

$$-\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$$

- When the object is situated in denser medium, the relation between μ_1, μ_2, R, u and v can be obtained by interchanging μ_1 and μ_2. In that case, the relation becomes

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R} \quad \text{or} \quad -\frac{\mu_1}{v}+\frac{\mu_2}{u}=\frac{\mu_2-\mu_1}{R}$$

These formulae are valid for both convex and concave spherical surfaces

Lens

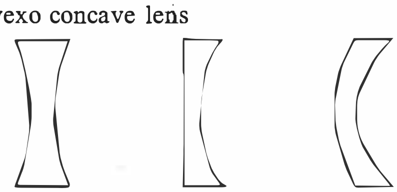
- A lens is a portion of a transparent refracting medium bound by two spherical surfaces or one spherical surface and the other plane surface.
- Lenses are divided into two classes :

  Convex lens or converging lens
  Concave lens or diverging lens
- Convex lens or converging lens : When a lens is thicker in the middle than at the edges it is known as convex lens or converging lens. These are of three types :

  Double convex lens or biconvex lens
  Plano convex lens
  Concavo convex lens

Biconvex lens | Plano convex lens | Concavo convex lens

- Concave lens or diverging lens : When the lens is thicker at the edges than in the middle it is known as concave lens or diverging lens. These are of three types :
  - Double concave lens or biconcave lens
  - Plano concave lens
  - Convexo concave lens



Biconcave lens | Plano concave lens | Convexo concave lens

Sign Conventions

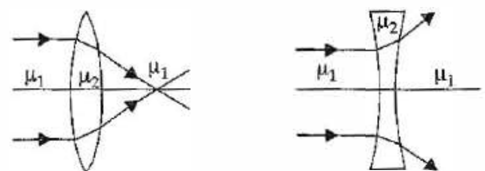
- The sign conventions for thin lenses are the same as those of spherical mirrors except that instead of the pole of the mirror, we now use optical centre of a lens.

Lens Maker's Formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

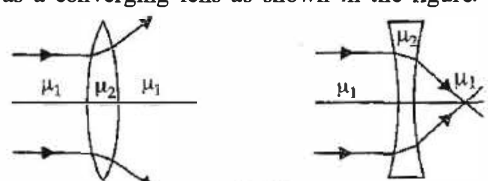
where R_1 and R_2 are radii of curvature of the two surfaces of the lens and μ is refractive index of material of lens w.r.t. medium in which lens is placed.

- This formula is valid for thin lenses. It is valid for both convex and concave lenses.
- As per sign convention, for a convex lens, R_1 is positive and R_2 is negative and for a concave lens. R_1 is negative and R_2 is positive.
- When the refractive index of the material of the lens is greater than that of the surroundings, then biconvex lens acts as a converging lens and a biconcave lens acts as a diverging lens as shown in the figure.



$\mu_2 > \mu_1$

- When the refractive index of the material of the lens is smaller than that of the surrounding medium, then biconvex lens acts as a diverging lens and a biconcave lens as a converging lens as shown in the figure.



$\mu_2 < \mu_1$

Thin Lens Formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

where
u = distance of the object from the optical centre of the lens
v = distance of the image from the optical centre of the lens
f = focal length of a lens
f is positive for converging or convex lens and f is negative for diverging or concave lens.

Linear Magnification

$$m=\frac{\text{size of image }(I)}{\text{size of object }(O)}=\frac{v}{u}.$$

- m is positive for erect image and m is negative for inverted image.

Power of a Lens

$$P=\frac{1}{\text{focal length in metres}}.$$

- The SI unit of power of lens is dioptre (D).
  1 D = 1 m^{-1}.
  For a convex lens, P is positive.
  For a concave lens, P is negative.
  When focal length (f) of lens is in cm, then
  $$P = \frac{100}{f\text{ (in cm)}}\text{ dioptre.}$$

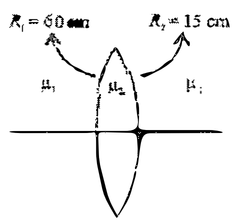
Combination of Thin Lenses in Contact

- When a number of thin lenses of focal length f_1, f_2, ...etc. are placed in contact coaxially, the equivalent focal length F of the combination is given by
  $$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...$$
- The total power of the combination is given by
  $P = P_1 + P_2 + P_3 + ...$
- The total magnification of the combination is given by
  $m = m_1 \times m_2 \times m_3 ....$
- When two thin lenses of focal lengths f_1 and f_2 are placed coaxially and separated by a distance d, the focal length of a combination is given by
  $$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}.$$
- In terms of power, $P = P_1 + P_2 - dP_1P_2$.

Illustration 8

Calculate the focal length of a biconvex lens if the radii of its surfaces are 60 cm and 15 cm, and index of refraction of the lens glass = 1.5.

Soln.:



Using the Lens maker's Formula,
$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$\frac{1}{f} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{60} - \frac{1}{-15}\right) = (0.5)\left[\frac{1}{60} + \frac{1}{15}\right]$$
$$\frac{1}{f} = (0.5)\left[\frac{1+4}{60}\right] \Rightarrow f = +24\text{ cm.}$$

Illustration 9

A magnifying lens has a focal length of 10 cm
(i) Where should the object be placed if the image is to be 30 cm from the lens?
(ii) What will be its magnification?

Soln.: $f = +10$ cm
$v = \pm 30$ cm

Case 1. $v = +30$ cm
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{30} - \frac{1}{u} = \frac{1}{10} \Rightarrow u = -15\text{ cm}$$
$$m = \frac{v}{u} = \frac{+30}{-15} = -2$$
If the object is placed 15 cm before the lens, a real image is formed 30 cm away from the lens. Its magnification is 2 and the image is inverted.

Case 2. $v = -30$ cm
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$
$$\frac{1}{-30} - \frac{1}{u} = \frac{1}{10} \Rightarrow u = -7.5\text{ cm}$$
$$m = \frac{v}{u} = \left(\frac{-30}{-7.5}\right) = +4.$$
If the object is placed, 7.5 cm before the lens, a virtual image is formed 30 cm from the lens. It is 4 times enlarged and erect.

Illustration 10

A lens has a power of + 5 dioptres in air. What will be its power if completely immersed in water? Given, $\mu_g = \frac{3}{2}$ and $\mu_w = \frac{4}{3}$.

Soln.: $$P_w = \frac{\mu_w}{f_w} = \mu_w\left[\frac{\mu_g}{\mu_w} - 1\right]\cdot\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(i)$$
$$P_{air} = \frac{1}{f_{air}} = [\mu_g - 1]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(ii)$$
eqn. (i) ÷ (ii), gives
$$\frac{P_w}{P_{air}} = \frac{(\mu_g - \mu_w)}{(\mu_g - 1)}$$
$$P_w = \frac{\left(\frac{3}{2} - \frac{4}{3}\right)}{\left(\frac{3}{2} - 1\right)} \times 5 = \frac{1/6}{1/2} \times 5$$
$$\Rightarrow P_w = \frac{2}{6} \times 5 = \frac{10}{6} = \frac{5}{3}\text{ D.}$$
The power of the lens gets altered inside water.

REFRACTION THROUGH A PRISM

- **Prism** : It is a homogeneous, transparent medium enclosed by two plane surfaces inclined at an angle. These surfaces are called the refracting surfaces and angle between them is known as the refracting angle or the angle of prism.
- The angle between the incident ray and the emergent ray is known as the angle of deviation.
- For refraction through a prism it is found that