$$\Rightarrow \mu \sin A \sqrt{\left(1-\frac{\sin^2 i}{\mu^2}\right)} > 1 + \cos A \sin i$$

$$\Rightarrow \sin A \cdot \sqrt{\mu^2 - \sin^2 i} > (1 + \cos A \sin i)$$

Squaring both sides,

$$\sin^2 A (\mu^2 - \sin^2 i) > (1 + \cos A \sin i)^2$$

$$\mu^2 \sin^2 A - \sin^2 A \sin^2 i > 1 + \cos^2 A \sin^2 i + 2 \cos A \sin i$$

$$\mu^2 \sin^2 A > 1 + (\cos^2 A + \sin^2 A) \sin^2 i + 2 \cos A \sin i$$

$$\mu^2 \sin^2 A > 1 + \sin^2 i + 2 \cos A \sin i$$

The greatest value of $\sin i = 1$

$$\Rightarrow \mu^2 \sin^2 A > 1 + 1 + 2 \cos A$$

$$\mu^2 \left(2^2 \sin^2 \frac{A}{2} \cos^2 \frac{A}{2}\right) > 2 (1 + \cos A)$$

$$4 \mu^2 \sin^2 \frac{A}{2} \cos^2 \frac{A}{2} > 4 \cos^2 \frac{A}{2} \Rightarrow \mu^2 > \frac{1}{\sin^2 \frac{A}{2}}$$

$$\mu > \operatorname{cosec} \left(\frac{A}{2}\right).$$

Illustration 13

Calculate the dispersive power of crown glass where $\mu_V = 1.522$ and $\mu_R = 1.514$.

Soln.: $\omega = \dfrac{\mu_V - \mu_R}{\mu - 1}$

Here, $\mu = \dfrac{\mu_V + \mu_R}{2} = \dfrac{1.522 + 1.514}{2} = 1.518$

$$\Rightarrow \omega = \frac{1.522 - 1.514}{1.518 - 1} \Rightarrow \omega_{crown} = 0.015.$$

SCATTERING OF LIGHT

- As sunlight travels through the earth's atmosphere, it gets scattered (changes its direction) by the atmospheric particles. Light of shorter wavelengths is scattered much more than light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as Rayleigh scattering.

Illustrations of Scattering of Light

- Blue colour of sky
- White colour clouds
- The sun looks reddish at the time of sun rise and sun set
- Danger signals are red.

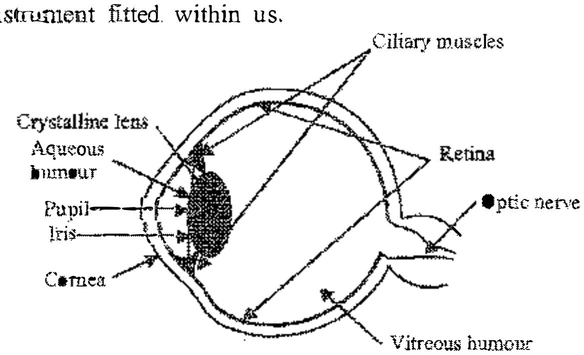
Rainbow

- Rainbow is a beautiful arc of seven colours seen in the sky after rainfall.
- The rainbow is an example of the dispersion of sunlight by the water drops in the atmosphere. This is a phenomenon due to combined effect of dispersion, refraction and reflection of sunlight by spherical water droplets of rain.
- To observe the rainbow, back of observer must be towards the sun.
- Generally, there are two kinds of rainbows
  - Primary rainbow
  - Secondary rainbow
- **Primary rainbow :** Primary rainbow occurs due to one internal reflection and two refractions from the water drops suspended in air. Violet colour is on the inner edge and red colour is on the outer edge.
- **Secondary rainbow :** Secondary rainbow occurs due to two total internal reflections and two refractions from the water drops suspended in air. Red colour is on the inner edge and violet colour is on the outer edge.
  The secondary rainbow is fainter than the primary rainbow.

