$n = 2$ for second bright fringe and so on
d = distance between two slits
D = distance of slits from the screen
x_n = distance of n^{th} bright fringe from the centre.

$$\therefore\ x_n = n\lambda\frac{D}{d}$$

- For destructive interference (i.e. formation of dark fringes).
  For n^{th} dark fringe,
  Path difference $= x_n\dfrac{d}{D} = (2n-1)\dfrac{\lambda}{2}$
  where
  $n = 1$ for first dark fringe,
  $n = 2$ for 2^{nd} dark fringe and so on.
  x_n = distance of n^{th} dark fringe from the centre
  $$\therefore\ x_n = (2n-1)\frac{\lambda}{2}\frac{D}{d}$$
- **Fringe width :** The distance between any two consecutive bright or dark fringes is known as fringe width.
  Fringe width, $\beta = \dfrac{\lambda D}{d}$
- Angular fringe width, $\theta = \dfrac{\beta}{D} = \dfrac{\lambda}{d}$
- If W_1, W_2 are widths of two slits, I_1, I_2 are intensities of light coming from two slits; a, b are the amplitudes of light from these slits, then
  $$\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2}$$
  $$\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2}.$$
  Fringe visibility, $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$
- When entire apparatus of Young's double slit experiment is immersed in a medium of refractive index μ, then fringe width becomes
  $$\beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}$$
- When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the interfering waves, fringe width remains unaffected but the entire pattern shifts by
  $$\Delta x = (\mu - 1)\,t\,\frac{D}{d} = (\mu - 1)\,t\,\frac{\beta}{\lambda}$$
  This shifting is towards the side in which transparent plate is introduced.

Coherent sources

- The sources of light, which emit continuous light waves of the same wavelength, same frequency and in same phase or having a constant phase difference are known as coherent sources.
- Two independent sources of light cannot be coherent.

Illustration 16

The interference pattern of two identical slits separated by a distance d = 0.25 mm is observed on a screen at a distance of 1 m from the plane of the slits. The slits are illuminated by monochromatic light of wavelength 589.3 nm (sodium D) travelling perpendicular to the plane of the slits. Bright bands are observed on each side of the central maxima. Calculate the separation between adjacent bright bands?

Soln.: Fringe width $(\beta) = \dfrac{D\lambda}{d} = \dfrac{(1\,\text{m})(589.3\times10^{-9}\,\text{m})}{(0.25\times10^{-3})\,\text{m}}$
$= 2 \times 10^{-3}$ m = 2 mm.

Illustration 17

In a *YDSE*, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths λ = 750 nm and λ' = 900 nm. At what distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?

Soln.: The n^{th} bright fringe of the λ pattern and the n'^{th} bright fringe of the λ' pattern are situated at

$$y_n = n\cdot\frac{D\lambda}{d} \text{ and } y_n' = n'\frac{D\lambda'}{d}.$$

As this coincide, $y_n = y_n'$

$$\Rightarrow \frac{nD\lambda}{d} = \frac{n'D\lambda'}{d}$$

$$\Rightarrow \frac{n}{n'} = \frac{\lambda'}{\lambda} = \frac{900}{750} = \frac{6}{5}$$

hence the first position where overlapping occur is

$$y_5' = y_6 = \frac{nD\lambda}{d} = \frac{6(2\text{m})(750\times10^{-9}\text{m})}{(2\times10^{-3}\text{m})} = 4.5 \text{ mm.}$$

Illustration 18

The intensity of the light coming from one of the slits in a *YDSE* is double the intensity from the other slit. Find the ratio of maximum intensity to minimum intensity in the interference fringe pattern observed.

Soln.: $\dfrac{I_{max}}{I_{min}} = \dfrac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$

Now, $I_1 = 2I_2 \Rightarrow \dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{2I_2}+\sqrt{I_2}}{\sqrt{2I_2}-\sqrt{I_2}}\right)^2 = \left(\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = 34.$

DIFFRACTION OF LIGHT

- It is the phenomenon of bending of light around corners of an obstacle or aperture in the path of light. On account of this bending, light penetrates into the geometrical shadow of the obstacle. The deviation is more effective when the dimensions of the obstacle or aperture are comparable to the wavelength of light. This is, therefore, the essential condition for diffraction of light.

- Diffraction of light is of two types :
  Fresnel diffraction
  Fraunhofer diffraction
- **Fresnel diffraction :** In this case, the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction.
- **Fraunhofer diffraction :** In this case, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction.

