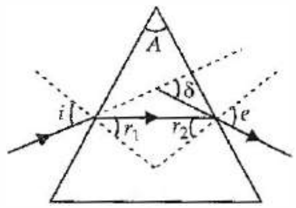


$\delta = i + e - A$ where $A = r_1 + r_2$

When A and i are small

$\therefore\ \delta = (\mu - 1)A$

In a position of minimum deviation, $\delta = \delta_m$, $i = e$, and $r_1 = r_2 = r$

$$\therefore\ i = \left(\frac{A + \delta_m}{2}\right) \text{ and } r = \frac{A}{2}$$

- The refractive index of the material of the prism is

$$\mu = \frac{\sin\left[\frac{(A+\delta_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$

This is known as prism formula

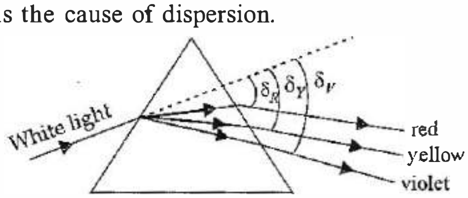
where A is the angle of prism and δ_m is the angle of minimum deviation.

DISPERSION OF LIGHT

- It is the phenomenon of splitting of white light into its constituent colours on passing through a prism. This is because different colours have different wavelengths ($\lambda_R > \lambda_V$). According to Cauchy's formula

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

where A, B, C are arbitrary constants. Therefore, μ of material of prism for different colours is different ($\mu_V > \mu_R$). As $\delta = (\mu - 1)A$, therefore different colours turn through different angles on passing through the prism. This is the cause of dispersion.



Angular dispersion

- The difference in deviation between any two colours is known as angular dispersion.

Angular dispersion $\delta_V - \delta_R = (\mu_V - \mu_R)A$

where μ_V and μ_R are the refractive index for violet and red rays.

Mean deviation, $\delta = \dfrac{\delta_V + \delta_R}{2}$.

$$\text{Dispersive power, } \omega = \frac{\text{angular dispersion } (\delta_V - \delta_R)}{\text{mean deviation } (\delta)}$$

$$\omega = \frac{\mu_V - \mu_R}{(\mu - 1)},$$

where $\mu = \dfrac{\mu_V + \mu_R}{2}$ = mean refractive index

Dispersion without deviation

- Suppose we combine two prisms of refracting angles A and A', and dispersive powers ω and ω' respectively in such a way that their refracting angles are reversed with respect to each other.
- For no deviation, the condition is

$\delta + \delta' = 0$

$$(\mu - 1)A + (\mu' - 1)A' = 0 \text{ or } A' = -\frac{(\mu - 1)A}{(\mu' - 1)}$$

Under this condition, net angular dispersion produced by the combination

$$= (\delta_V - \delta_R) + (\delta'_V - \delta'_R) = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$$

Deviation without dispersion

The condition for no dispersion is

$$(\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0 \text{ or } A' = -\frac{(\mu_V - \mu_R)A}{(\mu'_V - \mu'_R)}$$

Under this condition, net deviation produced by the combination is

$= \delta + \delta' = (\mu - 1)A + (\mu' - 1)A'$

Illustration 11

A ray of light is incident on one face of a prism at an angle of 60°. The refractive index of the prism is 1.5 and angle of prism is 60°. Find the angle of emergence and the angle of deviation.

Soln. : $\sin r_1 = \dfrac{\sin i}{\mu} = \dfrac{\sin 60^\circ}{1.5} = \dfrac{\sqrt{3}}{2 \times 1.5} = 0.577$

$r_1 = \sin^{-1}(0.577) = 35^\circ$, $r_2 = A - r_1 = 60^\circ - 35^\circ = 25^\circ$

$\sin e = \mu \sin r_2 = 1.5 \sin(25^\circ) = 1.5 \times 0.423$

$\sin e = 0.634 \Rightarrow e = \sin^{-1}(0.634) = 39^\circ$

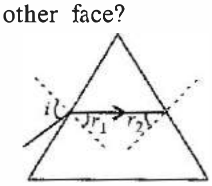
Now, $i + e = A + \delta$

$\Rightarrow \delta = i + e - A = 60^\circ + 39^\circ - 60^\circ = 39^\circ$.

Illustration 12

What is the required condition, if the light incident on one face, does not emerge from the other face?

Soln.: For no emergence, $r_2 > \theta_c$



$A - r_1 > \theta_c$

$\sin(A - r_1) > \sin\theta_c$

$$\sin A \cos r_1 - \cos A \sin r_1 > \frac{1}{\mu}$$

$$\sin A\,[\cos r_1] - \cos A\left[\frac{\sin i}{\mu}\right] > \frac{1}{\mu}$$

$$\Rightarrow \mu \sin A\sqrt{1 - \sin^2 r_1} - \cos A \sin i > 1$$