OPTICAL INSTRUMENTS

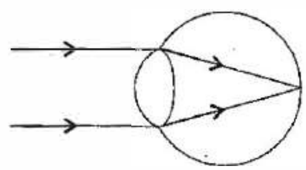
Human Eye

Eye is one of the most important optical biological instrument fitted within us.



- Light enters through the curved front surface called cornea. Then it passes through the pupil which is the central hole in iris. The size of pupil can change under control of muscles. Thus, amount of light or intensity of light entering the eye is controlled by the size of pupil.
- Ciliary muscles control the curvature of the lens in the eye and change the effective focal length of crystalline lens of the eye. When muscles are fully relaxed, focal length is maximum which decreases when muscles are strained.
- Light is focused by the eye lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. It contains cells in the shapes of rods and cones, which sense light intensity and colour respectively, and transmit electric signals via the optic nerve to the brain which finally process this information.
- For image to be clear, it must be formed on retina. Thus, image distance is fixed for clear vision and is equal to the distance of the retina from the eye lens. It is about 2.5 cm for a grown up person.

- Focal length of eye lens is maximum when ciliary muscles are fully relaxed and is equal to the distance between eye lens and retina *i.e.*, 2.5 cm for a grown up person.



So, when $u_{max} = \infty$ and $v = +2.5$ cm

Then $\frac{1}{f_{max}} = \frac{1}{v} - \frac{1}{u_{max}} = \frac{1}{2.5} - \frac{1}{\infty} = \frac{1}{2.5} - 0 = \frac{1}{2.5}$

or $f_{max} = +2.5$ cm

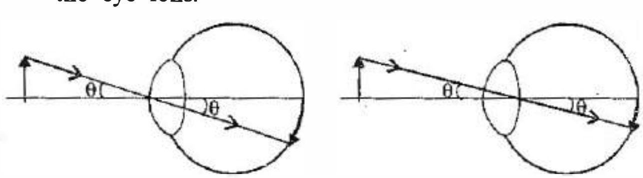
- The closest distance for which the lens can focus light on the retina is called the **least distance of distinct vision D,** or the **near point,** which is 25 cm for normal vision. So, focal length of eye lens is minimum when ciliary muscles are fully strained and object is at near point.

So, when $u_{min} = -25$ cm and $v = +2.5$ cm

Then $\frac{1}{f_{min}} = \frac{1}{v} - \frac{1}{u_{min}} = \frac{1}{2.5} + \frac{1}{25} = \frac{11}{25}$

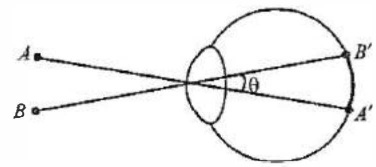
or $f_{min} = \frac{25}{11} = +2.27$ cm

- So, focal length of eye lens can change within the range of 2.27 cm to 2.5 cm by the action of ciliary muscles, in order to maintain the same image distance (2.5 cm). This property of eye is called accommodation. The ability of the eye by virtue of which it can adjust its focal length to see objects at infinity to at a closest distance of 25 cm from it, is called **power of accommodation.**
- **Visual angle** is the angle subtended by the object at the eye lens.



Size of the object, as observed by the eye depends upon the visual angle. When the object is close, visual angle is large and object appears large. When object is far, visual angle is small and same object appears to be smaller. Visual angle is maximum when the object is at the least distance of distinct vision D.

- **Resolving power of eye** is the reciprocal of the smallest angle θ subtended by two close objects, so that they appear separately visible, when observed through the eye. This angle θ is called limit of resolution of eye and is less than 1 minute or 1/60°.