Diffraction due to a Single Slit

- The diffraction pattern produced by a single slit of width a consists of a central maximum bright band with alternating bright and dark bands of decreasing intensity on both sides of the central maximum.
- Condition for n^{th} secondary maximum is

  Path difference $= a\sin\theta_n = (2n+1)\dfrac{\lambda}{2}$

  where $n = 1, 2, 3, \ldots\ldots$
- Condition for n^{th} secondary minimum is

  Path difference $= a\sin\theta_n = n\lambda$

  where $n = 1, 2, 3, \ldots\ldots$
- Width of secondary maxima or minima

  where $\beta = \dfrac{\lambda D}{a} = \dfrac{\lambda f}{a}$

  a = width of slit

  D = distance of screen from the slit

  f = focal length of lens for diffracted light

  Width of central maximum $= \dfrac{2\lambda D}{a} = \dfrac{2f\lambda}{a}$
- Angular fringe width of central maximum $= \dfrac{2\lambda}{a}$.
- Angular fringe width of secondary maxima or minima $= \dfrac{\lambda}{a}$

Illustration 19

Light of wavelength 580 nm is incident on a slit of width 0.30 mm. The observing screen is placed 2.0 m from the slit. Find the positions of the first dark fringes and the width of the central bright fringe.

Soln.: For first dark fringe,

$$\sin\theta = \pm\frac{\lambda}{a} = \pm\frac{5.8\times10^{-7}\,\text{m}}{0.3\times10^{-3}\,\text{m}} = \pm 1.9\times10^{-3}$$

The positions of the first minima, measured from the central axis are

$y_1 = \pm D\tan\theta \approx \pm D\sin\theta$

As θ is small, $\tan\theta = \sin\theta$

$$\Rightarrow y_1 = \pm D\left(\frac{\lambda}{a}\right) = \pm 2\,(1.9\times10^{-3}\,\text{m}) = \pm 3.8\times10^{-3}\,\text{m}$$

The positive and negative signs correspond to the first dark fringes on either side of the central bright fringe. Hence, the width of the central bright fringe is given by $2\,|y_1| = 7.6\times10^{-3}$ m $= 7.6$ mm.

Note that this value is much greater than the width of the slit (0.3 mm)

Illustration 20

In a single slit diffraction experiment, first minimum for red light (660 nm) coincides with the first maximum of another wavelength λ'. Then find the value of λ'.

Soln.: For minima, $d\sin\theta = n\lambda$

Here $n = 1$, $d\cdot\left(\dfrac{y_1}{D}\right) = 1\cdot(6600\text{ Å})$

$$y_1 = \frac{D}{d}\cdot(6600\text{Å})$$

Now, first maximum is approximately between the first minima and second minima.

$$y_I = \left(\frac{y_1+y_2}{2}\right) = \left(\frac{1+2}{2}\right)\cdot\frac{D\lambda'}{d}$$

$$\text{As } y_1 = y_I \Rightarrow \frac{D}{d}(6600\text{Å}) = \left(\frac{3}{2}\right)\frac{D}{d}\lambda'$$

$$\lambda' = \frac{2\times6600\text{ Å}}{3} = 4400\text{ Å}$$

Fresnel distance

- It is the minimum distance a beam of light has to travel before its deviation from straight line path becomes significant.

  Fresnel distance, $Z_F = \dfrac{a^2}{\lambda}$

Resolving power

- It is the ability of an optical instrument to produce distinctly separate images of two close objects *i.e.* it is the ability of the instrument to resolve or to see as separate, the images of two close objects.

Limit of resolution

- The minimum distance between two objects which can just be seen as separated by the optical instrument is known as the limit of resolution of the instrument. Smaller the limit of resolution of the optical instrument, greater is its resolving power and vice-versa.

Rayleigh's criterion of limiting resolution

- According to Rayleigh, two nearby images are said to be resolved if the position of the central maximum of one coincides with the first secondary minimum of the other and vice versa.

Resolving power of a microscope

- It is defined as the reciprocal of the minimum distance d between two point objects, which can just be seen through the microscope as separate.

$$\text{Resolving power} = \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}$$

where μ is refractive index of the medium between object and objective lens, θ is half the angle of cone of light from the point object, d represents limit of resolution of microscope and $\mu\sin\theta$ is known as the numerical aperture.

Resolving power of a telescope

- It is defined as reciprocal of the smallest angular separation ($d\theta$) between two distant objects, whose images are just seen in the telescope as separate.

$$\text{Resolving power} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

where D is diameter or aperture of the objective lens of the telescope, $d\theta$ represents limit of resolation of telescope.

POLARIZATION OF LIGHT

- The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to direction of wave motion is known as polarization of light.
- The plane in which vibrations of polarized light are confined is known as plane of vibration.
- A plane which is perpendicular to the plane of vibration is known as plane of polarization.

Angle of Polarization

- The angle of incidence for which an ordinary light is completely polarized in the plane of incidence when it gets reflected from a transparent medium.

Laws of Malus

- According to law of Malus, when a beam of completely plane polarized light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of the angle (θ) between plane of transmission of analyser and polarizer

i.e. $I \propto \cos^2\theta$

Brewster's law

- According to Brewster's law, when unpolarized light is incident at polarizing angle (i_p) on an interface separating a rarer medium from a denser medium of refractive index μ, such that

$\mu = \tan i_p$

then light is reflected in the rarer medium is completely polarized. The reflected and refracted rays are perpendicular to each other.