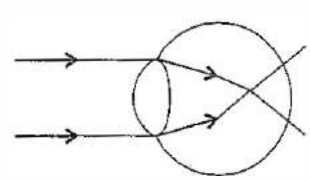
- **Persistence of vision :** If the time interval between two consecutive light pulses is less than 1/16 s, then eye cannot distinguish between them and the light pulses are then observed to be continuous. This is called persistence of vision.
- **Defects of Eye**

Defects of eye are mainly of four types:

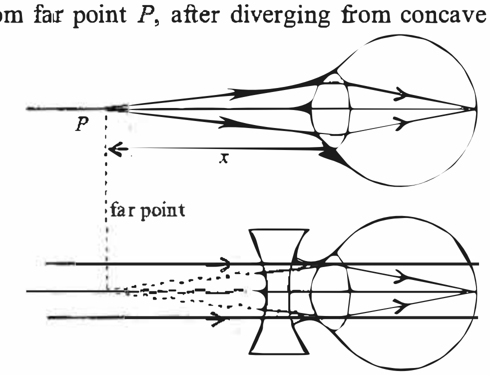
- Myopia
- Hypermetropia
- Presbyopia and
- Astigmatism

(a) **Myopia or Near Sightedness :** In this defect of eye, the eye lens becomes too thick and cannot focus the image of distant objects on the retina, due to which eye is not able to observe distant objects clearly. In this defect, lens converges incident light of distant object to a point well before the retina, and maximum focal length is less than distance between lens and the retina *i.e.*, less than 2.5 cm.

Far point P of a myopic eye is the farthest distance of the object from the eye, of which clear image is focused on retina.



This defect is removed by introducing an appropriate concave lens between the eye and the object. Concave lens with right diverging effect focuses the image of distant object on the retina, and the parallel rays from infinity, appear to be coming from far point P to the eye lens. If the far point P is at distance x from eye lens, then the incident rays from infinity should appear to be coming from far point P, after diverging from concave lens.



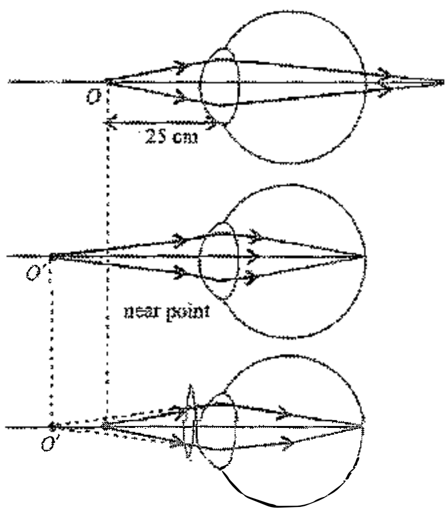
So, for $u = -\infty$, $v = -x$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-x} - \frac{1}{\infty} = -\frac{1}{x} + 0 \quad \text{or} \quad f = -x \text{ m}$$

This gives the focal length and $P = -\frac{1}{x}$ D gives the power of the concave lens required to correct the defect of the given myopic eye.

(b) **Hypermetropia (Far sightedness) :** In this defect the eye lens becomes too thin and cannot focus the image of nearby objects on the retina, due to which eye is not able to observe near by objects clearly. In this defect, eye converges incident light of near by object to a point behind the retina, and minimum focal length is more than distance between the lens and the retina *i.e.*, more than 2.5 cm.

Near point D of a hypermetropic eye is the closest distance of the object from the eye, of which clear image is focused on retina.



This defect is removed by interposing an appropriate convex lens between the eye and the object. Convex lens with the right converging effect focuses the image of nearby object on the retina and the ray from the object at the least distance of distinct vision (25 cm), appear to be coming from near point P to the eye lens.

If the near point is at distance y from the eye lens, then the incident rays from the object at the least distance of distinct vision (25 cm), appear to be coming from near D, after converging from convex lens. So, for $u = -0.25$ m, $v = -y$ m

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = -\frac{1}{y} + \frac{1}{0.25} = -\frac{1}{y} + 4 \quad \text{or} \quad f = \frac{y}{4y-1}\ \text{m}$$

This gives the focal length and $P = \frac{4y-1}{y}$ D gives the power of the convex lens required to correct the defect of the given hypermetropic eye.

(c) **Presbyopia :** It is an old age disease. At old age, ciliary muscles loose their elasticity and cannot change the focal length of eye lens effectively. Due to this eye lens looses its power of accommodation, and person can then suffer from both myopia and hypermetropia.

This is overcome either by using two separate spectacles, one for myopia and another for **hypermetropia** or by using a single spectacle having bifocal lens.

(d) **Astigmatism :** It is the defect of eye which occurs when the cornea is not spherical in shape. For example, if the cornea have a larger curvature in the vertical plane than in the horizontal plane, then on looking at a horizontal line, focusing in the vertical plane is needed for a sharp image. But due to astigmatism, lines in one direction are well focused, while those in perpendicular direction are not. It is corrected by a lens with one cylindrical surface. A cylindrical surface focuses rays in one plane but not in a perpendicular plane. By choosing the radius of curvature and axis direction of the cylindrical surface, astigmatism can be corrected. Astigmatism can occur along with myopia or hypermetropia.

Simple Microscope

- It is also known as magnifying glass or simple magnifier. It consists of a convergent lens with object between its focus and optical centre and eye close to it. The image formed by it is erect, virtual, enlarged and on same side of lens between object and infinity.
- Magnifying power

  $$M = \frac{\text{angle subtended by image at the eye}}{\text{angle subtended by the object at the eye}} = \frac{\tan\beta}{\tan\alpha} = \frac{\beta}{\alpha}$$

  where both the object and image are situated at the least distance of distinct vision.
- When the image is formed at infinity (far point),

  $$M = \frac{D}{f}$$
- When the image is formed at the least distance of distinct vision D (near point),

  $$M = 1 + \frac{D}{f}$$

Compound Microscope

- It consists of two convergent lenses of short focal lengths and apertures arranged co-axially. Lens (of focal length f_o) facing the object is known as objective or field lens while the lens (of focal length f_e) facing the eye, is known as eye-piece or ocular. The objective has a smaller aperture and smaller focal length than eye-piece.
- Magnifying power of a compound microscope

  $M = m_o \times m_e$
- When the final image is formed at infinity (normal adjustment),

  $$M = \frac{v_o}{u_o}\left(\frac{D}{f_e}\right)$$

  Length of tube, $L = v_o + f_e$

- When the final image is formed at least distance of distinct vision,

$$M = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$$

where u_o and v_o represent the distance of object and image from the objective lens, f_e is the focal length of an eye lens.

Length of the tube, $L = \mu_0 + \left(\frac{f_e D}{f_e + D}\right)$

Astronomical Telescope (Refracting type)

- It consists of two converging lenses. The one facing the object is known as objective or field lens and has large focal length and aperture while the other facing the eye is known as eye-piece or ocular has small focal length and aperture.
- When the final image is formed at infinity (normal adjustment),

$M = \frac{f_o}{f_e}$

Length of tube, $L = f_o + f_e$

- When the final image is formed at least distance of distinct vision,

$M = \frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$

Length of tube, $L = f_o + \frac{f_e D}{f_e + D}$

Reflecting Type Telescope

- Reflecting type telescope was designed by Newton in order to overcome the drawbacks of refracting type telescope. In a reflecting type telescope, a concave mirror of large aperture is used as objective in place of a convex lens. It possesses a large light gathering power and a high resolving power. Due to this, it enables us to see even faint stars and observe their minute details.
- In normal adjustment

Magnifying power, $M = \frac{f_o}{f_e} = \frac{\left(\frac{R}{2}\right)}{f_e}$

where R is the radius of curvature of concave mirror.

- Reflecting type telescope is free from chromatic aberration because light does not undergo refraction.
- By using paraboloidal mirror, spherical aberrations can be eliminated in reflecting type telescope.

Illustration 14

A compound microscope has an objective of focal length 1 cm and an eyepiece of focal length 2.5 cm. An object has to be placed at a distance of 1.2 cm away from the objective for normal adjustment (far point adjustment). (a) find the angular magnification (b) find the length of the microscope tube.

Soln.: (a) Using lens equation for the objective,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v_o} - \frac{1}{(-1.2\,\text{cm})} = \frac{1}{(1\text{cm})}$$

$\Rightarrow v_o = 6$ cm.

$$M_{\text{far point}} = -\left(\frac{v_o}{u_o}\right)\left(\frac{D}{f_e}\right) = -\left(\frac{6\,\text{cm}}{1.2\,\text{cm}}\right)\left(\frac{25\text{cm}}{2.5\,\text{cm}}\right) = -50.$$

(b) For far point adjustment, the first image formed by the objective should be formed at the focal length of the eye-piece.

$\Rightarrow L = v_o + f_e = 6$ cm + 2.5 cm = 8.5 cm.

Illustration 15

An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eyepiece is 36 cm and the final image is formed at infinity. What are the focal length of the objective and eyepiece?

Soln.: For astronomical telescope, $|m| = \left(\frac{f_o}{f_e}\right) = 5$

and $L = f_o + f_e = 36$

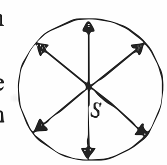
$\Rightarrow 5f_e + f_e = 36$ or $f_e = 6$ cm $\Rightarrow f_o = 30$ cm

WAVEFRONT AND HUYGEN'S PRINCIPLE

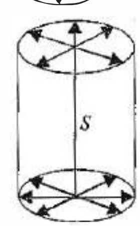
A source of light sends the disturbance in all the directions and continuous locus of all the particles vibrating in same phase at any instant is called as wavefront. Phase speed is the speed with which a wavefront moved outwards from the source.

For example, when we throw a stone in still water, circular ripples are produced. Each circular ripple is a wavefront of water waves generated. Generally the wavefronts have the shape similar to the source.

For example, a point source produces spherical wave fronts, a line source produces cylindrical wave fronts and a parallel beam of light have plane wavefronts. A wavefront always lies normal to the direction of propogation of waves *i.e.*, normal to the rays of light.

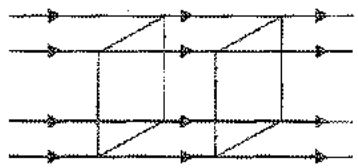


- **Spherical wavefront :** For a point source all such points which are equidistant from point source will lie on a sphere.



- **Cylindrical wavefront :** For a source of light linear in shape, such as finerectangular slit, locus of all such points which are equidistant from linear source will be a cylinder.

- **Plane wavefront :** For a parallel beam of light or a small portion of a spherical or cylindrical wavefront at large distance from source will be a plane wavefront.



- **Ray of light :** Whatever is the shape of a wavefront, the disturbance travels outwards along straight lines emerging from the source, *i.e.*, the energy of a wave travels in a direction perpendicular to the wavefront.

An arrow drawn perpendicular to a wavefront in the direction of propagation of a wave is called a ray.

If we measure the separation between a pair of wavefronts along any ray, it is found to be a constant.
This illustrates two general principles :
1. Rays are perpendicular to wavefronts.
2. The time taken for light to travel from one wavefront to another is the same along any ray.

In case of a plane wavefront, the rays are parallel (figure a)

In case of spherical wavefront, the rays either converge to a point (figure b) or diverge from a point (figure c).

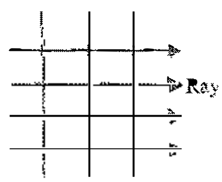


Figure (a)

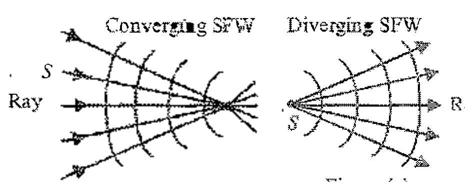


Figure (b) Figure (c)

Huygens' Principle :

Huygens principle is the basis of wave theory of light. It is useful for finding the position and construction of new wavefront of any moment. It also tells how a wavefront propagates through a medium.

It states that "Every point on the given primary wavefront acts as a source of secondary wavelets, sending out disturbance in all directions in a similar manner as the original source of light does. The new position of the wavefront at any instant called secondary wavefront is the envelope of the secondary wavelets at that instant"

Let us discuss the same principle in points.

(i) Each point on a wavefront acts as a fresh source of new disturbance, called secondary waves or wavelets.

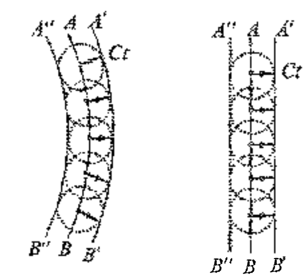
(ii) The secondary wavelets spread out in all directions with the speed of light in the given medium.

(iii) The new wavefront at any later time is given by the forward envelope (tangential surface in the forward direction) of the secondary wavelets at that time.

(iv) The secondary waves or wavelets have same frequency, wavelength as original waves but has reduced intensity.

Huygens' Construction

- In order to geometrically construct the position or a new wavefront after time t, we use Huygen's principle. As every point acts as new source on primary wavefront, AB, so by taking any point on primary wavefront as centre, we draw small spheres of radius ct showing envelope of secondary wavelets in time t. Now a tangent to these sphere $A'B'$ gives the position of new secondary wavefront after time t.



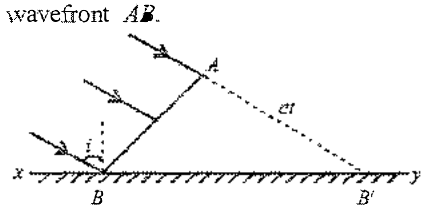
Huygens' geometrical construction for the propagation of (a) spherical, (b) plane wavefront.

- Huygen argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction. Thus, Huygens could explain the absence of back wavefront $A''B''$.
- Voigt and Kirchoff mathematically proved that the contribution of wavelet in a direction making an angle θ with the normal to the wavelet is proportional to $\frac{1}{2}(1 + \cos\theta)$.

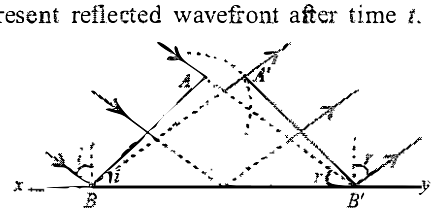
  So, contribution of the wavelet in backward direction at $\theta = 180°$ is zero.

Laws of Reflection by Huygens' Principle

- Let us consider a plane wavefront AB incident on the plane reflecting surface xy. Incident rays are normal to the wavefront AB.



Let in time 't' the secondary wavelets reaches B' covering a distance ct. Similarly from each point on primary wavefront AB, secondary wavelets start growing with the speed 'c'. To find reflected wavefront aftertime 't', let us draw a sphere of radius 'ct' taking 'B' as centre and now a tangent is drawn from B' on the sphere the tangent $B'A'$ represent reflected wavefront after time t.



For every point on wavefront AB, a corresponding point lie on the reflected wavefront $A'B'$.

So, comparing two triangle $\Delta BAB'$ and $\Delta B'A'B$

We find that

$$AB' = A'B = ct$$
$$BB' = \text{common}$$
$$\angle A = \angle A' = 90^\circ$$

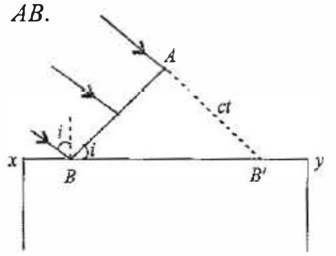
Thus two triangles are congruent, hence $\angle i = \angle r$

This proves first law of reflection.

Also incident rays, reflected rays and normal to them all lie in the same plane. This gives second law of reflection.

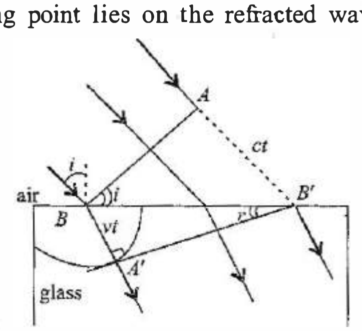
Laws of Refraction by Huygens' Principle

Let us consider a plane wavefront AB incident on the plane refracting surface xy. Incident rays are normal to the wavefront AB.



Let in time t the secondary wavelets from A reaches B' covering a distance ct. Similarly from every point on primary wavefront AB, secondary wavelets start growing which travel with speed c in air and with speed 'v' in denser medium.

To find refracted wavefront after time 't' let us draw a sphere of radius 'vt' in the denser medium, taking B as centre and now a tangent is drawn from B' on the sphere. The tangent $B'A'$ represent refracted wavefront after time 't'. For every point on primary wavefront AB, a corresponding point lies on the refracted wavefront $A'B'$.



In $\Delta ABB'$ and $\Delta A'B'B$

Snell's law can be proved

$$\frac{\sin i}{\sin r} = \frac{ct / BB'}{vt / BB'} = \frac{c}{v} = {}_a\mu_g$$

So, first law of refraction can be proved.

Also, the incident ray, refracted rays and normal to the rays, all lie in the same plane. This gives the second law of refraction. When a wave passes from one medium to another then change in speed v take place, wavelength λ also changes, whereas its frequency υ remains the same.

INTERFERENCE OF LIGHT

- It is the phenomenon of redistribution of energy on account of superposition of light waves from two coherent sources. Interference pattern produce points of maximum and minimum intensity. Points where resultant intensity is maximum, interference is said to be constructive and at the points of destructive interference, resultant intensity is minimum.

Conditions for sustained interference of light

- The two sources should continuously emit waves of the same wavelength or frequency.
- The amplitudes of waves from two sources should preferably be equal.
- The waves emitted by the two sources should either be in phase or should have a constant phase difference.
- The two sources must lie very close to each other.
- The two sources should be very narrow.

Intensity distribution

- If a, b are the amplitudes of interfering waves due to two coherent sources and ϕ is constant phase difference between the two waves at any point P, then the resultant amplitude at P will be

$$R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

If $a^2 = I_1$; $b^2 = I_2$, then

Resultant intensity $I = R^2 = a^2 + b^2 + 2\,ab\cos\phi$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

If $I_1 = I_2 = I_0$, then

$$I = I_0 + I_0 + 2I_0\cos\phi = 4I_0\cos^2\frac{\phi}{2}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

When $\cos\phi = 1$; $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

When $\cos\phi = -1$, $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

If $I_1 = I_2 = I_0$, then

$I_{max} = 4I_0$; $I_{min} = 0$.

$$I = 4I_0\cos^2\frac{\phi}{2}$$

If the sources are incoherent, $I = I_1 + I_2$

Young's Double slit Experiment

- Young's double slit experiment was the first to demonstrate the phenomenon of interference of light. Using two slits illuminated by monochromatic light source, he obtained bright and dark bands of equal width placed alternately. These were called interference fringes.
- For constructive interference (i.e. formation of bright fringes)

  For n^{th} bright fringe,

  Path difference $= x_n \dfrac{d}{D} = n\lambda$

  where $n = 0$ for central bright fringe

  $n = 1$ for first bright fringe,